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Regular Dilations and The Douglas-Foiaş Model

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ABSTRACT

As shown by Douglas and Foiaş (A Classification of Multi-Isometries, to appear) every completely non-unitary (c.n.u.) bi-isometry $V = (V_1, V_2)$ is "modeled" by a certain isometric pair on $H^2(\mathcal{E})$ in terms of two operators in the Hilbert space $\mathcal{E}$, U unitary and P orthogonal projection, called its unitary invariants. A relating result (Dan Popovici-On the structure of c.n.u. bi-isometries via the Douglas-Foiaş model, *Acta Sci. Math.*, **66**(2000), to appear) gives characterizations for different parts of the double-commuting isometric pair $V : V_i$ pure iff $U^{3-2i} \mid \ker[(i-1)I + (3-2i)P]$ pure, V_i pure and V_{3-i} unitary iff $P = (2-i)I\ (i = 1,2),\ V$ bi-shift iff U bilateral shift with defect space $UP\mathcal{E} \ominus P\mathcal{E},\ V$ has no bi-shift part iff $PU = UP$. It is our aim in this paper to obtain structure results relative to this model for the minimal regular (respectively $*$-regular) isometric dilation (introduced by Brehmer and Sz.-Nagy) $V_T = (V_T^{(1)}, V_T^{(2)})$ (respectively V_{T^*}) of a given bi-contraction $T = (T_1, T_2)$ (respectively T^*) on a Hilbert space $\mathcal{H}$.

Remark firstly that a minimal isometric dilation of T is c.n.u. iff $T_1T_2 \in C_{\cdot 0}$ ($C_{\alpha,\beta},\ \alpha, \beta \in \{\cdot, 0, 1\}$ represent the Sz.-Nagy-Foiaş classes of contractions defined in terms of punctual convergence). Consider contractions $S_T^{(i)} \in \mathcal{L}(\mathcal{D}_{T_{3-i}}),\ S_T^{(i)} D_{T_{3-i}} h := D_{T_{3-i}} T_i h$ and unitary operators $R_T^{(i)} \in \mathcal{L}(\mathcal{D}, \mathcal{D}_{S_T^{(i)}}),\ R_T^{(i)} \Delta_T^{1/2} h = D_{S_T^{(i)}} D_{T_{3-i}} h,\ h \in \mathcal{H},\ i = 1,2$ ($\Delta_T := I - T_1^*T_1 - T_2^*T_2 + T_1^*T_2^*T_1T_2$, $\mathcal{D} = \overline{\Delta_T \mathcal{H}}$, $D_Z = (I - Z^*Z)^{1/2}$ the defect operator and $\mathcal{D}_Z = \overline{D_Z \mathcal{H}}$ the defect space of a contraction Z). The unitary invariants of V_{T^*} are given by the formulas $U = A_1 + A_2^*$, $A_i = (A_{jk}^{(i)})_{j,k \in \mathbb{Z}}$, $A_{10}^{(i)} = D_{S_T^{(i)}} R_T^{(i)},\ A_{11}^{(i)} = S_T^{(i)*},\ A_{20}^{(i)} = -S_T^{(i)} R_T^{(i)},\ A_{21}^{(i)} = D_{S_T^{(i)*}},\ A_{j\,j-1}^{(i)} = I_{\mathcal{D}_{S_T^{(i)*}}}\ (j \geq 3),\ A_{jk}^{(i)} = 0$ (for other (j,k)), $i = 1,2$ and $P = (P_{jk})_{j,k \in \mathbb{Z}}$, $P_{jj} = I_{\mathcal{D}_{S_T^{(2)*}}}\ (j \leq -2)$, $P_{-1\,-1} = I_{\mathcal{D}_{T_1}},\ P_{jk} = 0$ (for other (j,k)) on $\ell^2_{\mathbb{Z}_-^*}(\mathcal{D}_{S_T^{(2)*}}) \oplus \mathcal{D}_{T_1} \oplus \mathcal{D} \oplus \mathcal{D}_{T_2} \oplus \ell^2_{\mathbb{Z}_+^*}(\mathcal{D}_{S_T^{(1)*}})$. Having as a starting point a paper of D.Gaşpar and N.Suciu (On the geometric structure of regular dilations, *Op.Theory: Adv. and Appl.*, **103**(1998), 105-120) we can obtain corresponding results for V_T.

Finally, as applications, we characterize the membership of V_T and V_{T^*} to some special classes of bi-isometries in terms of T. Thus $V_T^{(i)}$ is unitary iff $T_i,\ S_T^{(i)}$ are co-isometries and $V_T^{(i)}$ is pure iff $T_i,\ S_T^{(i)} \in C_{\cdot 0},\ i = 1,2$. Moreover V_{T^*} is bi-shift, V_T is unitary iff T_i are co-shifts and $S_T^{(i)}$ co-isometries $i = 1,2$, V_{T^*} is bi-shift, $V_T^{(1)}$ is unitary and $V_T^{(2)}$ is shift iff T_1 is co-shift, $T_2 \in C_{00},\ S_T^{(1)}$ co-isometry, $S_T^{(2)} \in C_{\cdot 0}$. Similar characterizations are obtained for other remarkable parts of V_T and V_{T^*}.

Keywords: *bi-contraction, isometric dilation, regular, $*$-regular, unitary, completely non-unitary*

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