3rd European Congress of Mathematics
Poster sessions

Section 14: Mathematical Physics | Poster number 502

Entropic uncertainty relations in quantum mechanics

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ABSTRACT

Let A and B denote two noncommuting Hermitian operators (observables) in Hilbert space. An entropic uncertainty relation (EUR) for this pair is a nontrivial inequality of the form $H(A) + H(B) \geq H_{AB}$, where H_{AB} is a constant while $H(A)$ and $H(B)$ are the information entropies corresponding to the probability distributions for the possible outcomes of measurements of A and B, respectively, in an arbitrary quantum state. For instance, the optimal EUR for the position-momentum pair is $H(X) + H(P) \geq 1 + \ln \pi$, as conjectured by Hirschman and Everett (1957) and proved by Beckner and Bialynicki-Birula and Mycielski (1975) using Beckner's formula for the (p, q)-norm of the Fourier transform. In finite-dimensional Hilbert space, the existence of EURs for arbitrary pairs of discrete spectrum observables with no common eigenstates was first proved by Deutsch (1983), who found that $H(A) + H(B) \geq -2 \ln((1 + c)/2)$ with the constant c depending only on the eigenvectors $\{|a_i\rangle\}$ of A and $\{|b_j\rangle\}$ of B ($c \equiv \max_{i,j} |\langle a_i|b_j\rangle|$). Using the M. Riesz convexity theorem (1926), Maassen and Uffink (1988) obtained a stronger inequality, $H(A) + H(B) \geq -2 \ln c$, which, however, is still not optimal in the general case.

According to Shannon's information theory, entropy is the *only* rigorous quantitative measure of the uncertainty or lack of information associated to a random variable. EURs thus provide a rigorous mathematical formulation for the uncertainty principle of quantum mechanics, unlike the standard Heisenberg UR (Deutsch 1983). EURs also provide a natural link between the uncertainty principle and the branches of quantum physics that make use of information theory, such as quantum communication, quantum computation and quantum cryptography.

Here we review some recent results on EURs, obtained by the presenter author during the last years. As regards EURs for pairs of observables, we have found (i) the explicit form of the overlap c appearing in the Deutsch and Maassen-Uffink EURs for arbitrary angular momentum observables, and (ii) the optimal EUR for arbitrary observables in two-dimensional Hilbert space. We also have showed that the entropic formulation of the uncertainty principle can be extended to sets of more than two mutually noncommuting observables, for which one can find EURs that are stronger than those derived from the EURs existing for each pair in the set; and, for particular classes of states (e.g. pure states) there are also nontrivial *upper* bounds on the sum of the entropies, i.e. "entropic certainty relations" (ECRs). In particular, we have derived nontrivial EURs and ECRs for sets of $N + 1$ complementary observables in N-dimensional Hilbert space, the optimal EURs for the $N = 2$ and $N = 3$ cases (a conjecture is formulated for the $N = 5$ case), and the optimal ECR for the $N = 2$ case.

Keywords: *Uncertainty principle; Quantum measurements; Information entropy; Complementary observables; Inequalities*

Mathematics Subject Classification: *81P15; 81Q10; 94A17; 47Axx*

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