

Intersections of fixed subgroups of a free group

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ABSTRACT

Let F be a finitely generated free group, and let n denote its rank. A subgroup H of F is said to be *auto-fixed* if there exists a set S of automorphisms of F such that H is precisely the set of elements fixed by every element of S ; similarly, H is *1-auto-fixed* if there exists a single automorphism of F whose set of fixed elements is precisely H . We conjecture that the families of auto-fixed and 1-auto-fixed subgroups of F coincide. And, as a first step towards the conjecture, we show that each auto-fixed subgroup of F is a free factor of a 1-auto-fixed subgroup of F . We show also that if (and only if) $n \geq 3$, then there exist free factors of 1-auto-fixed subgroups of F which are not auto-fixed subgroups of F . In the natural way, the notions of *mono-fixed* and *endo-fixed* subgroups of F can be defined. And the two results above are also true, changing auto-fixed subgroups to mono-fixed subgroups or to endo-fixed subgroups.

By a well-known result of M. Bestvina, M. Handel, W. Imrich and E. Turner, a 1-endo-fixed subgroup of F has rank at most n . And, by results due to W. Dicks, E. Ventura and G. Bergman, every endo-fixed subgroup of F also does. Hence, the conjecture above is true in the maximum-rank case. We further prove that if H is a maximum-rank 1-auto-fixed subgroup of F , then the group of automorphisms of F which fix every element of H is free abelian of rank at most $n - 1$.

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