Non-commutative Algebra, WS 19/20

Lectures: W. Crawley-Boevey
Exercises: A. Hubery

Exercises 6

Let K be a field and let Q be the quiver

$$2 \xrightarrow{b} 1 \circlearrowleft a$$

We give this quiver a grading by setting $\deg(a) = 1$ and $\deg(b) = 0$.

1. Let A be the quotient $KQ/(a^2, ab)$.
   (a) Compute the algebra $\tilde{A}$, living in degrees $[-2, 0]$.
   (b) Compute the Auslander-Reiten quiver for $\tilde{A}$.
   (c) Hence compute the Auslander-Reiten quiver for A.

2. Let B be the quotient $KQ/(a^3, ab)$.
   (a) Compute the algebra $\tilde{B}$, living in degrees $[-3, 0]$.
   (b) Compute the Auslander-Reiten quiver for $\tilde{B}$.
   (c) Hence compute the Auslander-Reiten quiver for B.

3. Let C be the quotient $KQ/(a^2)$.
   (a) Compute the algebra $\tilde{C}$, living in degrees $[-3, 0]$.
   (b) Compute the Auslander-Reiten quiver for $\tilde{C}$.
   (c) Hence compute the Auslander-Reiten quiver for C.
   (d) There are two non-isomorphic indecomposable C-modules having the same dimension vector. Give the matrices for the corresponding representations.

4. Compute the Auslander-Reiten quiver of the algebra D given as

$$\text{quiver} \quad \begin{tikzcd} 4 \arrow[r, "s"] \arrow[d, "q"] & 3 \arrow[d, "r"] \\ 2 \arrow[r, "p"] & 1 \end{tikzcd} \qquad \text{relations} \quad pq.$$

To be handed in by 2nd December.