Calculations in exceptional groups, revisited

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This is a sequel of (I think) 3 talks on the subject I gave at the seminar in 1992-2003. It goes about the use of minimal modules for Chevalley groups of types F_4, E_6, E_7 and E_8 over a commutative ring R to prove results in structure theory and K-theory. I call such proofs, based on the reduction of rank, geometric, as opposed to arithmetic proofs, based on the reduction of dim(R).

The first generation geometric proofs (Stepanov, Plotkin and myself) invoked the presence of VERY large classical subgroups (such as A_5 or D_5 in E_6, A_7 or D_6 in E_7 and, finally, D_8 in E_8), used quadratic equations defining the orbit of the highest weight vector, and depended on a rather delicate equilibrium of signs.

In 2003 I reported on the second generation proofs (joint with my Gavrilovich), which worked for E_6 and E_7, only depended on the presence of A_2 for reduction to a maximal parabolic, and only used some LINEAR equations, coming from the Lie algebra.

After recalling briefly the basics about the geometry of minimal modules, I would like too tell about recent progress, a similar (but much tougher!) proof for F_4 (joint with Nikolenko) and an A_3-proof, which allows in the cases of E_6 and E_7 to reduce to a SUBMAXIMAL parabolic. This is exactly what is needed to START producing van der Kallen's "another presentations" for Steinberg groups modeled on those groups, but it's a long way ahead to prove all lemmas necessary to establish correctness and compatibility.