Optimization and Dynamics

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Tutorials: Siyu Liang - Postbox: 207 in V3-126/128

Exercise Sheet 9

Deadline: Wednesday 27.06.19
(in the tutorial or 3 p.m. in the postbox)

Exercise 31 (Theorem 8.10). *10 points*
Consider the following IVP

$$\begin{cases} x'(t) = x(t) \cdot \cos(t) \\ x(0) = 1 \end{cases}$$

Show that there exists a unique solution on the interval $[-\frac{1}{2}, \frac{1}{2}]$ *and determine the solution.*

Exercise 32 (Proposition 9.4). *10 points*
Let A *and* B *be commuting* $d \times d$ *matrices and* S *an invertible* $d \times d$ *matrix. Prove the following identities:*

a) $e^{A+B} = e^A e^B$ *(Hint*[1] *or Hint*[2]*)*
b) $(e^A)^{-1} = e^{-A}$
c) $(e^A)^k = e^{kA}$, $k \in \mathbb{Z}$ *(Hint*[3]*)*
d) $e^{SAS^{-1}} = Se^AS^{-1}$

Exercise 33 (Proposition 9.7, Proposition 9.16). *10 points*
Let $A = \begin{pmatrix} -1 & 2 \\ -4 & 5 \end{pmatrix}$.

a) Write down the characteristic equation of A *and show that* A *fulfils it.*
b) Diagonalise A *and hence find* A^{12} *and* e^A *by using Proposition 9.7.*
c) Find the polynomial q *such that* $e^A = q(A)$ *by using Proposition 9.16.*

[1]Hint: Show $Be^{At} = e^{At}B$ and consider $F(t) = e^{(A+B)t} - e^{At}e^{Bt}$ and its derivative $F'(t)$
[2]Hint: Make use of the binomial theorem
[3]Hint: Case differentiation and induction are helpful