GoTh Workshop

Groups of Thompson and their relatives

The workshop takes place in **Building 03, room 106**, and all talks will be streamed at Zoom room **613 2292 0012**. For the password, fill in the blank: This is a workshop on _ _ _ _ _ _ _ _ groups (first letter uppercase). Times in Central European Summer Time (CEST).

Monday, Sept. 18

Sean Cleary (9:30 – 10:30)

Groups in the Thompson family, trees, and rotations

The original Thompson's groups of F, T, and V, introduced in the 1960s, have grown into a large family of generalizations. There are many ways of describing the original groups, and tree descriptions work effectively for a range of goals there. There are many directions for generalizing these groups, some of which have natural tree-based descriptions and associated normal forms connected to those. For the purposes of understanding the metrics on these groups, sometimes the tree-based descriptions work effectively and sometimes there are limitations on how strong these connections are. Rotations of trees are small changes to trees from which more complicated transformations can be constructed and the number of rotations required to transform one tree to another is related to metric questions in groups of the Thompson family.

Tatiana Nagnibeda (11:00 – 12:00)

On maximal subgroups of infinite index in finitely generated groups

Margulis and Soifer proved that in a finitely generated linear group all maximal subgroups are of finite index if and only if the group is solvable; otherwise there exist uncountably many maximal subgroups of infinite index, all of them isomorphic to a free group of infinite rank. In the talk we will discuss maximal and weakly maximal subgroups in some non-linear finitely generated groups.

Collin Bleak (14:00 – 15:00)

3/2's generation for Thompson-esque groups

We borrow from the theory of finite simple groups, and ask: Does every non-trivial element f of a simple group G admit a co-generator? That is, an element g s.t. $G = \langle f, g \rangle$? In the case of R. Thompson's groups T and V, the answer is "Yes". Indeed, for $G \in \{T, V\}$ we have the stronger result that there is an element z so that for any non-trivial element $f \in G$ we can find an element $c \in G$ with $G = \langle f, z^c \rangle$. We will also discuss some developing work generalising these results to the wide class of finitely generated vigorous simple groups.

Joint with Casey Donovan, Scott Harper, James Hyde, and Rachel Skipper.

Jennifer Taback (15:00 – 15:30, remote speaker)

Conjugation curvature in solvable Baumslag–Solitar groups

Bar Natan, Duchin and Kropholler introduced conjugation curvature as a discrete Ricci curvature for Cayley graphs of finitely generated groups. In joint work with Alden Walker, we show that the solvable Baumslag–Solitar groups $\mathtt{BS}(1, n)$ have sets of elements of positive, negative and zero conjugation curvature, and that these sets have positive density in the group. To prove this, we use a lattice-based approach to produce a geodesic representative for every element of the group, from which we derive a word-length formula, all with respect to the standard generating set for $\mathtt{BS}(1, n)$. A subset of these geodesic representatives forms a regular language, and in subsequent work we use this language to give a new proof of the growth rate of $\mathtt{BS}(1, n)$.

Gili Golan (16:00 – 16:30, remote speaker)

On maximal subgroups of Thompson's group F

We study subgroups of Thompson's group F by means of a 2-automaton associated with them. We prove that every maximal subgroup of F of infinite index is closed, that is, it coincides with the subgroup of F accepted by the 2-automaton associated with it. It follows that every finitely generated maximal subgroup of F is undistorted in F. We also prove that F has infinitely many non-isomorphic maximal subgroups of infinite index.

Tuesday, Sept. 19

Nancy Guelman (9:30 – 10:30)
Uniform simplicity for subgroups of piecewise continuous bijections of the unit interval

A group G is uniformly simple if there exists a positive integer N such that for any $f, g \in \backslash\{\mathrm{id}\}$, the element g can be written as a product of at most N conjugates of f or f^{-1}.

We provide conditions which guarantee that a subgroup G of the group of piecewise continuous bijections of $I = [0, 1)$ is uniformly simple.

Ross Geoghegan (11:00 – 12:00)
Is the group F really infinite-dimensional?

Thompson's group F is infinite-dimensional. But I'd call that "weakly infinite-dimensional". So what is the meaning of "strongly infinite-dimensional"? This is relevant to all Thompson-like groups, but it is particularly motivated by the search for an invariant that would distinguish F from $F \times \mathbb{Z}$ up to quasi-isometry. A topological discovery in the 1970's, not usually known to group theorists, suggests where to seek the answer. I'll explain.

Francesco Matucci (14:00 – 15:00)
Finite Germ Extensions

We introduce a family of groups of homeomorphisms of a topological space obtained from a group of piecewise linear homeomorphisms by adding finitely many singularities and we prove results about their simplicity, abelianizations and finiteness properties.

This family arose naturally in the process of solving a Kourovka notebook question by Bridson and De la Harpe asking whether there exists a finitely presented group containing the additive group $\mathbb{Q}$ of rational numbers.

Among the examples we construct, we describe two groups $T\mathcal{A}$ and $V\mathcal{A}$ that are simple, two-generated, finitely presented and contain, respectively, all countable torsion-free abelian groups and all countable abelian groups, explicitly realizing the Boone–Higman embedding theorem. Moreover, we show that they have type F_∞.

We also discuss how our results can be applied to other related groups, such as some Nekrashevych groups, a class of groups which are generated by Thompson groups $V_{n,r}$ and suitable self-similar groups.

This is joint work with Jim Belk and James Hyde.

Fabienne Chouraqui (15:00 – 15:30)
Connections between the Yang-Baxter equation and Thompson's group F

The quantum Yang-Baxter equation is an equation in mathematical physics and it lies in the foundation of the theory of quantum groups. One of the fundamental problems is to find all the solutions of this equation. Drinfeld suggested the study of a particular class of solutions, derived from the so-called set-theoretic solutions. A set-theoretic solution of the Yang-Baxter equation is a pair (X, r), where X is a set and

$$r : X \times X \to X \times X\,, \quad r(x, y) = (\sigma_x(y), \gamma_y(x))$$

is a bijective map satisfying $r^{12}r^{23}r^{12} = r^{23}r^{12}r^{23}$, where $r^{12} = r \times Id_X$ and $r^{23} = Id_X \times r$. We define non-degenerate involutive partial solutions as a generalisation of non-degenerate involutive set-theoretical solutions of the quantum Yang-Baxter equation (QYBE). The induced operator is not a classical solution of the QYBE, but a braiding operator as in conformal field theory. We show that there is a connection between partial solutions and the Thompson's group F. This raises the question of whether there are further connections between partial solutions and Thompson's groups in general.

Rachel Skipper (16:00 – 16:30, remote speaker)
Maximal Subgroups of Thompson's group V

Maximal subgroups of a group provide a range of information about the group. First, maximal subgroups correspond to primitive actions of a group. Secondly, in a finitely generated group every proper subgroup is contained in a maximal one. In this talk, we will discuss some ongoing work with Jim Belk, Collin Bleak, and Martyn Quick to understand and classify maximal subgroups of Thompson's group V.

Wednesday, Sept. 20

Jim Belk (9:30 – 10:30)
Boone–Higman for hyperbolic groups

The 1973 Boone–Higman conjecture asserts that a finitely generated group has solvable word problem if and only if it embeds into a finitely presented simple group. In joint work with Collin Bleak, Francesco Matucci, and Matthew Zaremsky, we prove that this conjecture holds for hyperbolic groups, i.e., every hyperbolic group embeds into a finitely presented simple group. The construction involves a generalization of Röver–Nekrashevych groups as well as twisted Brin–Thompson groups.

Conchita Martínez-Perez (11:00 – 12:00)
Subgroups of some groups of piecewise linear homeomorphisms of $\mathbb{R}$

There are many well-known obstructions for a group to be isomorphic to a subgroup of Thompson group F. In this talk we will consider some more general subgroups of piecewise linear homeomorphisms of $\mathbb{R}$ that contain F, such as the index two subgroup of $\mathrm{Aut}(F)$ consisting of orientation preserving homeomorphisms and see which of these obstructions remain and which do not. This is a joint work (in pregress) with Susan Hermiller.

Valeriano Aiello (12:00 – 12:30)
F, Jones' representations, maximal subgroups, knots, and all that.

In 2014 Vaughan Jones introduced several unitary representations of the Thompson groups and a method to construct knots from their elements. Thanks to these representations, interesting subgroups cropped up and gave rise to infinite index maximal subgroups. As for the knot construction, this prompted the beginning of a new theory analogous to that of braids and knots, but with the Thompson groups replacing the braid groups. I will report on some work on these projects.

Thursday, Sept. 21

Yash Lodha (9:30 – 10:00, remote speaker)
Finitely presented simple left orderable groups, and docile monsters

I will describe two recent constructions of groups of homeomorphisms of the real line:

1. Finitely presented simple left orderable groups (joint work with Hyde).
2. Finitely presented left orderable groups with the property that all their actions on $\mathbb{R}$ are globally contracting (joint work with Fournier-Facio and Zaremsky).

James Hyde (10:00 – 10:30)
Finitely presented simple groups of homeomorphisms of the reals

I will give an introduction to Thompson's groups F and T and then build and discuss three families of examples of finitely generated simple groups of homeomorphisms of the real line and one family that is also finitely presented. This represents joint work with Yash Lodha and Cristobal Rivas.

Robert Bieri (11:00 — 12:00)
Groups of rearrangements of tessellations (joint work with Heike Sach)

The process of cutting a regular tessellation along tile edges into a finite number of nice pieces, and rearranging them to a new neat covering of the old positions, defines a permutation of the tile-centers. The topic here is the structure and finiteness properties of the permutation groups we find, based on the standard tessellation of the Euclidean n-space by unit cubes.

Particularly happy made us the observation that when we applied this to the regular tessellation of the hyperbolic plane by ideal triangles, we recovered Thompson's groups! This shows that Thompson's groups are the hyperbolic cousins of our generalized Houghton groups and might open a door for more.

Louis Funar (14:00 – 15:00)
Thompson groups and smooth mapping class groups

We show that Thompson groups arise as groups of homeomorphisms of Cantors subsets of manifolds which extend to ambient diffeomorphisms. This strengthtens the relationship between Thompson groups, their braidings and mapping class groups of infinite type surfaces.

Olga Varghese (15:00 – 15:30)
Profinite flexibility and rigidity of right-angled Coxeter groups

We show that every right-angled Coxeter group (RACG) is profinitely rigid amongst all Coxeter groups. On the other hand we exhibit RACGs which have infinite profinite genus amongst all finitely generated residually finite groups. Along the way we show that the Higman–Thompson groups V_{2n} are generated by 4 involutions generalising a classical result of Higman for Thompson's group V.

Alina Vdovina (16:30 – 17:30)
Buildings, C^*-algebras and new higher-dimensional non-residually finite and Thompson-like groups

We present explicit constructions of new infinite families of CW-complexes of arbitrary dimension with buildings as the universal covers. These complexes give rise to new families of C^*-algebras, classifiable by their K-theory. The underlying building structure allows explicit computation of the K-theory.

We will also present the first infinite series of quaternionic groups of finite characteristic and non-residually finite groups acting on cubes complexes and new higher-dimensional generalizations of the Thompson groups. Such types of groups are usually difficult to distinguish, but the K-theory of C^*-algebras gives new complete invariants.

We plan to finish with a long list of open problems which can be tackled by our methods.

Friday, Sept. 22

Dessislava Kochloukova (9:30 – 10:30)
Higher dimensional algebraic fiberings of group extensions

In a joint work with S. Vidussi we prove some conditions for the existence of higher dimensional algebraic fibering of group extensions. This leads to various corollaries on incoherence of groups and some examples of algebraic fibers of type FP_n but not FP_{n+1}. One of the main tools used is the Bieri–Strebel–Neumann–Renz invariants, including higher dimensional ones. If the time permits we will discuss the pro-p case of the same type of problems.

Dilshan Wijesena (11:00 – 11:30)
Classifying Pythagorean representations of Thompson's groups

The analytical properties of Richard Thompson's groups F, T and V have been challenging experts for many decades. One reason for this is because very little is known about its representation theory. Luckily, thanks to the novel technology of Vaughan Jones, a rich family of so-called Pythagorean unitary representation of Thompson's groups can be constructed by simply specifying a pair of finite-dimensional operators satisfying a certain equality. These representations can even be extended to the celebrated Cuntz algebra and carry a powerful diagrammatic calculus which we use to develop techniques to study their properties. This permits to reduce very difficult questions concerning irreducibility and equivalence of infinite-dimensional representations into problems in finite-dimensional linear algebra. This provides a new rich class of irreducible representations of F. Moreover, we introduce the Pythagorean dimension which is an invariant for Pythagorean representations. For each dimension d, we show the irreducible classes form a moduli space of a real manifold of dimension $2d^2 + 1$. Finally, we introduce a new kind of tensor product for which Pythagorean representations of F, T, V and representations of the Cuntz algebra are closed under.

Se-Jin Kim (11:30 – 1:00)
C^*-simple groups and partial dynamical systems

A group is said to be C^*-simple if the reduced group C^*-algebra, that is, the operator norm closure of the left regular representation, is simple as an algebra. C^*-simple groups tend to be

very far away from being amenable. For example, the free groups, Tarski's monster groups, and Thompson's group V are known to be C^*-simple. Furthermore, a result of Le Boudec–Matte Bon shows that Thompson's group F is non-amenable if and only if F is C^*-simple if and only if T is C^*-simple. Our goal in this talk is to describe dynamically how C^*-simplicity arises in a group and propose a basic question about how C^*-simple groups interact with partial dynamical systems.

Shayo Olukoya (14:00 – 14:30)
Automorphisms of the Higman – Thompson groups

We begin with the seminal article of M. Brin "The chameleon groups of Richard Thompson: Automorphisms and dynamics" which characterises the automorphisms of Thompson's group F and T. The follow-on article of Brin and Guzman "Automorphisms of generalized Thompson Groups", extends the techniques of Brin to study automorphisms of generalisations $F_n, F_{n,\infty}, T_{n,n-1} \dots$ of F and T. Both of these articles build upon ideas in the article "On groups of PL-homeomorphisms of the real line" of R. Bieri and R. Strebel. We summarise the techniques and ingredients for characterising automorphisms of F and T and their generalisations and indicate where things break down for the Higman-Thompson groups $G_{n,r}$ (generalisations of Thompson's goup V).

We then discuss recent breakthroughs on automorphsims of the $G_{n,r}$'s. We provide all necessary definitions, and carefully introduce the key idea of "synchronizing rational homeomorphisms" which provide a characterisation of automorphisms of $G_{n,r}$. We compare the techniques and tools here with those of Brin and Guzman and highlight some emergent themes.

José Burillo (14:30 –15:30)
Grid diagrams for higher-dimensional Thompson's groups

Elements in higher-dimensional Thompson's groups can be represented by tree-pair diagrams the usual way, but considering two types of carets corresponding to subdivisions in the different directions. This leads to having new relations, and also to some elements having more than one reduced diagram representing them. This is common when trees can have two types of carets. In this talk we will introduce grid diagrams for the elements of kV, show that every element has a unique reduced grid diagram representing it, and prove some metric inequalities for the distance in these higher-dimensional groups based on this unique diagram. This is joint work with Brita Nucinkis.