Parabolic Group Actions with a Dense Orbit and Fans

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Parabolic groups and their linear actions occure in many branches of mathematics. Whereas for reductive groups many problems are well understood there is not much known for parabolic groups. One standard example is the action of a parabolic group on the Lie algebra of its unipotent radical and the derived Lie algebras. It turns out that those problems are equivalent to a classification problem for modules over certain quasi-hereditary algebras. The existence of a dense orbit is then equivalent to the existence of a module without selfextension, a purely homological property. To such a module one can associate a cone, and the set of all those cones form a fan. We explain the connection of the dense orbit problem for parabolic groups with the structure of the fan in detail.