

Comparing shapes of manifolds: applications of Size Theory to computer vision

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ABSTRACT

The problem of comparing two closed manifolds \mathcal{M} and \mathcal{N} by computing the infimum of an operator Θ defined on a suitable set H of homeomorphisms from \mathcal{M} to \mathcal{N} is a classic object of study in many fields of Geometry.

A simple way of defining the operator Θ is the following one. For each manifold \mathcal{M} we choose a convenient function $\varphi_{\mathcal{M}} : \mathcal{M} \rightarrow \mathbb{R}$ related to the structure we are interested in, and compute how much each homeomorphism $f : \mathcal{M} \rightarrow \mathcal{N}$ “changes” such a function: this measure is the value taken by our operator Θ at f , i.e. $\Theta(f) \stackrel{\text{def}}{=} \max_{P \in \mathcal{M}} |\varphi_{\mathcal{M}}(P) - \varphi_{\mathcal{N}}(f(P))|$. The usual task is to make $\Theta(f)$ as small as possible, and to take its infimum for $f \in H$ as a pseudodistance d between the considered manifolds.

The functional Θ allows us to compare some kinds of structures on manifolds (e.g., Riemannian structures) by using suitable real-valued functions.

The purpose of comparing “shapes” of manifolds for solving computer vision problems has made the pseudodistance d interesting from an applicative point of view, together with the “twin” and strictly related concepts of *size function* and *size homotopy group*.

References

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