

On The Spectral Radius of Homogeneous Order-Preserving Functions

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ABSTRACT

For functions $F : \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n$, where \mathbb{R}_+^n is the cone of nonnegative vectors in \mathbb{R}^n , that are homogeneous and order preserving, we give an asymptotic characterization of the (cone) spectral radius. More precisely we show that

$$\rho(F) = \lim_{k \rightarrow \infty} \max_{1 \leq i \leq n} (F^k(x))_i^{\frac{1}{k}}, x \in (\mathbb{R}_+^n)^\circ$$

As an immediate conclusion we get that the maximum component of the cycle time vector $\chi(F) = \lim_{k \rightarrow \infty} (F^k(x))_i^{\frac{1}{k}}$, whenever it exists, is equal to their spectral radius.

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