3rd European Congress of Mathematics
Poster sessions

Section 08: Ordinary Differential Equations and Dynamical Systems — Poster number 390

On Fuchs-Takano's Differential Equation

Norihiro Misaki, College of Science and Technology, Nihon University.

ABSTRACT

We consider Fuchs-Takano's differential system (FTS)

$$\begin{cases} x^2\dfrac{\partial^2 u}{\partial x^2} + xp_1(x,y)\dfrac{\partial u}{\partial x} + q_1(x,y)\dfrac{\partial u}{\partial y} + r_1(x,y)u = 0 \\ x\dfrac{\partial^2 u}{\partial x \partial y} + xp_2(x,y)\dfrac{\partial u}{\partial x} + q_2(x,y)\dfrac{\partial u}{\partial y} + r_2(x,y)u = 0 \\ \dfrac{\partial^2 u}{\partial y^2} + xp_3(x,y)\dfrac{\partial u}{\partial x} + q_3(x,y)\dfrac{\partial u}{\partial y} + r_3(x,y)u = 0, \end{cases}$$

where p_j's,q_j's and r_j's $(j = 1, 2, 3)$ are holomorphic on the region

$$D = \{x; 0 < |x| < r\} \times \{y; |y| < r\} \subset \mathbf{C}^2.$$

Integrability conditions are assumed.

A solution w of Fuchsian equation has the form $w = x^\rho \varphi(x)$ around the singular point $x = 0$, where the exponent ρ is a constant and φ is holomorphic around $x = 0$. But when the systems of two complex variables x and y are treated, the exponents have been defined on each line $y = constant$. So they depend on the values of y.

We show that we can define the exponents for the system FTS as constants and that FTS has the solutions of such the type $u = x^\rho \varphi(x, y)$ with constant ρ and a holomorphic function $\varphi(x, y)$.

Keywords: *partial differential equations, complex variables,*

Mathematics Subject Classification: *34A20, 34A30, 35N10, 32A99*

Contact Address: `misaki@penta.ge.cst.nihon-u.ac.jp`