Cohomological Invariant Theory (for Algebraic Groups)

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The first fundamental theorem of invariant theory says that if a reductive group acts algebraically on a finitely generated commutative k-algebra, then the ring of invariants $H^0(G, A)$ is also a finitely generated k-algebra. Here k is the ground field. Our conjecture says that more generally for any geometrically reductive group scheme G acting on A, the full cohomology ring $H^*(G, A)$ is finitely generated. This would generalize theorems of Evens 1961 and of Friedlander-Suslin 1997. So far we can only handle SL2, (and in characteristic two SL3) but there is more evidence than that. We run into problems of homological algebra of the category of strict polynomial bifunctors.