
Optimization and Dynamics

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Summer semester 2019

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Exercise Sheet 8

Deadline: Wednesday 19.06.19
(in the tutorial or 3 p.m. in the postbox)

Exercise 28 (Chapter 7).

10 points

Consider the logistic map with parameter $a = 4$ i.e.

$$f: \mathbb{R} \rightarrow \mathbb{R}$$
$$x \mapsto f(x) = 4x(1 - x)$$

- a) Verify that f has sensitive dependence on the initial condition on $[0, 1]$ by using graphical analysis.
- b) Verify that f is topologically mixing on $[0, 1]$ by using graphical analysis.
- c) Conclude that f is chaotic on $[0, 1]$.

Exercise 29 (Section 8.1.).

10 points

Solve the following differential equations with corresponding initial conditions:

- a) $x'(t) = (x(t))^2$ with $x(0) = x_0 > 0$ in $0 \leq t < \frac{1}{x_0}$
- b) $x'(t) = 1 + x(t)$ with $x(0) = 9$
- c) $x'(t) = -2t \cdot x(t)$ with $x(0) = 1$

Exercise 30 (Definition 8.5).

10 points

Determine which of the following functions fulfils a Lipschitz condition on the set $\Omega := [0, 1] \times [0, 1]$.

- a) $f(t, x) = t^2 + x^2$
- b) $f(t, x) = \sin(t) \cdot \cos(t)$
- c) $f(t, x) = |t - x|$