3rd European Congress of Mathematics
Poster sessions

Section 09: Partial Differential Equations | Poster number 48

Uniformly Convergent Difference Scheme for a Stiff Elliptic Reaction-Diffusion Problem with Singular Source Term

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ABSTRACT

Consider two dimensional elliptic problem in a rectangle $\Omega = (-1, 1) \times (0, 1)$.

$$-\frac{\partial}{\partial x}\left(p^x(x,y)\frac{\partial u(x,y)}{\partial x}\right) - \frac{\partial}{\partial y}\left(p^y(x,y)\frac{\partial u(x,y)}{\partial y}\right) + q(x,y)u(x,y) = f(x,y), \tag{1}$$

with Dirichlet boundary conditions.

Within the domain we suppose there is a regular interface Γ across which the solution $u(x, y)$ or some of its derivatives are known to be discontinuous and the source term can be also discontinuous or even singular. We deal with the case when the diffusion parameters p^x, p^y are small in some parts of Ω. For simplicity we take $\Gamma = \{x = 0,\, |y| < 1\}$,

$$p^x(x,y) = p^y(x,y) \equiv \begin{cases} 1, & (x,y) \in \Omega^- = \{-1 < x < 0,\, |y| < 1\}, \\ \varepsilon^2, & (x,y) \in \Omega^+ = \{0 < x < 1,\, |y| < 1\}, \end{cases} \tag{2}$$

and jump conditions

$$[u]_\Gamma = u(+0, y) - u(-0, y) = 0, \tag{3}$$

$$-\left[\frac{\partial u}{\partial n_a}\right]_\Gamma = -\varepsilon^2 \frac{\partial u(+0,y)}{\partial x} + \frac{\partial u(-0,y)}{\partial x} = Q(y). \tag{4}$$

Problems of type (1) - (4) at small ε are often called "stiff'. They arise in many applications: heat-mass transfer, nuclear reactors, geofiltration, etc.

Since the diffusion coefficients in Ω^+ are small boundary layers appear around the boundary of Ω^+. The singularity of the domain and the discontinuity conditions (3), (4) cause also corner layers around corners of Ω^+ and two intersection points of $\partial\Omega^- \cap \Gamma$. It is well known that for problems with boundary and interior layers the standard numerical methods on uniform meshes are not ε-uniformly convergent. In order to solve numerically (1) – (4) we use finite volume method on a special condensed "Shishkin" mesh around the boundary of Ω^+. The mesh that arise is not overlapping and we use interpolation to overcome this difficulty. Due to stiffness of the coefficients the matrix that we get is badly scaled but simple diagonal preconditioning improves the condition number. Uniformly convergence in a energetic norm is proved. Numerical results confirming the theoretical estimates are given.

Keywords: *elliptic reaction-diffusion problems, asymptotic analysis, difference schemes, finite volume method, adaptive meshes*

Mathematics Subject Classification: *65N06, 65N15*

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