On some geometric discretisation of Yang-Mills equations

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ABSTRACT

Following [1] we construct a gauge–invariant discrete model of Yang–Mills equations on 4–dimensional sphere S^4.

Let A denote a connection 1–form on S^4 taking values in the Lie algebra $su(2)$ of the Lie group $SU(2)$. It is well known that the 2–form $F = dA + A \wedge A$ determines the curvature form of the connection A. Consider the equations

$$d_A F \equiv dF + A \wedge F - F \wedge A = 0, \tag{1}$$

$$d_A * F = 0, \tag{2}$$

where d_A is the covariant exterior derivative and $*$ is the metric adjoint operation (Hodge star). Equations (1) and (2) are called the Yang–Mills equations. Equation (1) is known as the Bianci identity. In the 4–dimensional case Yang–Mills theory can be regarded as a nonlinear generalization of Hodge theory.

We define a discrete analog of the covariant exterior differentiation operator d^c_A by using the formalism described in [2] for the Laplace operator in Euclidean space. We apply this operator to construct a model satisfying the gauge invariance principle. In addition a discrete analog of gauge transformations is introduced. To define a discrete analog of the metric adjoint operator we use a construction based on a double complex. Then the $*$ operation is defined in such a way that $** = 1$, i. e., it is more like its continual analog. First, introducing discrete analogs of the connection form A and the curvature form F we construct a discrete model of Equations (1) and (2) on the double complex. Then this model is generalized on a combinatorial analog of the 4–sphere. However, in comparison with the continual theory we obtain the gauge invariance of the Yang–Mills equations under some necessary and sufficient condition.

Finally, we consider difference analogs of the self–dual equation $F = *F$ and the anti–self–dual equation $F = - * F$. We obtain a finite system of nonlinear matrix equations on the 4–sphere. Properties are discussed.

References

[1] Sushch V. N. Gauge-invariant discrete models of Yang-Mills equations. *Mathematical Notes.* Vol. 61. No 5. P. 621–631. 1997.

[2] Dezin A. A. Multidimensional analysis and discrete models. *CRC Press.* Boca Raton. 1995.

Keywords: *Yang-Mills equations, discrete model, gauge-invariance, difference self-dual equation*

Mathematics Subject Classification: *35Q40, 53.99, 39A10*

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