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# Optimization and Dynamics

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Tutorials: Siyu Liang - Postbox: 207 in V3-126/128

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## Exercise Sheet 10

Deadline: Wednesday 03.07.19  
(in the tutorial or 3 p.m. in the postbox)

**Exercise 34** (Chapter 9).

10 points

Let  $A = \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix}$

a) Find  $e^A$ .

b) Hence solve the IVP  $x' = Ax$  with initial condition  $x(0) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$  and check finally that the solution solves the IVP.

**Exercise 35** (Chapter 9).

10 points

Consider the system

$$\begin{cases} x_1' &= 2x_1 + ax_2 \\ x_2' &= x_1 - 4x_2. \end{cases}$$

For which values of  $a \in \mathbb{R}$  is the fixed point  $x = 0$  asymptotically stable?

**Exercise 36** (Example 10.12).

10 points

Find a Lyapunov function for the system

$$\begin{cases} x' &= -2xy - 2y^2 \\ y' &= x^2 - y^3 + xy \end{cases}$$

and hence show that the origin is a stable fixed point of the system. Is it asymptotically stable?