The Hamiltonian Seifert Conjecture: Examples and Open Problems

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Abstract. Hamiltonian dynamical systems tend to have infinitely many periodic orbits. For example, for a broad class of symplectic manifolds almost all levels of a proper smooth Hamiltonian carry periodic orbits. The Hamiltonian Seifert conjecture is the existence problem for regular compact energy levels without periodic orbits.

Very little is known about how large the set of regular energy values without periodic orbits can be. For instance, in all known examples of Hamiltonian flows on $\mathbb{R}^{2n}$ such energy values form a discrete set, whereas "almost existence theorems" only require this set to have zero measure. We describe constructions of Hamiltonian flows without periodic orbits on one energy level and formulate conjectures and open problems.

1. Introduction

Hamiltonian flows of proper smooth functions on $\mathbb{R}^{2n}$ or on many other symplectic manifolds are known to have periodic orbits on almost all energy levels; see [2, 15, 17]. On the other hand, there exist examples of proper smooth Hamiltonians on $\mathbb{R}^{2n}$ with one regular level carrying no periodic orbits, [4, 7, 14].

In this notice we address the question of how large the set $\mathcal{AP}_H$ of regular values of H without periodic orbits can be. We refer to this problem as the Hamiltonian Seifert conjecture (see [23, 24]). Below we recall some results and notions concerning existence of periodic orbits and constructions of examples without periodic orbits. We also formulate conjectures on the size of the aperiodic set $\mathcal{AP}_H$. Very little is known on this problem and the conjectures made below are based exclusively on supporting evidence rather than on the lack of examples.

Another subject we briefly touch upon is Hamiltonian systems describing the motion of a charge in a magnetic field. (See [5] for an introduction and a survey of results.) These systems play an important role in the constructions of "counterexamples" to the Hamiltonian Seifert conjecture. A detailed review of these counterexamples in the context of dynamical systems can be found in [8].

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Two relevant questions are entirely omitted from our discussion: the existence of periodic orbits on contact type energy levels (see, e.g., [2, 15, 17, 26, 34]) and the dynamics of the transition from low to high energy levels for twisted geodesic flows (see, e.g., [29] and [5]).

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2. General Existence Results for Periodic Orbits of Hamiltonian Systems

Let W be a symplectic manifold and $H\colon W \to \mathbb{R}$ a proper smooth Hamiltonian. One cannot guarantee that a given regular level of H carries a periodic orbit even when $W = \mathbb{R}^{2n}$, as we will see in section 3. However, for a broad class of symplectic manifolds it is true that periodic orbits exist on almost all levels of H. (Here “almost all” is understood in the sense of measure theory.)

Theorem 2.1. (Almost Existence of Periodic Orbits) *Let P be a compact symplectic manifold with $\pi_2(P) = 0$. Then for $W = P \times \mathbb{R}^{2l}$ with the split symplectic structure, almost all energy levels of a proper smooth Hamiltonian $H\colon W \to \mathbb{R}$ carry periodic orbits.*

This theorem is proved for $W = \mathbb{R}^{2n}$ by Hofer, Zehnder, and Struwe (see [16, 17, 33]) and in the general case by Floer, Hofer, and Viterbo, [2].

Analogues of theorem 2.1 hold for many other manifolds W, for example, for $W = \mathbb{CP}^n$, [15]. (See also [26].) The theorem also holds for $W = P \times D^2(r)$, where $\pi_2(P)$ is not necessarily trivial, provided that H is proper as a function to its image and that the radius $r > 0$ is less than a certain constant $m(P, \omega)$ depending on the symplectic topology of P, [15, 28].

The Hofer-Zehnder capacity c_{HZ} provides a convenient setting for working with the almost existence problem. Let (W, ω) be a symplectic manifold. Consider the class $\mathcal{H}$ of smooth non-negative functions on W such that

1. a function $H \in \mathcal{H}$ vanishes on some open set, depending on H;
2. a function $H \in \mathcal{H}$ assumes its maximal value $m(H)$ on a complement to a compact set, which again may depend on H; and
3. the Hamiltonian flow of $H \in \mathcal{H}$ does not have non-constant periodic orbits of period $T \leq 1$.

By definition, $c_{\mathrm{HZ}}(W, \omega) = \sup m(H)$ over all $H \in \mathcal{H}$. Clearly, c_{HZ} takes values in $(0, \infty]$. The capacity c_{HZ} is monotone with respect to inclusions and homogeneous of degree one with respect to conformal symplectic diffeomorphisms (in contrast with the symplectic volume, which is homogeneous of degree n, where $2n$ is the dimension). In addition, it satisfies the following normalization condition:

$c_{\rm HZ}(D^{2n}(r)) = c_{\rm HZ}(D^2(r)\times\mathbb{R}^{2n-2}) = \pi r^2$, where $D^{2k}(r)$ is the ball of radius r in $\mathbb{R}^{2k}$ equipped with the standard symplectic structure. (The proof of the normalization condition is non-trivial. See [17] for more details and further references.)

In what follows, we say that W has bounded capacity if for every open set $U \subset W$ with compact closure, $c_{\rm HZ}(U) < \infty$. Note that a non-compact manifold W with $c_{\rm HZ}(W) = \infty$ can have bounded capacity (e.g., $W = \mathbb{R}^{2n}$). However, $c_{\rm HZ}(W) < \infty$ does imply, by monotonicity, that W has bounded capacity.

Proposition 2.2. (Hofer-Zehnder, [17]**)** *Assume that W has bounded capacity. Then the almost existence theorem holds for every smooth function H on W such that the sets $\{H \le a\}$ are compact.*

This proposition is a relatively straightforward consequence of the definition of the Hofer-Zehnder capacity, see [17, Section 4.2]. However, proving that W has bounded capacity essentially amounts to showing that periodic orbits exist on a dense set of levels, which is quite non-trivial already for $\mathbb{R}^{2n}$. The manifolds $W = P\times\mathbb{R}^{2n}$ (with $\pi_2(P) = 0$), $\mathbb{CP}^n$, and also $P\times D^2(r)$ (with $r < m(P,\omega)$) have bounded capacity, [2, 16, 17, 15]), as do all manifolds for which the almost existence theorem has been established. (It seems to be unknown whether or not $\mathbb{T}^{2n}$ with the standard symplectic structure has finite capacity for $n > 1$. However, almost existence has been proven for $H\colon \mathbb{T}^{2n}\to\mathbb{R}$ satisfying some additional conditions; see, e.g., [18].)

Almost existence of periodic orbits (theorem 2.1) does not extend to all compact symplectic manifolds:

Example 2.3. (Zehnder's torus, [36]**)** Let $2n \ge 4$. Consider the torus $W = \mathbb{T}^{2n}$ with an irrational translation-invariant symplectic structure $\omega_{\rm irr}$. Choose a Hamiltonian H on W so that every level $\{H = c\}$ with $c \in (0.5, 1.5)$ is the union of two standard embedded tori $\mathbb{T}^{2n-1}\subset\mathbb{T}^{2n}$. Since $\omega_{\rm irr}$ is irrational, the Hamiltonian flow of H on $\mathbb{T}^{2n-1}$ is quasiperiodic. Thus, none of the levels $\{H = c\}$ with $c \in (0.5, 1.5)$ carries a periodic orbit.

According to M. Herman, [12, 13], this phenomenon is stable: the flow of a sufficiently $C^{2n+\delta}$-small perturbation of H still has no periodic orbits in the energy range $(0.6, 1.4)$, provided that $\omega_{\rm irr}$ satisfies a certain Diophantine condition.

As a consequence of proposition 2.2, $c_{\rm HZ}(\mathbb{T}^{2n},\omega_{\rm irr}) = \infty$, even though $\mathbb{T}^{2n}$ is compact. Note also that in Zehnder's example the symplectic form is not exact on the energy levels without periodic orbits. However, many flows for which almost existence has been established (e.g., on $P\times\mathbb{R}^2$) also have this property. Thus, although it feels that non-existence of periodic orbits in Zehnder's example is forced by the topology of $\omega_{\rm irr}$, it is not clear how to make rigorous sense of this statement.

Now we are in a position to state the main question considered in this paper. We call a value a of H *aperiodic* if the level $\{H = a\}$ carries no periodic orbits.

> Let W be a symplectic manifold of bounded capacity and H a smooth proper function on W. How large can the set $\mathcal{AP}_H$ of regular aperiodic values of H be?

As we show in the next section, the set $\mathcal{AP}_H$ can be non-empty for an appropriately chosen function H on $\mathbb{R}^{2n}$, and hence, by Darboux's theorem, on any symplectic manifold.

3. The Hamiltonian Seifert Conjecture

3.1. Results

The *Seifert conjecture* is the question, posed by Seifert in [32], whether or not every non-vanishing vector field on S^3 has a periodic orbit. Of course, in this question the sphere S^3 can be replaced by another manifold, and additional restrictions can be imposed on the vector field. For example, the Seifert conjecture can be formulated for C^1- or C^∞-smooth or real analytic vector fields, divergence-free vector fields, etc. Note that the Seifert conjecture, when interpreted as above, is a question rather than a conjecture. Yet, we will refer to negative answers to this question as counterexamples to the Seifert conjecture.

The Seifert conjecture has a rich history extending for over than forty years. Here we mention only some of the relevant results. The first breakthrough was due to Wilson, [35], who constructed a C^∞-smooth vector field without periodic orbits on S^{2n+1}, $2n+1 \geq 5$. A C^1-smooth non-vanishing vector field without periodic orbits on S^3 was found by Schweitzer, [31], and a C^2-vector field by Harrison, [10]. Finally, a real analytic non-vanishing vector field on S^3 without periodic orbits was constructed by K. Kuperberg, [22]. The reader interested in a detailed discussion and further references should consult [8, 20, 21, 23, 24].

For Hamiltonian flows, the Seifert conjecture can be formulated in a number of ways; see [8]. For example, one may ask if there is a proper smooth function on a given symplectic manifold (e.g., $\mathbb{R}^{2n}$), having a regular aperiodic value, or, more generally, as in the previous section, if there exists a smooth proper function H with a given set $\mathcal{AP}_H$. Here we state and discuss known results and make some conjectures.

Recall that a *characteristic* of a two-form η of rank $(2n-2)$ on a $(2n-1)$-dimensional manifold is an integral curve of the line field formed by the null-spaces $\ker\eta$. The Hamiltonian Seifert conjecture (in a form slightly weaker than considered above) can be stated as whether or not in a given symplectic manifold there exists a regular compact hypersurface without closed characteristics.

Let $i\colon M \hookrightarrow W$ be an embedded smooth compact hypersurface without boundary in a $2n$-dimensional symplectic manifold (W,ω).

Theorem 3.1. ([4, 7, 14]) *Assume that $2n \geq 6$ and that $i^*\omega$ has a finite number of closed characteristics. Then there exists a C^∞-embedding $i'\colon M \hookrightarrow W$, which is C^0-close and isotopic to i, such that ${i'}^*\omega$ has no closed characteristics.*

An irrational ellipsoid M in the standard symplectic vector space $\mathbb{C}^n$ is the unit energy level of the quadratic Hamiltonian $H_0 = \sum \lambda_j |z_j|^2$, where the eigenvalues λ_j are positive and rationally independent. The level M carries exactly n periodic orbits. Applying theorem 3.1, we obtain the following

Corollary 3.2. *For $2n \geq 6$, there exists a C^∞-function $H\colon \mathbb{R}^{2n} \to \mathbb{R}$, C^0-close and isotopic (with a compact support) to H_0, such that the Hamiltonian flow of H has no closed trajectories on the level set $\{H = 1\}$.*

Remark 3.3. The method used in [4, 7] to construct such a function H relies on a Hamiltonian version of Wilson's plug. The horocycle flow (see example 4.1) is employed to interrupt periodic orbits of the flow of H_0 on the level $\{H_0 = 1\}$. The function H differs from H_0 only in n small balls with centers on these periodic orbits. A simple outline of the proof of theorem 3.1, some generalizations, and the list of all known Hamiltonian flows with $\mathcal{AP}_H \neq \emptyset$ can be found in [8]. Here we just mention that irrational ellipsoids are not the only hypersurfaces in $\mathbb{R}^{2n}$ with a finite number of closed characteristics. Examples of such non-simply connected hypersurfaces are found by Laudenbach, [25].

The construction given in [14] uses plugs arising as symplectizations of Wilson's plug for $2n \geq 8$ and Harrison's plug [10] for $2n = 6$. (Hence, the $C^{3-\epsilon}$-smoothness of the flow obtained by this method in dimension six.)

3.2. Conjectures and open problems

Returning to the question of the possible size of the set $\mathcal{AP}_H$, we primarily focus in this section on proper Hamiltonians H on $\mathbb{R}^{2n}$. (See the next section and [8] for results and conjectures for some other symplectic manifolds.) Recall that an upper bound on this set is given by theorem 2.1: the set $\mathcal{AP}_H$ must have zero measure. By theorem 3.1 and corollary 3.2, $\mathcal{AP}_H$ can be non-empty.

There is almost no doubt that the method used in [4, 7] to modify the function H_0 can be applied, literally without any change, to a sequence of energy levels a_k converging to some value a_0. The resulting function H can then be smoothened along the level $\{H = a_0\}$ at the expense of turning a_0 into a critical value of infinite order degeneration. Thus, $\mathcal{AP}_H$ can contain a sequence converging to a singular, infinitely degenerate, value of H.

The next step is more subtle. It appears plausible that the function H_0 can be modified as in the proof of theorem 3.1 simultaneously for all energy values from a compact zero measure set $K \subset (0, \infty)$. The resulting function H will then be smoothly isotopic to H_0 and have K as the set of aperiodic values. This leads to the following

Conjecture 3.4. *For $2n \geq 6$, there exists a C^∞-function $H\colon \mathbb{R}^{2n} \to \mathbb{R}$, C^0-close and isotopic (with a compact support) to H_0, such that the Hamiltonian flow of H has no closed trajectories for energy values in the set K.*

Remark 3.5. This method is unlikely to be applicable to the construction of a function H with a dense set $\mathcal{AP}_H$. The difficulty can be best seen by considering the function H from corollary 3.2. The characteristic foliation of ω on the level $\{H = a\}$, where $a \neq 1$ and $a > 0$, is diffeomorphic to that on $\{H_0 = a\}$. (In fact, the forms $\omega|_{\{H=a\}}$ and $\omega|_{\{H_0=a\}}$ are conformally equivalent.) As a consequence, the flow of H on $\omega|_{\{H=a\}}$ has exactly n periodic orbits and these orbits are non-degenerate and hence stable under small perturbations. The "bifurcation" that happens at $a = 1$ is simply that the periods of these orbits go to infinity as $a \to 1$. Other systems obtained by this method from any function H_0 with non-degenerate orbits will exhibit a similar behavior: periodic orbits on $\{H = a\}$, where $a \notin \mathcal{AP}_H$, will be non-degenerate. On the other hand, for a system with a dense set $\mathcal{AP}_H$ every energy level must have only degenerate periodic orbits.

Remark 3.6. Neither the method from [4, 7] nor from [14] apply to four-dimensional manifolds. There are also serious difficulties in adapting Kuperberg's plug, [22], to the symplectic setting. A C^1-smooth divergence-free vector field on S^3 was found by G. Kuperberg, [20]. It is not known whether or not this vector field can be obtained by a C^2- (or even C^1-) embedding of S^3 into $\mathbb{R}^4$. Such an embedding would give a C^2-smooth "counterexample" to the Hamiltonian Seifert conjecture in dimension four.

4. A Charge in a Magnetic Field

The first example of a Hamiltonian flow with a compact energy level carrying no periodic orbits was found by Hedlund in 1936 when he proved that the horocycle flow is minimal, i.e., every integral curve is dense, [11]. To put Hedlund's result in the context of the Hamiltonian Seifert conjecture, consider a compact Riemannian manifold M equipped with a closed two-form σ. Denote by ω_0 the standard symplectic form on T^*M and by $\pi\colon T^*M \to M$ the natural projection. The form $\omega = \omega_0 + \pi^*\sigma$ on T^*M is symplectic (a twisted symplectic structure). The flow of the standard metric Hamiltonian H (the kinetic energy) on (T^*M, ω) describes the motion of a charge on M in the magnetic field σ. We will refer to this flow as a *twisted geodesic flow*. The behavior of twisted geodesic flows seems to depend on whether σ is allowed to degenerate or not. In what follows we focus entirely on the case where σ is non-degenerate, i.e., symplectic.

The study of twisted geodesic flows by methods of symplectic topology originates from [1], where V. Arnold showed that for $M = \mathbb{T}^2$ with a flat metric and a non-vanishing form σ every level of H carries periodic orbits. This theorem is extended to low energy levels on other compact surfaces in [3, 6]. (See [5] for a survey of related results.) The existence of periodic orbits on low energy levels has also been established for many manifolds M, when $2m = \dim M > 2$. For example, when σ is symplectic and compatible with the metric on M, every low energy level carries at least $\mathrm{CL}(M) + m$ periodic orbits, [19], and at least $\mathrm{SB}(M)$ periodic orbits when the orbits are non-degenerate, [9]. (Here $\mathrm{CL}(M)$ is the cup-length of M and $\mathrm{SB}(M)$ is the sum of Betti numbers of M.) Note also that this problem of existence of periodic orbits can be extended to a broader class of Hamiltonian flows to include the Weinstein-Moser theorem, [9, 19].

The following example shows that the above results do not generalize to all energy levels.

Example 4.1. (The horocycle flow) Let M be a compact surface of genus $g \geq 2$ equipped with a metric of constant negative curvature $K = -1$. Let σ be the area form on M. Then the Hamiltonian flow on the energy level $\{H = 1\}$ has no periodic orbits. In fact, the flow on this energy level is smoothly equivalent to the horocycle flow (see, e.g., [6]), which is minimal by a theorem of Hedlund, [11]. This example has (Hamiltonian) analogues in all even dimensions; see [8, Example 4.2].

It is also known that a neighborhood of M in T^*M with a twisted symplectic structure has finite Hofer-Zehnder capacity under some additional assumptions on M, [27]. Moreover, for some manifolds M, the twisted cotangent bundle has bounded capacity, [9, 18, 27]. This implies almost existence of periodic orbits for a certain class of twisted geodesic flows (e.g., on $\mathbb{T}^{2m}$ with any form σ).

Conjecture 4.2. *For every M and any symplectic form σ, a neighborhood of M in T^*M has finite Hofer-Zehnder capacity. Moreover, almost all levels of H carry periodic orbits. When σ is symplectic, every low energy level of H has a contractible periodic orbit.*

Remark 4.3. This conjecture is supported by some additional evidence; see, e.g., [30]. Interestingly, there seems to be no sufficient evidence that T^*M has bounded Hofer-Zehnder capacity in general. For example, it is not known whether in the setting of example 4.1 the sets $\{H < a\}$ with $a > 1$ have finite capacity.

Example 4.1 is the only known "naturally arising" example of a Hamiltonian flow with a regular compact aperiodic energy level. Furthermore, this is the only known example of a twisted geodesic flow with an aperiodic energy level. It is not known if there exists a twisted geodesic flow with more than one aperiodic energy level.

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