Workshop on Applied Linear Algebra in honor of Ludwig Elsner

January 21-23, 1999
Bielefeld
Hörsaal H15

Angelika Bunse-Gerstner, Universität Bremen
Volker Mehrmann, Universität Chemnitz
Reinhard Nabben, Universität Bielefeld
Matthias Pott, Universität Bielefeld
Wolf-Jürgen Beyn, Universität Bielefeld

ABSTRACTS

Subadditive and Superadditive Inequalities

T. Ando
Faculty of Economics
Hokusei Gakuen University
Sapporo 004-8631
Japan

ando@hokusei.ac.jp

Key identifying words: subadditive and superadditve inequalities, operator-monotone function.

It is known that if $f(t)$ is a non-negative, operator-monotone (that is, matrix-monotone of all orders) function on $[0, \infty)$ then the inequality

$$|||f(A+B) - f(A)||| \leq |||f(B)|||$$

holds for every pair of positive semi-definite matrices A, B and every unitarily invariant norm $||| \cdot |||$. In this talk we discuss inequalities of the form

$$|||f(A+B)||| \leq |||f(A) + f(B)|||.$$

Diagonals of Matrices

Rajendra Bhatia
Indian Statistical Institute
New Delhi 110 016
India

rbh@isid1.isid.ac.in

Key identifying words: diagonal, norm, Fourier series, inequalities

The diagonal of a matrix can be expressed as an average over unitary conjugates of the matrix.We will display such expressions for the main diagonal and other diagonals parallel to it.This leads to some interesting sharp bounds for the part of matrix obtained from it by retaining the diagonals and putting zeros elsewhere.

Canonical Forms of Positive Discrete–Time Linear Systems

Rafael Bru
Dept. Matemàtica Aplicada
Universitat Politècnica de València
46071 València
Spain

rbru@mat.upv.es

Key identifying words: positive systems, reachability, controllability, canonical forms.

In this paper, canonical forms of reachability, controllability and essential reachability of positive discrete-time linear control systems are studied. These properties are characterized in terms of directed graph of the state matrix. This work has been supported by Spanish DGES grant number PB97–0334.

Adaptive Multilevel Solvers

Wolfgang Dahmen
Institut für Geometrie und Praktische Mathematik
RWTH Aachen
52056 Aachen
Germany

dahmen@indy2.igpm.rwth-aachen.de

Key identifying words: multiscale techniques, wavelets

The essence of multiscale techniques and wavelet concepts is the ability of separating effects associated with different scales of resolution. Moreover, significant coefficients in a wavelet expansion indicate the location and type of singularities. The mathematical foundation lies in cancellation properties of wavelet-type functions and the fact that in some range weighted sequence norms of expansion coefficients are equivalent to function norms of Sobolev or Besov type. This accounts for the potential of such concepts with regard to the analysis as well as to the efficient numerical treatment of problems involving the interaction of a wide range of scales. It is indicated how to exploit these concepts for the development of numerical schemes for partial differential or integral equations which aim at achieving any desired accuracy at the expense of a possibly small number of adaptively generated degrees of freedom and a proportional amount of computational work.

Newton's method for a general rational matrix equation occuring in stochastic control

Tobias Damm
Institut für Dynamische Systeme
Universität Bremen
28334 Bremen
Germany

tobias@math.uni-bremen.de

Key identifying words: rational matrix equations, Riccati equations, Newton's method, positive operators, Krein-Rutman, stochastic control.

In this paper we regard a rather general class of rational matrix equations, which contains the continuous (CARE) and discrete (DARE) algebraic Riccati equations as special cases. The equation in question was encountered in [1] where an H^∞-type problem of disturbance attenuation for stochastic linear systems was studied.
We show that this equation can be solved by Newton's method and that the convergence is quadratic, provided a stabilizing solution exists. In doing so we especially give a unified approach to the iterative solution of both CARE and DARE. In particular the rate-of-convergence-result generalizes a result of L. Elsner. In most parts the argument follows the ideas exposed in [2]. But to reach the more general result we also make use of the Krein-Rutman theorem as well as a result from [3].

References

[1] A. El Bouhtouri, D. Hinrichsen, and A. J. Pritchard. On the disturbance attenuation problem for a wide class of time invariant linear stochastic systems. *To appear in Stochastics and Stochastics Reports.*

[2] P. Lancaster and L. Rodman. *Algebraic Riccati Equations.* Clarendon Press Oxford, 1995.

[3] H. Schneider. Positive operators and an inertia theorem. *Numerische Mathematik*, 7:11–17, 1965.

A pollard type result for restricted sums

J. A. Dias da Silva
Centro de Algebra da Universidade de Lisboa
Av. Gama Pinto 1699, Lisboa Codex
Portugal

perdigao@hermite.cii.fc.ul.pt

Key identifying words: Multilinear Algebra, Additive Number Theory, Control Theory.

Let $\mathbb{F}$ be an arbitrary field. Let p be the characteristic of $\mathbb{F}$ in case of finite characteristic and ∞ if $\mathbb{F}$ has characteristic 0. Let A be a finite subset of $\mathbb{F}$. By $\wedge^2 A$ we denote the set

$$\{a+b \mid a, b \in A \text{ and } a \neq b\}.$$

For $c \in \wedge^2 A$, let $\nu_c^{(R)}$ be one half of the cardinality of the set of pairs (a,b) satisfying $a \neq b$ and $a+b=c$. Denote by $\mu_i^{(R)}$ the cardinality of the set

$$\{c \in \wedge^2 A | \nu_c^{(R)} \geq i\}.$$

We prove that, for $t = 1, \ldots, \left\lfloor \frac{|A|}{2} \right\rfloor$,

$$\sum_{i=1}^{t} \mu_i^{(R)} \geq t \min\{p, 2(|A|-t)-1\}.$$

For $\mathbb{F} = \mathbb{Z}_p$ and $t=1$ we get the Erdős-Heilbronn conjecture. This is a joint work with Cristina Caldeira.

Krylov methods for P(A)x=b

Peter Fiebach
Bergische Universität GH Wuppertal
Fachbereich Mathematik
Gauß-Str. 20
42097 Wuppertal

peter@wmai07.math.uni-wuppertal.de

Key identifying words: Matrix polynomial, linear systems, Krylov subspaces, iterative methods.

We investigate two iterative methods for solving nonsingular linear systems

$$P(A)x = b, \qquad (*)$$

where $P(A) = \sum_{i=0}^{p} \gamma_i A^i$ denotes a matrix polynomial and A is hermitian. The approximate solutions are chosen from the Krylov subspaces $x^0 + K_j(A, r^0)$, with $r^0 = b - P(A)x^0$. Thus, only one matrix vector multiplication (*gemv*) is necessary to extend the Krylov space, whereas classical methods for solving $(*)$, like *CG* or *MINRES*, require p *gemv*s in each step. Both algorithms use short recurrences to compute iterates which minimize an appropriate residual norm. Theoretical result are given, and the efficiency of the methods is illustrated for linear systems from Tikhonov-Phillips regularization.

Some spectral properties of real Hankel matrices

Miroslav Fiedler
Institut of Mathematics
Czech Academy of Sciences
Zitna 25 115 67 Praha 1
Czech Republic

fiedler@math.cas.cz

Key identifying words: eigenvalues, Hankel matrices

Approaching the problem to completely characterize all possible n-tuples of eigenvalues of $n \times n$ real Hankel matrices, we first find two necessary conditions for such spectrum. For $n = 3$, one of them is also sufficient. Then we investigate and completely solve the problem for two special subclasses, Hankel-circulant and Hankel-skew-circulant matrices.

The limit of the spectral radius of nonnegative Toeplitz matrices

Shmuel Friedland
Department of Mathematics
Box 4348
Chicago, Illinois 60680
USA

friedlan@uic.edu

Key identifying words: nonnegative Toeplitz matrices

A bi-infinite sequence $...t_{-2}, t_{-1}, t_0, t_1, t_2, ...$ of nonnegative numbers defines a sequence of nonnegative Toeplitz matrices $T_n = (t_{ik}), n = 1, ...,$, where $t_{ik} = t_{k-i}, i, k = 1, \ldots, n$. We show that the limit of the spectral radius of T_n, as n tends to infinity, is given by $\inf\{\sigma(\xi) : \xi \in [0, \infty]\}$, where $\sigma(\xi) = \sum_{j \in \mathbf{Z}} t_j \xi^j$. A corresponding result holds in the case of block Toeplitz matrices, where the t_i are $p \times p-$nonnegative matrices and $\sigma(\xi)$ is the spectral radius of $\sum_{j \in \mathbf{Z}} \xi^j t_j$. This is a joint work with Ludwig Elsner.

Asynchronous Iterations – an Overview

Andreas Frommer
Department of Mathematics
University of Wuppertal
Germany

frommer@math.uni-wuppertal.de

Key identifying words: iterative solution methods, parallel systems

Asynchronous iterations arise naturally on parallel systems if synchronization points are eliminated in order to minimize processor idle times.

This talk will give an overview over old and new results on the convergence and the practical behaviour of asynchronous methods in various areas like linear systmes, odes and pdes. Ludwig Elsner and his group have made substantial contributions to the field.

On an asymptotic representation of coefficients of algebraic equation for sample points of Chebyshev Integration formula

Seiji Fujino
Hiroshima City University
3-4-1 Ozuka-Higashi, Asaminami-ku,
Hiroshima, 731-3194, Japan

fujino@ce.hiroshima-cu.ac.jp

Key identifying words: Chebyshev integration, asymptotic representation, algebraic equation.

In this article a verification of asymptotic representation of coefficients of algebraic equation for sample points of Chebyshev Integration formula will be reported. We suppose that $F_n(z)$ is an algebraic equation with zeros points as n sample points of Chebyshev Integration formula:

$$\int_{-1}^{1} f(x)dx \approx \frac{2}{n}\sum_{i=1}^{n} f(x_i). \tag{1}$$

Then $F_n(z)$ is represented as

$$F_n(z) \quad = \quad a_0 z^n + a_2 z^{n-2} + \cdots + \begin{cases} a_{n-1}z & (n : \text{odd}) \\ a_n & (n : \text{even}) \end{cases} \tag{2}$$

and the asymptotic representation of coefficients a_{2k} is shown as

$$\begin{cases} a_0 = 1 \\ a_{2k} = -\dfrac{n}{2k}\displaystyle\sum_{j=1}^{k}\frac{1}{2j+1}a_{2(k-j)} \qquad (k = 1, 2, 3, \cdots, \dfrac{n}{2}\ ). \end{cases} \tag{3}$$

Here we impose $\alpha = k/n$. The asymptotic representation for the coefficients a_k can be derived by Dr. S. Moriguchi as below. When k is even, the term $(\frac{\sqrt{1+v^2}}{e^\alpha v^{1-\alpha}})^n$ of the following equation is estimated as a major term and $0 \leq v \leq \infty$.

$$\begin{aligned} a_k &= 0 \qquad (k : \text{odd}) \\ &= (-1)^{1/2} \left(\frac{\sqrt{1+v^2}}{e^\alpha v^{1-\alpha}} \right)^n \frac{1}{\sqrt{n}} \sqrt{\frac{2}{\pi}} / \sqrt{\frac{1-\alpha}{v^2} - \frac{1}{1+v^2}} \quad (k : \text{even}) \end{aligned} \tag{4}$$

The purpose of this research is to verify the asymptotic representation for the coefficients a_k. When n is extremely large, we calculated them using multiple-precision arithmetics for preserving accuracy of calculation. Through the numerical experiments on a parallel computer we verified the validity of asymptotic representation for the coefficients of algebraic equation of 20480 degrees. This is a joint work with Harumi Ono and Akira Kojima.

On Orbital Matrices

Harald Gropp
Mühlingstr. 19
69121 Heidelberg
Germany

d12@ix.urz.uni-heidelberg.de

Key identifying words: graph, incidence matrix, orbital matrix, symmetric design, theorem of Bruck-Ryser-Chowla.

In this paper the state of knowledge on orbital matrices is discussed. Orbital matrices are certain generalizations of incidence matrices of symmetric designs such that the entries are nonnegative integers (not only 0 and 1). The corresponding theorem of Bruck-Ryser-Chowla also holds, but many more nonexistence results for parameter sets can be obtained otherwise, e.g. by using relations between linear algebra and graph theory.

Stabilized Superfast Toeplitz Solvers

Georg Heinig
Dept. of Mathematics and Comp.Sci.
Kuwait University
P.O.Box 5969 Safat 13060, Kuwait

georg@math-1.sci.kuniv.edu.kw

Key identifying words: structured matrices, fast algorithms.

In the talk we discuss the possibility to solve $n \times n$ Toeplitz systems of equations with less than $O(n^2)$ operations. Special attention is paid to the nonsymmetric case. The presented approach is based on an interpolational interpretation and includes several algorithms using pivoting techniques, divide-and-conquer strategies, cascade iterations and multi-grid ideas. This is a joint work with Peter Kravanja and Marc Van Barel.

On the spectral radius of multi-matrix functions

Daniel Hershkowitz
Department of Mathematics
Technion - Israel Institute of Technology
Haifa 32000, Israel

hershkow@techunix.technion.ac.il

Key identifying words: multi-matrix function, Hadamard functions, nonnegative H-matrices

Three problems that are related to the spectral radius $\rho(f(A_1, \ldots, A_k))$ of multi-matrix functions f of $n \times n$ matrices are discussed:

1. Characterization of the Hadamard functions f, defined by $f(A_1, \ldots, A_k) = (f(a_{ij}^1, \ldots, a_{ij}^k))_{i,j=1}^n$, satisfying either

$$\rho(f(A_1, \ldots, A_k)) \leq f(\rho(A_1), \ldots, \rho(A_k))$$

or

$$\rho(f(A_1, \ldots, A_k)) \geq f(\rho(A_1), \ldots, \rho(A_k)) \tag{5}$$

2. Characterization of the continuous Hadamard functions that preserve the nonnegative H-matrices. It is shown that these functions also satisfy (5).

3. Conditions for monotonicity of the spectral radius of the product $A_1 \cdots A_k$ with respect to the cycle lengths in the graphs of $A_1, \ldots, A_k$, whenever the matrices are one cycle matrices.

Applications of five diagonal matrices for the solution of ordinary differential equations by the method of grids

Hrant Hovhannissian
Northern Department of National Survey
for Seismic Protection
Leningradian st. 5a,
Gyumri, Shirak Region
Armenia

ndnssp@shirak.am

Multigrid Preconditioning and Toeplitz Matrices

Thomas Huckle
Fakultät für Informatik
TU München
Arcisstr. 21
80290 München
Germany

huckle@in.tum.de

Key identifying words: Multigrid methods, Toeplitz matrices, Preconditioning.

In this paper we discuss Multigrid methods for Topelitz matrices. Then the restriction and prolongation operator can be seen as projected Toeplitz matrices. Because of the intimate connection between such matrices and trigonometric series we can express the Multigrid algorithm in terms of the underlying functions with special zeroes This shows how to choose the prolongation/resrtriction operator in order to get fast convergence. This approach allows Multigrid methods for indefinite problems. Furthermore, we can define projections not only on problems of half size $n/2$, but on every size n/k for an $k \in N$. We can apply the derived method also to the non-constant-coefficient case for elliptic PDE.

Finite Rational Criteria for Quasidiagonalizability

Khakim Ikramov
The Moscow State University
Faculty of Computational Mathematics
and Cybernetics
119899 Moscow
Russia

ikramov@cmc.msk.su

Key identifying words: diagonalizable, quasidiagonalizable

A matrix $A \in M_n(\mathbf{C})$ is said to be *quasidiagonalizable* if A is similar to a block diagonal matrix D with the diagonal blocks of order one or two. A is *unitarily quasidiagonalizable* if the similarity matrix Q transforming A to D can be chosen to be unitary. Unitary quasidiagonalizability can be thought of as a matrix property closest to normality.

In this talk, various criteria for quasidiagonalizability employing only (a finite number of) arithmetic operations will be discussed.

Characterization of Determinantal Inequalities in Positivity Classes

Charles R. Johnson
Department of Mathematics
College of William and Mary
Williamsburg, VA 23 185
USA

crjohnso@MATH.WM.EDU

Key identifying words: determinantal inequalities, Hadamard's inequality,positivity classes, positive definite, totally positive, M-matrices, and inverse M-matrices

Beginning with Hadamard's inequality for positive definite matrices in 1893, there has been considerable interest in inequalities among products of principal minors in any matrix of given type, for both pure and applied reasons. This has resulted in the discovery of particular families of inequalities for positive definite, totally positive, M-matrices, and inverse M-matrices, etc, by Fischer, Koteljanski, Fan, Carlson, Marcus, Barrett and Johnson, etc over the last 100 years. Recently a program was begun to determine ALL such inequalities for each of given classes (and even all bounded ratios of products of principal minors). We report on the the current progress in this program that has not only discovered new families of inequalities but characterized all inequalities in a number of cases. Combinatorial and cone theoretic ideas play an important role.

An extension of the Courant-Fischer minimax principle

Michael Karow
Institut für Dynamische Systeme
Fachbereich 3
Universität Bremen
Postfach 330 440
28334 Bremen, Germany

karow@math.uni-bremen.de

Key identifying words: eigenvalues, Hermitian pencils, joint numerical range, convexity

Using the canonical form of Hermitian pencils we prove the following

Theorem 1. Let $H_0, H_1 \in \mathbb{F}^{n\times n}$ be Hermitian matrices, where $\mathbb{F} = \mathbb{R}, \mathbb{C}$ or $\mathbb{H}$. Here $\mathbb{H}$ denotes the division ring of real quaternions. For a ll $k \in \{1,\dots,n\}$ the following assertions are equivalent.

1. There exists a k-dimensional subspace U of $\mathbb{F}^n$ such that $x^*H_0x \geq 0$ and $x^*H_1x \geq 0$ for all $x \in U$.
2. $\lambda_k(\,(1-t)H_0 + tH_1\,) \geq 0$ for all $t \in [0,1]$,

where $\lambda_1(\cdot) \geq \lambda_2(\cdot) \geq \dots$ denote the eigenvalues of a Hermitian matrix in decreasing order.

As a corollary we obtain the following Courant-Fischer-like formula.

$$\min_{t\in[0,1]} \lambda_k(\,(1-t)H_0 + tH_1\,) = \max_{\dim U = k}\ \min_{x\in U\setminus\{0\}}\ \min\left\{\frac{x^*H_0x}{x^*x}, \frac{x^*H_1x}{x^*x}\right\}.$$

Some generalizations of this formula for $k = 1$ are given in the next theorem.

Theorem 2. Let K be a compact convex subset of $\mathcal{H}_n(\mathbb{F}) := \{ H \in \mathbb{F}^{n\times n} \mid H^* = H \}$, $n \in \mathbb{N}_{\geq 1}$, and suppose that one of the following conditions holds.

1. $\mathbb{F} = \mathbb{C}$ and $\dim K \leq 2$.
2. $\mathbb{F} = \mathbb{H}$ and $\dim K \leq 4$.
3. $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$ and $AB = BA$ for all $A, B \in K$.

Then

$$\min_{H\in K} \lambda_1(H) = \max_{\|x\|=1} \min_{H\in K} x^* H x = \max_{\|x\|=1} \min_{H\in \mathrm{ext}(K)} x^* H x,$$

where $\mathrm{ext}(K)$ denotes the set of extreme points of K and $\|\cdot\|$ denot es the Euclidean norm.

Discretized Gross-Pitaevskii Equations

Israel Koltracht
Department of Mathematics
University of Connecticut
Storrs, Connecticut 06269-3009
USA

kolt@math.uconn.edu

Key identifying words: nonlinear Schroedinger equations, Gross-Pitaevskii equations

We will discuss discretized nonlinear Schroedinger equations also known as Gross-Pitaevskii equations, which are used to model the Bose-Einstein condensation of atoms at near absolute zero temperatures.

The discretizations are obtained by approximating functions with corresponding interpolating polynomials at Legendre points. An example of a single Gross-Pitaevskii equation is as follows,

$$-\Delta u(x,y,z) + V(x,y,z)u(x,y,z) + ku^3(x,y,z) = \lambda u(x,y,z),$$

$$u(\infty) = 0, \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\ infty}^{\infty} u(x,y,z)^2 dxdydz = 1.$$

Its discretization in the case of cylindrical symmetry has the form,

$$AG + GB + V \odot G + kCG^3C = \lambda G, \quad \pi \sum_{i,j=1}^{n} |r_i| G_{i,j}^2 = 1.$$

Here G and λ are the unknown positive eigenmatrix and eigenvalue, $\odot$ denotes the componentwise, or Schur, matrix product, A, B, C, V are given matrices, and k is the parameter of nonlinearity.

We will describe some iterative methods for the numerical solution of such equations, and will show existence of a positive solution in the one variable case. The existence proof can be obtained from the monotone fixed point theory of L.V. Kantorovich, which will be briefly outlined.

Trace Inequalities for Hermitian Matrices and Their Application to the X-ray Analysis of Single Crystals

Arnold R. Kräuter
Institut für Mathematik und Angewandte Geometrie
Montanuniversität Leoben
Franz-Josef-Strasse 18
A-8700 Leoben
Austria

kraeuter@unileoben.ac.at

Key identifying words: trace, hermitian matrices, tensors of rank r, X-ray analysis of single crystals.

The selection of 'optimal' measurement directions in the determination of a tensor of rank r (e. g. in the investigation of some physical property of single crystals by means of X-rays) leads to the problem of finding sharp lower bounds for the trace of a class of hermitian matrices. An answer to this is provided by the following **Theorem** [4]. *For* $x, y \in \mathbf{C}^n$ *let* $(x, y) := x^*y$, *and let* $S := \{x \in \mathbf{C}^n : (x, x) = 1\}$.

Let $a \in S$ *be fixed and let* X *be a complex* $M \times n$ *matrix* $(2 \leq n \leq M)$. *Suppose that* n *of the rows* x_m *of* X *are linearly independent, that each* x_m *is in* S *and that it satisfies the condition* $|(x_m, a)| \geq c$ *for* $m = 1, ..., M$, *where* c *denotes a fixed real number with* $0 < c < 1$. *Finally, let* $P := X^*X/M$. *Then*
(a) P *is a positive definite hermitian* n*-square matrix;*
(b) *the following inequality holds:*

$$\operatorname{tr}(P^{-1}) \geq \begin{cases} n^2 & \text{if} \quad c < n^{-1/2}, \\ 1/c^2 + (n-1)^2/(1-c^2) & \text{if} \quad c \geq n^{-1/2}. \end{cases}$$

We present a detailed discussion of the case of equality in assertion (b) (including very recent results) as well as the abovementioned application. This is a joint work with Balder Ortner.

References

1. Z. Bai and G. H. Golub, Bounds for the trace of the inverse and the determinant of symmetric positive definite matrices. *Annals of Numerical Mathematics* **4,** 29 - 38 (1997).
2. C.-K. Li and R. Mathias, Matrix inequalities and partial isometries that arise in X-ray crystallography. *Linear Algebra and Its Applications* **254,** 303 - 314 (1997).
3. B. Ortner, On the selection of measurement directions in second-rank tensor (e. g. elastic strain) determination of single crystals. *Journal of Applied Crystallography* **22,** 216 - 221 (1989).
4. B. Ortner and A. R. Kräuter, Lower bounds for the determinant and the trace of a class of hermitian matrices. *Linear Algebra and Its Applications* **236,** 147 - 180 (1996).

On the Treatment of Bounded Domains and Boundary Conditions in Adaptive Wavelet Methods for Elliptic PDEs

Angela Kunoth
Institut für Geometrie und Praktische Mathematik
RWTH Aachen
52056 Aachen, Germany

kunoth@igpm.rwth-aachen.de

Key identifying words: Lagrange multipliers, boundary conditions, saddle point problem, Ladyšenskaja–Babuška–Brezzi (LBB) condition, fictitious domains, biorthogonal wavelets, adaptive methods.

For the numerical solution of elliptic partial differential equations, embedding the domain into a larger simple one, called fictitious domain, has been a favorable method, in particular, for domains with complicated boundaries. This approach in combination with the idea of explicitly treating essential boundary conditions by means of Lagrange multipliers decouples the differential operator from the boundary conditions as much as possible. In a weak saddle point formulation of the problem, stable discretizations on the domain and the boundary are then enforced by the corresponding Ladyšenskaja–Babuška–Brezzi (LBB) condition.

In my talk, I would like to present some new ideas how to use an adaptive wavelet-based method for this saddle point problem combined with a strategy how to satisfy the LBB condition for the problem at hand.

Some Recent Results on the Stability of Gyroscopic Systems

Peter Lancaster
Department of Mathematics and Statistics,
University of Calgary,
Calgary, AB T2N 1N4,
Canada

lancaste@ucalgary.ca

Key identifying words: gyroscopic systems, eigenvalues, stability, strong stability.

A differential equation of the form $M\ddot{x} + G\dot{x} + Kx = 0$ with real matrix coefficients $M > 0$, $G^T = -G$, $K^T = K$ is described as a *gyroscopic system.* We are mainly interested in the case $-K = C > 0$. Stability of the system can be completely understood in terms of the eigenvalue problem $L(\lambda)x = 0$, where $L(\lambda) = \lambda^2 M + \lambda G - C$, and a necessary condition for stability is that all eigenvalues (zeros of $\det L(\lambda)$), lie on the imaginary axis. Some conditions proposed for stability, or for instability, will be discussed, and classes of *strongly stable* systems will be described. Finally, some unifying analysis in the context of a two-parameter eigenvalue problem will be presented. This also admits the numerical computation of a (frequently precise) stability criterion as the solution of a convex optimization problem.

On the Distribution of the Subdominant Eigenvalue of Random Matrices

Michael Neumann
Department of Mathematics
University of Connecticut
Storrs, Connecticut O6269-3OO9
USA

neumann@math.uconn.edu

Key identifying words: Random Matrices, Eigenvalue distribution

In numerical experiments using MATLAB that we have carried out in connection with the convergence of powers of nonnegative matrices we observed that as the dimension n of the random matrix **grows**, the **ratio** of the modulus of the subdominant eigenvlaue to the spectral radius **diminishes**. We shall devote the talk to proving this and other related results.

This is a joint work with G. Goldberg, P. Okunev, and H. Schneider.

Markov Chains and extended convergence of SOR

Wilhelm Niethammer
Instiut für Praktische Mathematik
Universität Karlsruhe
76128 Karlsruhe
Germany

niethammer@math.uni-karlsruhe.de

Key identifying words: SOR-method, Markov chains

It is well–known that successive overrelaxation (SOR) can be used to compute the stationary distribution of a homogeneous Markov chain. In a long paper (LAA 154–156; 145–223 (1991)) Kontovasalis, Plemmons and Stewart showed together with other results that for p–periodic Markov chains convergence of SOR in an extended sense can happen for $\omega \notin (0, 2)$. The purpose of this lecture is to show how this astonishing result can be derived comparitively easy for two–periodic Markov chains from the well–known convergence properties of power iteration.

On the Effects of Nonnormality on Vector and Subspace Iterations

M.H.C. Paardekooper
Katholieke Universiteit Brabant
Subfaculteit Econometrie
Postbus 90153
NL-5000 Tiburg
The Netherlands

akelei@iaehv.nl

Key identifying words: nonnormal matrices, power method, subspace iteration

It is well known that in one step of the power method for normal matrices the decrease of the angle between the aimed eigenvector and an approximation is bounded by the quotient of the largest but one and the largest eigenvalue: $\tan\angle(Ax, v_)/\tan\angle(x, v_1) \leq |\lambda_1|/|\lambda_2|$. Not so for nonnormal matrices. Then a sharp bound for $\tan\angle(Ax, v_)/\tan\angle(x, v_1)$ depends also on A's departure of normality.
This talk discusses the effect of nonnormality on the power method and on the inverse iteration process. To get these bounds we have to restrict A's departure of normality.
Using wellknown results of P.Å. Wedin and G.W. Stewart for the perturbation of pseudo-inverse, we analyse one step of the subspace iteration in case of nonnormal matrices. Comparable with the vector iteration now again some limitation on both the gap between invariant subspace V_1 and approximation X and A's departure of normality is essential in order to get bounds for $\mathrm{gap}(AX, V_1)/\mathrm{gap}(X, V_1)$.
Computational experiments with both vector as subspace iteration are presented to highlight the effects of nonnormality.

Hartley Transform Representations of Inverses of Toeplitz-plus-Hankel Matrices

Karla Rost
Fakultät für Mathematik
Technische Universität Chemnitz
09107 Chemnitz
Germany

krost@mathematik.tu-chemnitz.de

Key identifying words: Toeplitz-plus-Hankel matrix, Cauchy matrix, Hartley transform, discrete Fourier transform.

Representations for inverses of Toeplitz-plus-Hankel matrices and more general Bezoutians involving only discrete Hartley transformations, diagonal matrices, and permutations are presented. These representations can be used for fast matrix-vector multiplication. The result is that multiplication of a column vector by an inverse of a Toeplitz-plus-Hankel matrix can be carried out with the help of 6 Hartley transformations plus 8 for preprocessing. This complexity estimate is significantly better than previous ones. This is a joint work with Georg Heinig.

Symmetrically Reciprocal Matrices and their Applications

Pal Rozsa
Tulipan utca 15
Budapest
H-1022 Hungary

rozsa@math.bme.hu

Key identifying words: symmetrically reciprocal matrices, transitive matrices, spectral characteristics

Square matrices with nonzero complex elements for which $a_{ij}a_{ji} = 1$ for $i \neq j$, and $a_{ii} = 1 (i, j = 1, 2, ..., n)$ hold, are called symmetrically reciprocal matrices (SRM). Square matrices with the property of $a_{ij}a_{jk} = a_{ik}$, for all i, j, k, will be called transitive matrices. It is easy to show that any transitive SRM is a one-rank matrix, furthermore any one-rank transitive matrix is a SRM and any one-rank SRM is a transitive matrix. Spectral characteristics of transitiv SRMs are described. Such matrices occur
i) in decision theory, where the elements of Saaty's Analytic Hierarchy Process (AHP) are positive real numbers,
ii) in macroeconomics,where in the coefficient matrix of the dynamic input-output analysis negative elements may also occur and
iii) in engineering, where the input spectral density matrix in vehicle system dynamics contain complex elements.

In applied problems, certain elements of these transitive SRMs are perturbed such that their symmetrically reciprocal property is preserved, however, their transitivity is lost. Spectral properties of the perturbed matrix is shown. This effect is proportional to the degree of nonlinearity of the multiple-input system that describes the dynamic excitations of the vehicle.

Olga Holtz disproves four conjectures Talk by Hans Schneider based on Not all GKK τ-matrices are stable Paper and results by Olga Holtz

Hans Schneider
Department of Mathemtics
University of Wisconsin–Madison
Madison, WI 53706, USA

hans@math.wisc.edu

Key identifying words: core linear algebra, eigenvalues, GKK matrices, τ-matrices, stability

Hermitian positive definite, totally positive, and nonsingular M-matrices enjoy many common properties, in particular

(A) positivity of all principal minors,

(B) weak sign symmetry,

(C) eigenvalue monotonicity,

(D) positive stability.

The class of GKK matrices is defined by properties (A) and (B), whereas the class of nonsingular τ-matrices is defined by (A) and (C). It was conjectured that

(A), (B) $\Longrightarrow$ (D)	(Carlson, 1974),
(A), (C) $\Longrightarrow$ (D)	(Engel/Schneider, 1976),
(A), (B) $\Longrightarrow$ (D)+	(Varga,1978),
(A), (B), (C) $\Longrightarrow$ (D)	(Hershkowitz, 1992).

A class of unstable GKK τ-matrices is determined and thus all four conjectures are disproved.

Structured Backward Errors for KKT Systems

Ji-guang Sun
Department of Computing Science,
University of Umeå
S-901 87 Umeå, Sweden

jisun@cs.umu.sea

Key identifying words: Karush-Kuhn-Tucker (KKT) systems

Karush-Kuhn-Tucker (KKT) systems are linear systems with coefficient matrices of the form

$$\begin{pmatrix} H & A \\ A^T & 0 \end{pmatrix},$$

where H is symmetric. A normwise structured backward error for KKT systems is defined, and a computable formula of the structured backward error is obtained. Simple examples show that the structured backward error may be arbitrarily larger than the unstructured ones in the worst case, and a stable algorithm for solving KKT systems is not necessarily strongly stable. Consequently, the computable formula of the structured backward error may be useful for testing the strong stability of practical algorithms for solving KKT systems

Block P-Matrices

Tomasz Szulc
Faculty of Mathematics and Computer Science
Adam Mickiewicz University
Matejki 48/49, PL 60-679 Poznań
Poland

tszulc@math.amu.edu.pl

Key identifying words: Block P-matrices, stability, Schur stability,

The notion of a block P-matrix has been introduced in [L.Elsner, T.Szulc: *Block P-matrices*, LAMA, to appear]. The talk will present some results about this class of matrices together with discussing a

variety of applications where it arises. In particular it will be shown that nonsingularity (stability, and Schur stability, respectively) of all convex combinations of nonsingular (stable, and Schur-stable, respectively) square matrices $A_1, \ldots, A_k$ can be characterized by referring to a block P-matrix.
The presented results are joint with L.Elsner.

On Symmetric Singular Systems

Néstor Thome
Dept. Matemàtica Aplicada
Universitat Politècnica de València
46071 València
Spain

njthome@mat.upv.es

Key identifying words: symmetric singular systems, equivalent systems.

In this work, we study properties of state–space symmetric singular systems. In particular, we obtain input–output equivalent systems, some of them without infinite poles. In addition, from the input–output equivalence, a model reduction of the initial system is derived.

This work has been supported by Spanish DGES grant number PB97–0334. The work of the third author was carried out while visiting the Universidad Politécnica de Valencia, the hospitality of which is gratefully acknowledged and was supported by a Grant of Agencia Española de Cooperación Internacional. This is a joint work with Rafael Bru and Carmen Coll.

Symmetric Schemes for Computing the Minimum Eigenvalue of a Symmetric Toeplitz Matrix

Heinrich Voss
TU Hamburg–Harburg
Arbeitsbereich Mathematik
D–21071 Hamburg
Germany

voss@tu-harburg.de

Key identifying words: Toeplitz matrix, minimum eigenvalue, projection method, symmetry.

In a recent paper (SIMAX 18, 521–534 (1997)) Wolfgang Mackens and the author presented a novel method for computing the minimum eigenvalue of a symmetric Toeplitz matrix. Similarly to an algorithm of Cybenko and Van Loan it is a combination of bisection and a root finding method of the secular equation using rational Hermitian interpolation. In a further paper (LAA 275-276, 401-415 (1998)) it was interpreted as a projection method, and it was accelerated substantially. Global and cubic convergence was proved. Taking advantage of the symmetry or skew-symmetry of the corresponding eigenvector both methods are improved considerably in this talk.

The ABC's and QR's of Eigenvalue Computation

David S. Watkins
Department of Mathematics
Washington State University
Pullman, WA 99164-3113
USA

watkins@wsu.edu

Key identifying words: Eigenvalues, QR algorithm, GR algorithms

The famous QR algorithm and its relatives comprise a rich and interesting family of methods for calculating eigenvalues of matrices. We will discuss several aspects of this family, including the general convergence theory of GR algorithms developed by Ludwig Elsner and the speaker, other contributions of Elsner in this area, and some of the speaker's recent work on the transmission of shifts and shift blurring.

Lagrangian Invariant Subspaces of Hamiltonian Matrices

Hongguo Xu
Fakultät für Mathematik
TU Chemnitz
D-09107 Chemnitz
FR Germany

hxu@mathematik.tu-chemnitz.de

Key identifying words: eigenvalue problem, Hamiltonian matrix, symplectic matrix, Lagrangian invariant subspace, algebraic Riccati equation, structure inertia index

The existence and uniqueness of Lagrangian invariant subspaces of Hamiltonian matrices is studied. Necessary and sufficient conditions are given in terms of the Jordan structure and certain sign characteristics that give uniqueness of these subspaces even in the presence of purely imaginary eigenvalues. These results are applied to obtain in special cases existence and uniqueness results for Hermitian solutions of continuous time algebraic Riccati equations. This is a joint work with Volker Mehrmann.

Projections with local minimality properties on finite dimensional classical Banach spaces and the matrices which represent them

M. Zippin
Mathematics Department
The Hebrew University
Jerusalem 91904
Israel

zippin@math.huji.ac.il

Key identifying words: Almost locally minimal projections. Orthogonally almost locally minimal projections. c-doubly stochastic projections.

A projection P on a Banach space X is called "almost locally minimal" if, for every $a > 0$ small enough, the ball $B(P, a)$ in the space $L(X)$ of all operators on X contains no projection Q with $\|Q\| \leq \|P\|(1 - Da^2)$ where D is a constant. A necessary and sufficient condition for P to be almost locally minimal is proved in the case of finite dimensional spaces. This criterion is used to describe almost locally minimal projections on ℓ_1^n. Examples of almost locally minimal projections are given and properties of their representing matrices are discussed.