3rd European Congress of Mathematics
Poster sessions

Section 13: Real Analysis Poster number 312

Comparison Theorems on Orders of Approximation of Fourier Expansions by Matrix Methods

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ABSTRACT

Let X be a Banach space with norm $\|\cdot\|$ and $(T_k)\,(k=0,1,...)$ be a total sequence of mutually orthogonal continuous projections on X. Then with each $x\in X$ one may associate its formal Fourier expansion $x\sim\sum_k T_k x$. Let $M^\varphi=(m_{nk})$ be a triangular matrix, generated by the continuous function $\varphi(t)$ on $[0,\infty[$, where $\varphi(0)=1$ and $\varphi(t)=0$, if $t\geq 1$, i.e.

$$m_{nk}=\begin{cases}\varphi(\frac{k}{n+1}) & (k\leq n),\\ 0 & (k>n).\end{cases}$$

If $\varphi(t)=1-t^r$, then M^φ is the method of Zygmund Z^r. In [2] (see Theorem 3) the orders of approximation of Fourier expansions by Z^r and M^φ are compared. Also the special case, if M^φ is the method of Rogosinski (i.e. $\varphi(t)=cos(\pi t/2)$ for $t\in[0,1]$ and $\varphi(t)=0$ for $t>1$) is studied in [2] and the special case $M^\varphi=Z^s$ in [1]. Here we shall represent some new conclusions from above-mentioned Theorem 3 from [2]. Namely, we shall consider the following cases:

1. $\varphi(t)=(1-t^2)^\delta$ for $t\in[0,1]$ and $\varphi(t)=0$ for $t>1$,
2. $\varphi(t)=1-6t^2+6t^3$ for $0\leq t\leq 1/2$, $\varphi(t)=2(1-t)^3$ for $1/2\leq t\leq 1$ and $\varphi(t)=0$ for $t\geq 1$,
3. $\varphi(t)=\pi t/2cot\pi t/2$ for $0<t\leq 1$, $\varphi(0)=1$ and $\varphi(t)=0$ for $t\geq 1$,
4. $\varphi(t)=(1-t)cos\pi t+sin\pi t$ for $t\in[0,1]$ and $\varphi(t)=0$ for $t>1$.

References

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[2] A. Aasma, Comparison of orders of approximation of Fourier expansions by different matrix methods, Facta Univ. (Nis) Ser. Math. Inform. 12 (1997), 233-238.

Keywords: *approximation orders, Fourier expansions, summability methods*

Mathematics Subject Classification: *40C05, 41A25*

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