Optimization and Dynamics

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Exercise Sheet 1

Deadline: Thursday 02.05.2019, 15 o'clock / 3 p.m.

Exercise 1 (Repetition of basic definitions). *10 points*

a) Let $(a_n)_{n\geq 1}$ be a sequence of real numbers. Give the definition of convergence to a limit $a \in \mathbb{R}$.

b) Consider the following sequences and find the limit or briefly state why the limit does not exist.

b1) $a_n := \frac{n(1-3n)}{n^2-5}$

b2) $a_n := (-1)^n \frac{n-1}{n+1}$

c) Let f be a mapping between two vector spaces over $\mathbb{R}$. Give the definition of a linear mapping.

d) Consider the following functions. Which of them are (bi-)linear ? Prove your statement.

d1) $f\colon \mathbb{R} \to \mathbb{R}$; $f(x) := 17x - \pi$

d2) $f\colon \mathbb{R}^2 \to \mathbb{R}$; $f(x, y) := xy$

d3) $f\colon \mathbb{R} \to \mathbb{R}$; $f(x) := \exp(x)$

Exercise 2 (Example 1.1 - Newton-Raphson method). *10 points*

a) Consider the function $f(x) = \ln(x) + x - 2.8$, $x \in \mathbb{R}$. Apply the Newton-Raphson method for the starting value $x_0 = 2$ and determine the root.

b) Consider the function $g(x) = x^3 - 2x + 2$, $x \in \mathbb{R}$. Show that g has at least one real root[1]. Then apply the Newton-Raphson method with initial value x_0. What happens?

Exercise 3. *10 points*

Solve the following differential equations and verify your results.

a) $x' = \frac{\mathrm{d}x}{\mathrm{d}t} = (1 + e^t)x^2$

b) $x' - x\cos(t) = \cos(t)$

[1] Hint: cubic equation