Symplectic and Contact Geometry and Hamiltonian Dynamics

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Abstract. This is an introduction to the contributions by the lecturers at the mini-symposium on symplectic and contact geometry. We present a very general and brief account of the prehistory of the field and give references to some seminal papers and important survey works.

Symplectic geometry is the geometry of a closed nondegenerate two-form on an even-dimensional manifold. Contact geometry is the geometry of a maximally nondegenerate field of tangent hyperplanes on an odd-dimensional manifold. The symplectic structure is fundamental for Hamiltonian dynamics, and in this sense symplectic geometry (and its odd-dimensional counterpart, contact geometry) is as old as classical mechanics. However, the science the present mini-symposium is devoted to is usually believed to date from H. Poincaré's "last geometric theorem" [70] concerning fixed points of area-preserving mappings of an annulus:

Theorem 1. (Poincaré-Birkhoff) *An area-preserving diffeomorphism of an annulus* $\mathbb{S}^1 \times [0;1]$ *possesses at least two fixed points provided that it rotates two boundary circles in opposite directions.*

This theorem proven by G. D. Birkhoff [16] was probably the first statement describing the properties of symplectic manifolds and symplectomorphisms "in large", thereby giving birth to *symplectic topology.*

In the mid 1960s [2, 3] and later in the 1970s ([4, 5, 6] and [7, Appendix 9]), V. I. Arnol'd formulated his famous conjecture generalizing Poincaré's theorem to higher dimensions. This conjecture reads as follows:

Conjecture 2. (Arnol'd) *A flow map* A *of a (possibly nonautonomous) Hamiltonian system of ordinary differential equations on a closed symplectic manifold* M *possesses at least as many fixed points as a smooth function on* M *must have critical points, both "algebraically" and "geometrically".*

The "algebraic" version of this conjecture means that the number of fixed points of A counting multiplicities is no less than the sum of the Betti numbers (over $\mathbb{Z}$) of manifold M. The "geometric" version states that the number of geometrically distinct fixed points of A is no less than the Lyusternik-Schnirel'man category of manifold M. For instance, a flow map of a Hamiltonian system on the torus $\mathbb{T}^{2n}$ possesses at least $2n+1$ geometrically distinct fixed points, and at least 4^n fixed points counting multiplicities. It is worthwhile to emphasize here that the "correct" higher-dimensional generalization of area-preserving two-dimensional mappings in this theory is symplectomorphisms rather than volume-preserving diffeomorphisms. More precisely, Conjecture 0.2 considers symplectomorphisms that are flow maps of (possibly nonautonomous) Hamiltonian systems (such symplectomorphisms are said to be *homological to the identity*).

The Arnol'd conjecture has affected greatly the development of the theory of symplectic manifolds in the subsequent years. The first noticeable step here was Ya. M. Èliashberg's proof [23] of this conjecture for all the two-dimensional surfaces. Of other important achievements in symplectic geometry and topology in the 1970s and early 1980s, one should mention A. Weinstein's results on Lagrangian submanifolds [80] and Èliashberg's theorem [24] on the so-called *rigidity*, or *hardness*, of symplectomorphisms (discussed previously by him and M. L. Gromov since the late 1960s):

Theorem 3. (Èliashberg-Gromov) *The group of symplectomorphisms of a closed symplectic manifold is C^0-closed in the group of all diffeomorphisms.*

Theorem 0.3 is often referred to as "the existence theorem of symplectic topology" [8]. It shows that symplectic geometry is an intrinsically topological science.

In their milestone paper [19], C. C. Conley and E. Zehnder proved Conjecture 0.2 for tori $\mathbb{T}^{2n}$ of all the even dimensions with the standard symplectic structure. They introduced a new technique of constructing a certain action functional on the space of contractible loops on the manifold. This technique can be regarded as a hyperbolic analogue of the Morse theory for positive functionals. Work [19], together with Gromov's celebrated paper [41] on the so-called pseudo-holomorphic curves (two-dimensional submanifolds that are symplectic analogues of geodesics) in a symplectic manifold, marked the beginning of the modern period of symplectic and contact topology, cf. [8]. In particular, Gromov [41] gave a new proof of the rigidity of symplectomorphisms and proved the following fundamental *nonsqueezing theorem*. Let $B^{2n}(R)$ denote the closed ball with center 0 and radius R in $\mathbb{R}^{2n}$ equipped with the standard symplectic structure.

Theorem 4. (Gromov) *There is no symplectic embedding $B^{2n}(R) \hookrightarrow B^2(r) \times \mathbb{R}^{2n-2}$ for $R > r$.*

This theorem shows that the symplectic invariants (called *symplectic capacities*) are essentially two-dimensional.

By now, symplectic/contact geometry/topology and the related aspects of Hamiltonian dynamics have turned into a vast and flourishing branch of mathematics which can definitely not be surveyed during 4.5 hours of the mini-symposium. The contributions collected here should therefore be thought of as just a certain "snapshot" of several active studies and interesting results in the field. Instead of trying to trace the development of the theory of symplectic and contact manifolds since 1985 or reviewing the state of the art, we will present here a brief account of each of the topics selected for the mini-symposium to give them a unity.

Two lectures are devoted to the Arnol'd conjecture discussed above. In the late 1980s, A. Floer published a series of very important papers (of which we cite here only three, [30, 31, 32]) where he, apart from other achievements, combined the variational approach by Conley and Zehnder [19] with Gromov's elliptic methods [41] and defined what has become known as the Floer (co)homology theory. This enabled him to prove Conjecture 0.2 for the so-called *positive*, or *monotone*, symplectic manifolds [32]. Afterwards, Floer's landmark result was generalized by H. Hofer and D. A. Salamon [45] and by K. Ono [69] to *semi-positive*, or *weakly monotone*, manifolds (in particular, to all the symplectic manifolds of dimensions ≤ 6), and by G. C. Lu [58, 59], to products of weakly monotone manifolds (and Calabi-Yau manifolds). Finally, a further extension of Floer's ideas and the theory of the so-called Gromov-Witten invariants have led K. Fukaya-K. Ono, H. Hofer-D. A. Salamon, J. Li-G. Liu-G. Tian, Y. B. Ruan, and B. Siebert to a proof of the Arnol'd conjecture for every closed symplectic manifold (for the case where all the fixed points are nondegenerate), we would confine ourselves with four references [34, 35, 56, 57]. The lecture by Salamon surveys this stream of studies in symplectic topology.

A quite different approach to the Arnol'd conjecture was proposed by B. Fortune [33] who proved it for projective spaces $\mathbb{C}\mathrm{P}^n$ with the standard symplectic structure [7, Appendix 3]. This proof was based on the fact that $\mathbb{C}\mathrm{P}^n$ is the reduced symplectic manifold of $\mathbb{C}^{n+1}$ under the Hopf $\mathbb{S}^1$-action and any Hamiltonian system on $\mathbb{C}\mathrm{P}^n$ is the Marsden-Weinstein reduction of an appropriate Hamiltonian system on $\mathbb{C}^{n+1}$. L. A. Ibort and C. Martínez Ontalba [49] showed that Fortune's method is in fact universal: the fixed point problem for a symplectomorphism (homological to the identity) of every closed symplectic manifold can be translated into a critical point problem with symmetry on loops in the space $\mathbb{R}^{2N}$ (for suitable N) endowed with the standard symplectic structure. All these questions are treated in Ibort's talk.

The lecture by P. Biran considers the interesting problem of *symplectic packing*: given a closed symplectic manifold M of dimension $2n$, what is the supremum $\nu_k(M)$ of volumes that can be filled by symplectic embeddings of k equal disjoint balls $B^{2n}(R)$ into M? This question was first addressed by Gromov [41] as an extension of the nonsqueezing phenomenon: whereas volume-preserving packing is obvious, there do exist obstructions to symplectic packing, and the latter turns out to be highly nontrivial already for the case $n = 2$ [14, 15, 60]. However, for every closed symplectic 4-manifold M with the symplectic structure representing a rational cohomology class, there exists an integer N such that for $k \geq N$, this manifold has a full packing: $\nu_k(M) = \mathrm{Volume}(M)$ [15].

In contrast to these three talks, the lecture by V. M. Zakalyukin is devoted to the *local* problem of generalized caustics. Let M be a symplectic manifold of dimension $2n$, and let functions $f_i\colon M \to \mathbb{R}$, $1 \leq i \leq m$, be independent and pairwise in involution ($m \leq n$). Their common level sets $f = c \in \mathbb{R}^m$ are coisotropic

$(2n-m)$-dimensional submanifolds of M. Given a Lagrangian submanifold $L \hookrightarrow M$, the set of values c for which L is not transversal to the fiber $f = c$ is called a *coisotropic caustic*. The singularities of "conventional" caustics ($m = n$) are well-studied [9], and the talk treats the case $m < n$. The singularities of caustics for $m < n$ were first examined in [81].

The next two lectures pertain to contact topology. H. Geiges' talk considers various constructions of contact manifolds, cf. [36]. A progress in constructing symplectic manifolds is exemplified by R. E. Gompf's method [40]. The lecture by Yu. V. Chekanov deals with Legendrian knots and their invariants. Here the problem is to determine when two topologically isotopic Legendrian knots in a contact 3-space are isotopic through contactomorphisms. For instance, two topologically trivial Legendrian knots can be transformed to each other by contact isotopies if and only if their Thurston-Bennequin invariants and Maslov numbers coincide respectively [27]. For an analogous problem for Lagrangian (two-dimensional) knots in symplectic 4-manifolds see, e.g., [25].

Finally, two lectures are devoted to the "core" Hamiltonian dynamics, to be more precise, to periodic and quasi-periodic motions in autonomous Hamiltonian systems. In the talk by V. L. Ginzburg, the speaker describes his constructions of smooth Hamilton functions $H\colon \mathbb{R}^{2n} \to \mathbb{R}$ such that the Hamiltonian flow afforded by H on the compact energy hypersurface $H = 1$ has no periodic trajectories (the symplectic structure on $\mathbb{R}^{2n}$ is assumed to be standard), see [37, 38, 39]. Such Hamiltonian systems provide counterexamples to the so-called *Hamiltonian Seifert conjecture*. Finally, À. Jorba's lecture studies the complicated "exponential" structure of the set of invariant tori (carrying quasi-periodic motions) near a given one in an analytic autonomous Hamiltonian flow, the relevant reference being [50]. This topic is within the framework of the KAM (Kolmogorov-Arnol'd-Moser) theory concerning quasi-periodic motions in generic dynamical systems.

For the basic ideas of the KAM theory, see [7, Appendix 8]; volume [17] presents a modern survey. Here we would like only to remark that whereas the KAM theory is always local with respect to the *action* variables, its global character with respect to the *angle* variables is best pronounced while considering coisotropic invariant tori of dimensions greater than the number n of degrees of freedom (see a bibliography and discussion in [17]). Indeed, an invariant torus of a Hamiltonian flow or symplectic diffeomorphism is automatically *isotropic* provided that this torus carries a quasi-periodic motion and the symplectic structure on the phase space is exact [17, 42]. Thus, coisotropic invariant KAM tori of dimensions $> n$ can occur for non-exact symplectic structures only (in particular, they are impossible in the local theory, e.g., near equilibrium/fixed points of Hamiltonian systems).

An interplay between a) Gromov's theory of pseudoholomorphic curves and Floer's homology theory and b) examining periodic orbits of Hamiltonian vector fields within energy surfaces is exemplified by paper [46], see also Hofer's plenary lecture [47] at the 23rd International Congress of Mathematicians.

As was already emphasized, the present mini-symposium covers unavoidably only a small fraction of modern symplectic and contact geometry and topology. Of the significant missed achievements, we would mention here only C. H. Taubes' results on a precise relation between the Seiberg-Witten invariants and the Gromov invariants for closed symplectic 4-manifolds [73, 74, 75, 76, 77, 78] (see also [20]) and S. K. Donaldson's works on symplectic Lefschetz pencils [21, 22] (see also [11]).

Of survey monographs on the field, one should first of all mention influential books [43, 62] as well as advanced textbooks [1, 12]. Monographs [26, 61] are devoted to special topics. Important contributions can be found in collections [10, 13, 18, 28, 29, 44, 48, 51, 55, 71, 72, 79]. Finally, we would like specifically to draw the reader's attention to the stimulating reviews of the field by F. Lalonde [52, 53, 54] and D. McDuff [63, 64, 65, 66, 67, 68].

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