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Rate of growth of hypercyclic entire functions

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ABSTRACT

By a well-known theorem of G. R. MacLane there exists an entire function f whose derivatives $D^n f = f^{(n)}$ form a dense set in the space $H(\mathbb{C})$ of entire functions. Such a function f is called hypercyclic for the operator D. In a previous paper the author has shown that $\frac{e^r}{\sqrt{r}}$ is the critical rate of growth for D-hypercyclic entire functions, in the following sense: For any function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\varphi(r) \to \infty$ as $r \to \infty$ there exists a D-hypercyclic entire function f with

$$|f(z)| = O\big(\varphi(r)\frac{e^r}{\sqrt{r}}\big) \quad \text{as} \quad |z| = r \to \infty, \tag{1}$$

while there can be no D-hypercyclic entire function f with

$$|f(z)| = O\big(\frac{e^r}{\sqrt{r}}\big).$$

Recently, Armitage has shown that, in fact, there is a dense set of D-hypercyclic functions f satisfying the growth condition (1).

In the present paper we consider, more generally, weighted shift operator $T : H(\mathbb{C}) \to H(\mathbb{C})$, $T(\sum_{n=0}^{\infty} c_n z^n) = \sum_{n=0}^{\infty} a_{n+1} c_{n+1} z^n$ on the space of entire functions; note that $a_n = n$ for the differentiation operator D. Suppose that $|a_n| \to \infty$ monotonically. Then it is known that T possesses hypercyclic functions, that is, functions f for which $\{T^n f : n \in \mathbb{N}_0\}$ is dense in $H(\mathbb{C})$. For such operators T we obtain the critical rate of growth for T-hypercyclic entire functions, thus generalizing our earlier result, and we also show that Armitage's addendum continues to hold for general T.

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