Invariant theory

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Place & Time, Exercises

Tuesday, 10.30 – 12.00 in U2-135
Thursday, 10.30 – 12.00 in V4-116

Every 2nd week there will be one exercise/tutorial session instead of a lecture. Exercise sheets will be handed out every two weeks.

Synopsis

The main subject of this course are group actions on algebraic varieties and their invariants. The first part of the course will be devoted to the case of finite groups. In this setting, we will introduce the concept of invariants and the related geometric concept of quotients. We will use this setting in order to introduce the basic machinery from commutative algebra and affine algebraic geometry. No prior knowledge of these subjects is required, though some familiarity will certainly be helpful. In particular, we will cover the following results:

- The Hilbert-Noether Theorem.
- The Noether bound.
- The Mollien Formula.
- The Theorem of Shephard-Todd.

The topics for the later parts of the course are not yet fixed, but we plan to cover actions of reductive groups, in particular, the finite generatedness of the ring of invariants and (perhaps) the counterexample of Nagata for the case of non-reductive groups.

In both parts we will try to include as much as possible of the invariant theory of "classical" groups, such as the symmetric groups or GL_n. Depending on time and interests of the audience, further topics can be discussed, such as:

- Computational and constructive aspects of invariant theory, in particular Gröbner basis methods.
- The concept of stability and good quotients of reductive group actions.
- Group schemes.
- Categorical properties of quotients.
- ...

Further proposals for topics to be covered in this course are welcome!

Literature

[1] M. F. Atiyah and I. G. Macdonald. *Introduction to Commutative Algebra.* Addison-Wesley Publishing Company, 1969.

[2] D. Benson. *Polynomial Invariants of Finite Groups.* Cambridge University Press, 1993.

[3] W. Crawley-Boevey. Lectures on representation theory and invariant theory. `http://www1.maths.leeds.ac.uk/~pmtwc/repinv.pdf`, 1990.

[4] I. Dolgachev. *Lectures on invariant theory*, volume 296 of *LMS Lecture Note Series.* Cambridge University Press, 2003.

[5] D. Eisenbud. *Commutative Algebra with a View Toward Algebraic Geometry*, volume 150 of *Graduate Texts in Mathematics.* Springer, 1995.

[6] W. Fulton. *Young Tableaux*, volume 35 of *London Mathematical Society Student Texts.* Cambridge University Press, 1997.

[7] H. Kraft. *Geometrische Methoden in der Invariantentheorie.* Aspekte der Mathematik. Friedr. Vieweg & Sohn, second edition, 1985.

[8] E. Kunz. *Einführung in die kommutative Algebra und algebraische Geometrie.* Vieweg, 1980.

[9] L. Manivel. *Symmetric Functions, Schubert Polynomials and Degeneracy Loci*, volume 6 of *SMF/AMS Texts and Monographs.* Providence, RI: American Mathematical Society, 2001.

[10] D. Mumford, J. Fogarty, and F. Kirwan. *Geometric Invariant Theory*, volume 34 of *Ergebnisse der Mathematik und ihrer Grenzgebiete.* Springer-Verlag, Berlin, Heidelberg, New York, 1994.

[11] M. D. Neusel and L. Smith. *Invariant theory of finite groups*, volume 94 of *Mathematical Surveys and Monographs.* American Mathematical Society, 2002.

[12] P. E. Newstead. *Lectures on introduction to moduli problems and orbit spaces*, volume 51 of *Tata Institute of Fundamental Research Lectures on Mathematics and Physics.* Springer, 1978.

[13] L. Smith. *Polynomial Invariants of Finite Groups*, volume 6 of *Research Notes in Mathematics.* A K Peters, 1995.

[14] B. Sturmfels. *Algorithms in Invariant Theory.* Texts and Monographs in Symbolic Computation. Springer, 1993.

[15] H. Weyl. *The classical groups.* Princeton Landmarks in Mathematics. Princeton University Press, 1997.