Geometry, Spectra and Topology:
An aperiodic afternoon

Donnerstag, 18. Juli 2019, Hörsaal 6

Natalie Priebe Frank, Vassar College, USA   **14:15 – 15:15 Uhr**

APPROACHES TO THE STUDY OF APERIODIC ORDER IN SEQUENCES AND TILINGS

Aperiodically ordered structures are of considerable interest as models of quasicrystals, which have been known to exist in the physical world since 1982. In this talk, we describe how to construct sequences and tilings with aperiodic order using self-similar methods. We then discuss how (and why) it makes sense to embed them in a topological space (the "hull") and considering the action of translation.
The dynamical system that results from this process can be analyzed in many ways, including measure-theoretically and topologically. Specific results from these two branches will be discussed in the following talks, but if time permits we will describe a promising new renormalization approach.

Christoph Richard, FAU, Erlangen   **15:30 – 16:30 Uhr**

MODULATED CRYSTALS AND ALMOST PERIODIC MEASURES

Modulated crystals and quasicrystals are both modulated quasicrystals, a class of point sets introduced by de Bruijn in 1987. With appropriate weight functions and modulation functions, modulated quasicrystals themselves constitute a substantial subclass of strongly almost periodic point measures. We re-analyse these structures using methods from modern mathematical diffraction theory, thereby providing a coherent view over that class. Similarly to de Bruijn's analysis, we find stability with respect to almost periodic modulations. This is joint work together with Jeong-Yup Lee (Soul), Daniel Lenz (Jena), Bernd Sing (Barbados) and Nicolae Strungaru (Edmonton).

16:30 – 17:00 Uhr   Coffee break

Lorenzo Sadun, University of Texas, USA   **17:00 – 18:00 Uhr**

TILINGS, SAMPLING, AND MASS TRANSPORT

Aperiodic tilings provide a compelling background for studying problems in sampling and mass transport. If $f(x)$ is a function that is defined locally by a tiling T, at what rate do sample averages of f converge to the ergodic average? If $f(x)$ and $g(x)$ are two locally defined mass distributions with the same density, when it is possible to do a bounded transport from one to the other? When can this transport be done in a way that is locally determined by the structure of the tiling, or at least can be approximated by such a locally determined process? These problems are closely related. I will explain how the top-dimensional Čech cohomology of the tiling space defined by T controls both Birkhoff sums and mass transport. I will present some recent results about this cohomology group and close with some open problems. This is joint work with Michael Kelly.