

Infinitestage games of search and completion

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ABSTRACT

The subject of this report is a broad class of infinitestage game processes, which have received little attention so far. We shall call these processes games of search and completion [B. I. Model', "Games of search and completion", In: Journal of Mathematical Sciences, Vol. 80, No 2, Plenum Publishing Corporation, New York (1996), pp. 1699-1744.]

The games of search and completion can be represented as follows. Suppose we are given the interval $(0, 1)$. The points of $(0, 1)$ are divided during the game into labeled and unlabeled. Before the game, m points from $(0, 1)$ are labeled (m is a natural number). Two players take part in the game, making moves in turn. On the first move, the first player searches through the labeled points, adding k of them to the unlabeled ones, $k < m$. On the second move, the second player augments the set of remaining $m - k$ labeled points, adding k unlabeled points to it, etc. (the move number runs through the set of natural numbers). We say that a point remains labeled after the end of the game if it remains labeled starting from some move. The first player tries to guarantee the least number of points that can remain labeled after the end of the game, whereas the goal of the second player is quite the opposite. We consider the most difficult case where the players know when choosing a move the current state and the number of the move.

With the use of the continuum hypothesis the following unexpected statement can be proved: the first player can guarantee that for all m and k at most one point will remain labeled after the end of the game.

The games of search and completion played jointly as well as simultaneous parties are of particular interest. We also consider the most difficult case where the players know when choosing a move the current state and the number of the move. With the use of the continuum hypothesis it is possible to reveal the unexpected possibilities of the first player, namely he can guarantee that not more than one point will remain labeled after the end of the simultaneous parties. Thus, the least guaranteed result of the simultaneous parties turns out to be not worse than the least guaranteed result of any constituent game.

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