J-invariant of linear algebraic groups

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Let G be a linear algebraic group over a field F and X be a projective homogeneous G-variety such that G splits over the function field of X. In the present paper we introduce an invariant of G called J-invariant which characterizes the splitting properties of the Chow motive of X. This generalizes the respective notion invented by A. Vishik in the context of quadratic forms. As a main application we obtain a uniform proof of all known motivic ecompositions of generically split projective homogeneous varieties (Severi-Brauer varieties, Pfister quadrics, maximal orthogonal Grassmannians, G_2- and F_4-varieties) as well as provide new ones (exceptional varieties of types E_6, E_7 and E_8). We also discuss applications to canonical dimensions and splitting properties of the group G.