Optimization and Dynamics

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Exercise Sheet 6

Deadline: Wednesday 05.06.19
(in the tutorial or 3 p.m. in the postbox)

Exercise 20 (Example 4.14, Example 4.15)**.** *10 points*

a) Consider the family of dynamical systems defined by

$$f_a(x) = x - (x^2 - a)(x^2 - 4a)\,,$$

as in Example 4.14. Show that the fixed point $x = 0$ is neither attracting nor repelling in the case $a = 0$ and hence it is unstable in the case $a = 0$.

b) Consider the family of dynamical systems defined by

$$f_a(x) = x - x(x^2 - a)(x^2 - 4a)\,,$$

as in Example 4.15. Show that the fixed point $x = 0$ is attracting and stable in the case $a = 0$.

Exercise 21. *10 points*

Let A be a 2×2 matrix given by

$$A := \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

and let

$$x_0 := A\,; \quad x_n := A^{n+1} \text{ for } n \in \mathbb{N}\,.$$

Identify A^2, A^3 and A^4. Find an expression for x_n in the form $x_n = f(n)\,x_0$ for all $n \in \mathbb{N}$ and prove it by mathematical induction.

Exercise 22 (Definition 5.17)**.** *10 points*

Consider the two dimensional linear dynamical system $x_{n+1} = Ax_n$ given by the matrix

$$A = \begin{pmatrix} 4 & -3 \\ \frac{3}{2} & -\frac{3}{2} \end{pmatrix}\,.$$

Determine the eigenvalues and eigenvectors of A. Then find the stable and unstable subspaces of the dynamical system.