Higher Preprojective Algebras — Bad Driburg, August 26–30, 2013

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Higher preprojective algebras are currently intensely studied and more and more properties and examples, in Algebra, Geometry, and Singularity Theory, come to the fore. One may rightly ask why they are so prevalent — and why they were not discovered much earlier!

The point of these three lectures is first to provide an abundant source of examples for such algebras coming from geometry, second to show how they apply to classifications of maximal Cohen–Macaulay modules, and then to study (the use of) their Hochschild cohomology.

Lecture 1: Higher Preprojective Algebras from Geometry. Tilting in geometric triangulated categories. Global and Orlov spectrum. Fano-type categories and higher preprojective algebras of representation infinite or τ_n–finite algebras. Examples.

Some pertinent literature: [5–11, 16, 18, 19, 27–30]

Lecture 2: Higher Preprojective Algebras and Maximal Cohen-Macaulay Modules. Recap of the McKay correspondence. Representation infinite algebras and maximal Cohen-Macaulay modules. Examples from geometry.

Some pertinent literature: [1, 2, 12, 17, 20–26, 30]

Lecture 3: Higher Preprojective Algebras and Hochschild Cohomology. A bit of a grab bag: Koszulity of finely graded higher preprojective algebras. Periodic resolutions and self-injective higher stably preprojective algebras.

Some pertinent literature: [3, 4, 13–15]

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