REPRESENTATION THEORY
EXERCISES 5

HENNING KRAUSE
JAN GEUENICH

1. Let $\mathcal{A}$ be an exact category. Prove that the composite

$$\mathcal{A} \to \mathbf{C}(\mathcal{A}) \to \mathbf{K}(\mathcal{A}) \to \mathbf{D}(\mathcal{A})$$

of canonical functors is fully faithful.

2. Verify the following facts about morphisms $f\colon X \to Y$ in triangulated categories:

(a) f admits a kernel iff f admits a cokernel iff f has the form

$$X' \oplus K \xrightarrow{\left(\begin{smallmatrix} f' & 0 \\ 0 & 0 \end{smallmatrix}\right)} Y' \oplus C$$

for some isomorphism f'.

(b) f admits a kernel and a cokernel if f is a monomorphism or epimorphism.

(c) f is an isomorphism iff the cone of f is zero iff f is a monomorphism and an epimorphism.

3. Let $\mathcal{A} = \operatorname{Mod}\Lambda$ be the module category of a ring Λ and let X be a complex in $\mathcal{A}$. Viewing Λ as a complex in $\mathcal{A}$ concentrated in degree zero, show that there exists a canonical isomorphism:

$$\operatorname{Hom}_{\mathbf{D}(\mathcal{A})}(\Lambda, X) \;\cong\; H^0 X$$

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