

## Structure of contractible locally $C^*$ -algebras

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### ABSTRACT

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A locally  $C^*$ -algebra (inverse limit of  $C^*$ -algebras) is contractible iff it is topologically isomorphic to the cartesian product of a family of full matrix algebras. This result answers in positive a conjecture posed by A.Ya. Helemskii in a conversation with the author. The general question concerning the structure of an arbitrary contractible Arens-Michael algebra (inverse limit of Banach algebras), came out from a relevant result of J.L. Taylor [3; Proposition 5.7] and it is still open, even in the normed case. The problem is solved for commutative Arens-Michael algebras (Taylor-Helemskii [1; Theorem IV. 5.27]) and for semiprime Fréchet Arens-Michael algebras having a nice geometrical property, the so-called approximation property of A. Grothendieck (Selivanov [2; Theorem 5]). Our result can not be taken from the preceding ones, since an arbitrary locally  $C^*$ -algebra is neither commutative nor Fréchet; take, for instance, the cartesian product of an uncountable family of full matrix algebras (where at least one of them consists of matrices of size  $> 1$ ). Notice that such an algebra is always contractible [3; p. 181].

### References

- [1] A.Ya. Helemskii, *The Homology of Banach and Topological Algebras*; Kluwer Academic Publ., Dordrecht, 1989
- [2] Y.V. Selivanov, *Fréchet algebras of global dimension zero*; Proceedings of the IIIrd International Conference on Algebra, held in Kranoyarsk, Russia, August 23–28, 1993. Walter de Gruyter, 1996, 225–236.
- [3] J.L. Taylor, *Homology and cohomology for topological algebras*; Advances in Math. 9 (1972), 137–182.

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