Gender and Mathematics Education

an ICMI Study in Stiftsgården Åkersberg, Höör, Sweden 1993

Edited by
Barbro Grevholm
Gila Hanna

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CONTENTS

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FOREWORD

This volume brings together a number of different perspectives on the part played by gender in mathematics education. In the past two decades there has been an upsurge of interest in educational issues related to gender and mathematics education, a greater understanding of these issues, and a resulting desire on the part of both men and women to redress the welldocumented gender imbalance in mathematics learning and in mathematics-related careers. Because these issues are common to all countries, whether developed or underdeveloped, an international gathering of researchers seemed particularly appropriate for a fruitful exchange of ideas on mathematics education and gender. This volume represents the proceedings of that conference, held in Höör, Sweden, on 7-12 October 1993.

The conference was part of a series of studies organized by the International Commission on Mathematical Instruction (ICMI) and designed to address key issues in mathematical education. It is particularly gratifying that ICMI, in dedicating one of its studies to gender issues, has assigned to these issues a position of prominence. This sends a strong message to the international mathematics education community that gender inequity is a serious educational issue, one that needs to be addressed with some degree of urgency.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

In preparation for this conference a call for papers was published, together with a discussion document setting out the scientific agenda, in a dozen scholarly mathematics education journals and in the newsletters of prominent national and international mathematics education organisations. The call for papers attracted a distinguished group of mathematicians and mathematics educators eager to discuss gender issues, some quite controversial, from the historical, philosophical, social and educational perspectives. Most of the participants were members of two very active organisations devoted to promoting women's participation in mathematics: The International Organisation of Women and Mathematics Education (IOWME) and the Association for Women in Mathematics (AWM). The conference was also very successful in attracting younger researchers and graduate students thus providing a forum for discussions which engaged scholars of all ages.

The present volume of proceedings contains, plenary speeches, a set of valuable research papers, summaries of panel discussions and reports from working groups.

Acknowledgments:

We wish to express our thanks to the members of the international program committee, Carlos Bosch, Geoffrey Howson, Christine Keitel-Kreidt, Gilah Leder, and Mogens Niss. The work of the local organising committee was also absolutely essential to the success of the conference, and we wish to express our sincere gratitude to Gerd Brandell, Bengt Johansson, Lisbeth Lindberg, Bo Rosén, Bo Sjöström, and Göran Wanby.

We also wish to express our thanks to the following bodies for their most valuable financial support:

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Barbro Grevholm and Gila Hanna

AN ICMI STUDY ON GENDER AND MATHEMATICS EDUCATION: KEY ISSUES AND QUESTIONS

Gila Hanna
Joyce Nyhof-Young
The Ontario Institute for Studies in Education,Toronto

Rationale for the Study

The study proposed in this discussion paper is based on a simple premise: there is no physical or intellectual barrier to the participation of women in mathematics, science, or technology. Having said this, we must ask ourselves: why don't they participate more? Here there is no simple explanation. For if there are no physical or intellectual barriers, there must be social and cultural barriers that account for their underrepresentation. For the most part, these barriers have not been raised intentionally. They are an integral part of a social order that carries with it discrimination. The perspective of this study is that discrimination on the basis of gender is no longer acceptable. Judge Rosalie S. Abella, an advisor to the Ontario government, has posed the problem as follows:

> Systemic discrimination requires systemic remedies. Rather than approaching discrimination from the perspective of the perpetrator and the single victim, the systemic approach acknowledges that by and large the systems and practices we customarily and often unwittingly adopt may have an unjustifiably negative effect on certain groups of society. The effect of the system on the individual or group, rather than its attitudinal sources, governs whether or not a remedy is justified.
>
> Remedial measures of a systemic and systematic kind are meant to improve the situation for individuals who, by virtue of belonging to and being identified with a particular group, find themselves unfairly and adversely affected by certain systems of practices. (CAUT, 1991, p. 12)

Statistics on the participation of women at the tertiary level in general and in mathematics, science, and technology in particular strengthen the case for a social, systemic viewpoint. We have to ask why women specifically avoid mathematics and sciences. Taking Canadian data as an example, we note that although women are attending universities in unprecedented numbers (and earning more than 50% of all bachelor's degrees in Canada), they are overrepresented in the humanities and underrepresented in mathematics and science. The proportion of women undergraduate students in the mathematical and physical sciences increased from 19.4% to 28.5% in the years 1971-87, and in engineering and applied science it

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

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increased from 1.2% to 12.2%. This constitutes very modest progress, when one compares it to the progress women have made as students in other traditionally male-dominated professions. Over the same period (1971-87), the proportion of women among those obtaining a bachelor's degree in law increased from 9.4% to 46.7%, while the proportion in medicine went from 12.8% to 41.7%. At the doctoral level, though women have increased their participation they are still underrepresented in mathematics and science.

Two decades of research on the problem of gender imbalance in higher mathematics, and in mathematics-related careers, have consistently found that when gender-related differences in achievement are present they are rather small. Or put in other terms, achievement *per se* does not account for the large discrepancies in enrolment in higher level mathematics courses and in the election of mathematics-related careers. This finding is perplexing in light of what we find in the media on girls and mathematics and science.

In the United States and Canada, and perhaps in Europe too, a lot of publicity has been given to their supposed inferiority in these subjects. Articles have appeared in popular magazines claiming that women are inferior in what they have referred to as "cognitive abilities", "spatial skills", or "aptitude for mathematics". It has also been claimed that women are incapable of grasping mathematics or science because they are "emotionally minded". It is hardly surprising that such messages in the popular press influence girls to believe in their inherent inability to succeed in mathematics, and thus discourage them from taking up mathematics or other branches of science.

Such claims are usually based upon studies of achievement. Yet, as stated above, most studies that have found achievement differences in favour of boys have found very small differences that are not educationally significant. The more important point is that the popular press, and indeed many of the researchers, have confounded achievement with aptitude, ignoring other factors. The truth is that we do not really know how to measure aptitude, or even whether aptitude alone is a determining factor in achievement. Some research suggests that learners' attitudes towards learning and their career aspirations are powerful determinants of achievement.

Whereas studies that show lower achievement for girls often receive wide publicity, studies that show the opposite may not. Research on the International Educational Association (IEA) mathematics results from 20 countries at the Grade 8 level (age 13) shows that boys and girls are about equal in achievement, and that the differences among countries are much larger than any differences within countries (Hanna, 1989).

Another study that challenges the popular notion of girls and lower mathematics achievement is one by Alan Feingold (1988). In reviewing the research results on cognitive gender differences for a period of 30 years in the United

States, Feingold shows that differences had actually declined over the three decades preceding his study. Clearly the research message is that the problem of gender differences and mathematics achievement, and of gender-based inequities in mathematics-related careers, is a socially constructed one.

At the same time numerous studies have been done that indicate what can be done at the level of societies and of education systems to counteract the development of gender inequities. This discussion paper is an attempt to summarize key questions in one segment of the literature on retaining girls and women in mathematics and science – namely, analyses of gender issues in mathematics education. It is hoped that the identification of the relevant questions will focus attention on key gender-related issues in mathematics education for the 1990s and beyond.

Factors Creating Gender Inequities in Mathematics

Attitudes

Femininity and masculinity are socially developed constructs that are reinforced by the interactions of children with each other and with adults. Implicit and explicit assumptions and messages about female and male intelligence, needs, and inclinations seem to affect attainment in mathematics. To a certain extent, gender differences in mathematics performance might be a reflection of differences in attitudes towards mathematics.

Girls tend to avoid mathematics courses when they are no longer compulsory. It appears that the attitudes females have towards mathematics, their feelings as learners of the subject, and the values that shape their attitudes determine whether or not they persist in mathematics course-taking. Girls who are aware that mathematics will be relevant to their lives and useful in their future careers are far more likely to remain in mathematics courses.

The larger question in this context pertains to socialization. What is its role in the observed differences in attitudes towards mathematics? More specifically, the following questions are helpful:

- Is there an implicit message in society that competence in mathematics is more important for the attainment of boys' career ambitions than it is for girls?
- How can we increase the confidence of females in their ability to do mathematics?
- Do specific teaching approaches and learning modes lead to more positive attitudes to mathematics?
- How does understanding the similarities between male and female achievement and attitudes help practitioners establish a basis for resolving inequities?

Culture

Ethnomathematics recognizes the influence of sociocultural factors on the teaching and learning of mathematics. Documentation exists that the emphasis placed within schools on the application of mathematics differs markedly within countries and from country to country and that this emphasis affects student performance. We have much to learn from this research, especially if we include consideration of the following additional questions:

- How informative are or what do we have to learn from international performance comparisons?
- Are there cultural patterns, such as social customs, family customs, customs in our educational system, and customs specific to mathematics, that discourage girls and women from pursuing mathematics?
- What difficulties in mathematics do females and males from minority groups face?
- What methods of encouraging, recruiting, and retaining women and minorities are used by different cultural and national groups?

Mathematics as a Discipline

Recently, the existence of gender biases in the practice of mathematics has been studied extensively from several different perspectives, including a feminist one. The questions emanating from this line of research are worth examining. Some essential questions are:

- What are the consequences in the theory and discourse of mathematics of the fact that it was constructed in predominantly patriarchal societies?
- Does the nature/structure/language of mathematics have a bias that promotes gender imbalances?
- What is the nature of the different areas of mathematics that appears to encourage (or not, as the case may be) students to persevere?
- What features of mathematics as a discipline (e.g., the contribution it can make to developing creativity and enjoyment, and its value in developing reasoning powers) can be emphasized to make it more relevant to both genders?

Manifestations of Gender Inequities

Jobs and Careers

Historically women have been seriously underrepresented in mathematics and related fields. This does not appear to be due to lower levels of achievement. Gender-related differences in mathematics achievement, when they are found, are very small and thus do not account for these large participation discrepancies. Even though more women have chosen to pursue careers in mathematics and science in the past decade, there is still a concern over their low representation in mathematics, engineering, and the natural sciences.

Educators need to pursue an understanding of the factors that account for the discrepancies in involvement in higher level mathematics courses and to develop strategies that will help both genders stay in mathematics courses and thus keep open the full spectrum of career and job options. Research still needs to be done around the following questions:

- Do social perceptions (media publicity, etc.) discourage girls from choosing careers that require mathematical skills?
- How can (female) students be helped to see that mathematics can also contribute to the solution of problems they will meet out of school and to job opportunities?
- Should the privileged position of mathematics as a screening device for professions be challenged?
- Why hasn't the preparation in mathematics translated into greater numbers of female science and engineering majors?
- How can the visible proportion of women in mathematics and related fields be increased so that these options and occupations become part of female students' accepted range of choices?
- How can women's opportunities for careers in scientific and technical professions be expanded? Conversely, should women go into mathematicsrelated fields, given the nature of the present system?

Girls and Technology

The technological environment can, and does, affect student attitudes and their conceptions of what comprises desirable knowledge and understanding. In 1990, Ursula Franklin noted that the practices used in technology define its content and "when certain technologies and tools are predominantly used by men, then maleness becomes part of the definitions of those technologies." As a result, many female students do not appear to hold a worldview that includes technology as relevant to their lives or as appropriate for them.

Few educators would disagree that schools must be more responsive to the science/technology thrust of our contemporary world and to the related educational needs of all students. However, international investigations have noted consistent gender inequalities in the technological education. Important questions for educators to discuss include:

- How does the considerable and growing impact of technology on schools and its changing role affect the education of females?
- How can we foresee and influence how technology changes their education?
- Can we influence the designers and producers of technology, and hence how girls are educated, by setting technological goals (e.g., development of technical hardware for educational purposes)?
- How are the areas of computer studies and mathematics to be made more relevant/accessible to girls?
- How can the computer be used as a learning and teaching aid? What are the effects of certain implementations on the cognitive development of the learner?

- What are epistemological changes due to the use of computers?

Foci for Change

Curriculum

To achieve gender equality in mathematics education, educators need to look at the development, content, and presentation of the mathematics curriculum within its general educational context.

In this regard it is helpful to find examples of success in teaching mathematics to all students (and to be aware of criteria used to denote the term "success") and to learn from these successes. Some worthwhile questions for consideration are:

- Given the pattern of lower rates of female participation in elective math-ematics courses, and the fact that mathematics is critical to careers at technical, professional, and managerial levels, to what extent would it be appropriate to make mathematics a compulsory subject in schools?
- What would a gender neutral curriculum and pedagogy look like?
- Would single-sex education benefit students who tend to opt out of mathematics?
- Should different mathematics curricula be provided for different groups of students?
- Does the mathematics curriculum fail to deal with topics of particular concern to girls and women?
- Why do specific mathematics topics seem easier to one group of students than another?
- What are the essentials which must be contained in mathematics curricula?
- How can the different components of curriculum – instructional methods, assessment programs, and resources produced by teachers and by publishers – be designed so that the development of mathematics skills and knowledge becomes a prime aim for all children?
- How can the pace and range of work in the mathematics classroom be adapted to allow for increased understanding by all students?
- Does the mathematics curriculum necessarily have to be so overloaded that the quantity tends to control the pedagogy?

Assessment

Assessment is a crucial component of mathematics education. It generally functions to provide information to assist in decision making about individual students, classes, teachers, programs, or institutions. The kind of information sought, how it is gathered, and the form in which it is reported, all have a bearing on mathematics education.

Major challenges and questions exist within the realm of assessment as it relates to gender issues. A critical question, for example, is whether mathematics is taught equally well to different groups of learners. Important queries within this larger question include:

- What is mathematical ability and how can it be measured?
- What kinds of mathematical tasks are being assessed (short technical exercises, long tasks, extended problems, etc.)?
- Are the methods of assessment used more favourable to certain groups of students?
- How can we ensure that classroom materials and exam questions properly reflect gender equity? Should they include a wider range of human activities and interests than traditional materials and examinations?
- Is the range of experiences provided in the mathematics classroom (or elsewhere in the school) biased in favour of one group of students to the possible detriment of others?
- Are there examples of assessment practices that are known to have a positive or negative influence on instruction? What aspects should be maintained and encouraged?
- Are there examples of assessment practices that negatively influence instruction; for example, by focussing instruction on assessment and tests rather than on more general goals?
- How do different assessment modes influence the social environment in the classroom?

Teachers and the School

Teachers are one of the most important educational influences on students' learning of mathematics. The school environment or social context in which students learn mathematics is another critical factor, influencing how they learn, their expectations, their perceptions and misapprehension of mathematics and of schooling in general. More research is needed on how the ethos of the school and individual teachers shape or alter student attitudes towards mathematics.

With respect to teacher education, the general question remains of how to make teachers at all levels aware of, and hence eliminate, any gender bias in their current practices. More specifically, we need to ask the following questions:

- Do we need to improve inservice training? Should we increase incentives to groups to participate and the amount of time that we spend on the topic of gender awareness?
- Should more research be focussed on teachers – their conceptions of their roles both in the classroom and in society, their understanding of the educational process, their methods and teaching aids?

Research has been done on the critical factors in the school environment that reduce retention of females in mathematics courses. We need to continue to ask:

- How can pupils' (particularly girls') self-confidence in mathematics be increased?

- How can the learning climate for girls be improved?
- Does the learning climate for girls improve within single sex settings?
- How can the modes of classroom organization and teacher-pupil interactions that would benefit all children be encouraged and developed ?

Working with Parents

Sex-role stereotyping begins at birth, a fact alluded to in the earlier discussion of attitudes and the different socialization patterns of girls and boys in our culture. This stereotyping is reinforced as the child progresses through school by the differential expectations and treatment of boys and girls by teachers, counsellors, parents, peers, and also through instructional materials and the media. It is known that parents and educators can intervene to modify the influence of sex-role stereotyping and to provide an equitable education for all students.

As well as looking at the gender factor, researchers have studied how parental educational and occupational level affects their children's mathematics learning. And so the basic public and community issues pertain to how the dual disadvantages of sex-role stereotyping and social class can be overcome. More specific questions include:

- How can parents be sensitized to ways they can encourage and support their children in math/science fields?
- How can public awareness be increased, especially among parents, teachers, and counsellors, of the advantages of mathematics-related careers for women and their achievements in mathematics?
- How can schools take responsibility for informing the community about the importance of girls' participation in mathematics?
- How can the commitment of national and local governments to supporting mathematics education for girls and women be increased?

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Note: We wish to thank Leone Burton for suggesting five of the questions included in this paper.

OPENING ADDRESS

Kerstin Mattsson
National Agency for Education, Stockholm

Introduction

I am honoured to be asked to introduce this conference, arranged by the programme committee for the ICMI study on gender and mathematics education.

First, I want to welcome you to Sweden and to some interesting and hopefully challenging and brainstorming days. My welcome is not only my own – it is also a welcome on behalf of the Swedish National Agency for Education.

Purpose of the Conference and Its Importance

The whole conference has a clear focus – gender and mathematics education – and the point of departure is clear as well and articulated in the discussion material sent out: there is no physical or intellectual barrier to the participation of women in mathematics, science, or technology.

With these words in mind, there is no need for further justification of the conference. Nevertheless, I would like to give some personal reflections on the importance of your work and I will also try to provide a Swedish context for the conference. I will do this in a rather general way.

As I dare not delve too deeply into matters before so many experts, I will just try to give some additional support to your own conviction – that your task is important.

The Importance of Mathematics Education

The first question to answer is, why is mathematics education important? The second question to answer is, why is gender and mathematics education important? I assume that you all agree that the answers to these questions must contain the identification of the problems – presently and traditionally – connected to the subject.

First, then, the importance of mathematics education. Why is mathematics considered to be one of the crucial school subjects throughout the world? Why is it obviously so troublesome to most countries if they show bad figures in comparative studies in mathematics? Why is there an almost schizophrenic attitude connected to mathematics – as well as science – not only among many teachers? On one hand, everyone must and can learn maths; on the other hand, mathematics is and should be the most difficult subject – something for the best and most clever.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

Of course, there are several explanations.

The first reason is the traditional role of mathematics in schools – mathematics as the prime tool for developing a disciplined and logical thinking. One hundred and fifty years ago there was an intense fight over the status of Latin in schools. The fight did not end until 50 years later but by then mathematics – as well as science, perhaps – had won. No one was any longer convinced that Latin made students brilliant, disciplined, and logical. In fact, very few were convinced that any subject could have that impressive transfer effect. Nevertheless, when we look back, we can see – and have much evidence – that mathematics actually took over Latin's role. The value of mathematics education was much greater than merely to teach students to count and solve problems. Mathematics education in its simpler forms – counting – was for all children, but mathematics education in its deeper meaning was for the elite, just as Latin had been. The importance of mathematics education in this sense has, of course, been for good and for bad. The status of mathematics education has not only manifested itself in much time in the school timetables and in resources for research. It has made teachers self-confident; they have not been forced to fight for or justify their subject, at least not to the same degree as many others. However, I am convinced that the absence of fight has also been negative. It has helped to make mathematics education, generally, a subject not deeply discussed; it has helped to cover problems.The second answer to the question why mathematics education is important is that mathematics, as well as spoken and written language, is the basis for all communication. Without this basis it is very difficult to live a social, professional or private life as human beings. In our rather complex modern society, the demands for communicative qualifications are high for all of us, not only for a small elite. The communicative subjects are also important tools for all other school subjects. However, as you are experts on this topic, I don't have to mention it any further.

The third answer to the question has to do with the students. In one sense, and from my point of view, it is maybe the most important one. You could perhaps claim that no other subject has that power to influence students' self-concept. Mathematics education starts early in the school career. It is easy to identify as a school subject. Most children learn to count. Some have problems and must have extra support from the teachers. However, the consequences of their failures or successes are wider than results on tests, on marks, or their interest for further studies in the specific subject and ambitions. Mathematics education has an extraordinary power to teach the students who they are. It helps the student learn that "I have no problems in school, I'm stupid, I'm intelligent, I'm the kind that will leave school as soon as possible," and so forth. The meta-learning power of mathematics education is connected to the former answers – to the symbolic, cultural, and real value of mathematics. However, it also puts a heavy responsibility on teachers and researchers in mathematics education. All subjects are important, as are national, local, and individual results on all school subjects. However, when there are problems in mathematics education, when it fails to cope with the problems and difficulties so common in most subjects, one could argue that it is more alarming. Because of its symbolic, traditional, and real power mathematics education has probably a greater power than most subjects to differentiate the students not only by merits and aptitudes but by social background and gender. Beside the depressing figures of a considerable proportion of individuals with low self-esteem, we make students narrow their career ambitions more than they need.

The Specific Focus: Gender and Mathematics

With this short background I have tried to give the background to the second question: why is gender and mathematics education important. Mathematics as a school subject seldom attracts women, at least at higher levels. Few women choose a vocational career that includes mathematics or science. Of course, much has happened during the past decades, but still there are obvious patterns.

This conference will deal with the question of why it is so and how to cope with it. I will shortly deal with the question, "Why is this well-known pattern important." As a matter of fact, one could argue that women choose other subjects or careers as important. Without nurses, experts in social sciences, and other "female" careers and occupations we would have other problems. Irrespective of this, the gender-bound pattern is disturbing.

The demands for qualifications in all communication subjects is increasing for all kinds of careers. It will problably be even more difficult to eliminate mathematics in any career or occupation. Up to now a majority of women have been able to leave their studies in mathematics at an early stage in their career. In the same way many young men have left their studies in foreign languages, and too often in mathematics. If school continues to produce women and men with low self-esteem and bad results in these basic subjects it will cost too much, to the individual and to society.

Few women, at least in Sweden, choose a science career. The Swedish government has given strong attention to this and has also taken some initiatives to influence both education and students. No one believes in short-term campaigns any longer. We have tried that before, sometimes with instant effect, sometimes with no effect, but never with any lasting effect. Again, we must focus the teaching process and what happens in the schools. Again we must focus on mathematics – the subject with its traditional, symbolic, and real value and its meta-learning power. With the problems unsolved we could easily argue that a high proportion of women at least have no real choice of career. Too many have learnt that mathematics is not for them, that they must have or develop other ambitions. This makes this conference very important.

The Context of the Conference

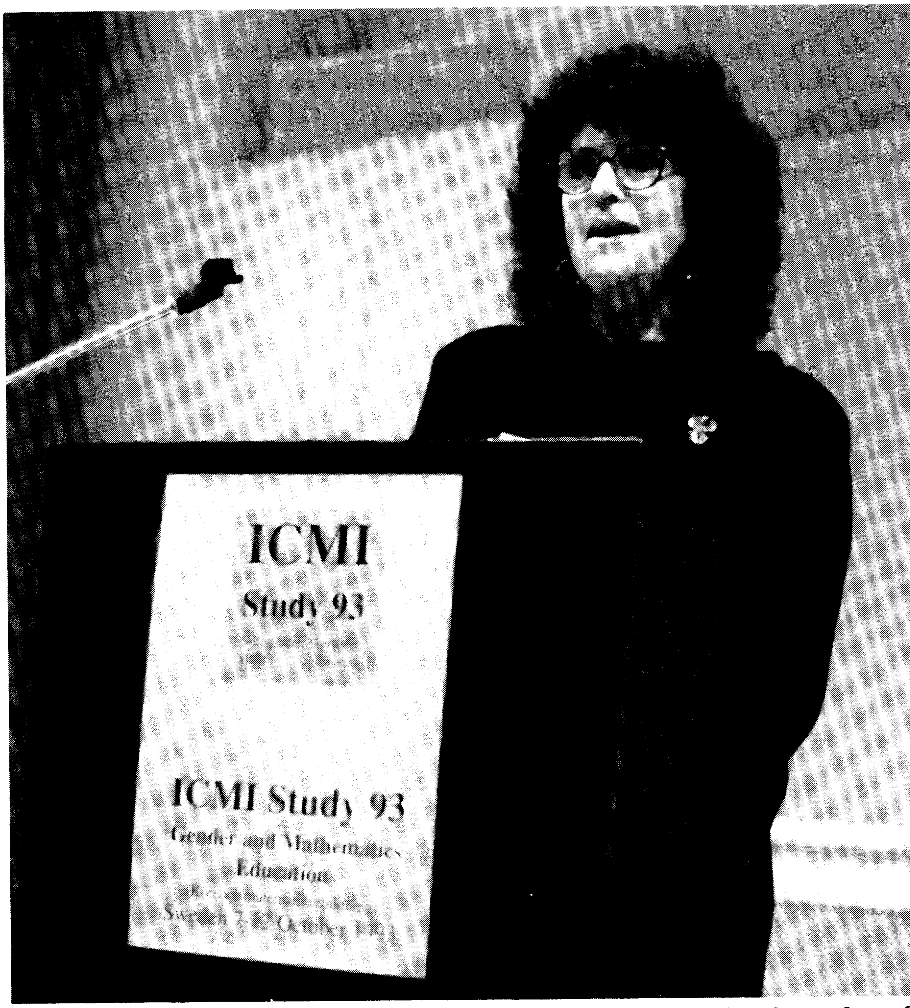
There are several other reasons why the topic of this conference is important. In Sweden, as in many other countries, the focus is on education and the need for change. The reform process on all levels in Sweden – the comprehensive as well as higher levels – has its parallell in many other countries. A reform process can mean a fundamental challenge and a source of inspiration to those involved. We also know that a nationally initiated reform process can become something much less in the classroom; it can mean minor adjustments rather than stimulating rethinking. What will happen depends on several factors. It depends on how the teachers themselves feel the need for change. However, it also depends on how different agents give input for change. There is a strong need for curricula that encourages rethinking; there is a strong need for new perspectives.

Sweden has for long had a rule-governed and highly centralised education system. We have started a process towards a decentralised system. Formerly every major reform process was defined and organised from the top, from the government and the national school authorities. In the long run we created a system that became too stable and too difficult to change. There was a great distance between those who planned the reform – its content and how to implement it – and those who in fact were responsible for something happening in the classroom. We have left this system. The responsibility of the teachers and school leaders is clearer. The national agency can no longer dictate how to implement reform in the classroom. There is a greater freedom for the teachers to find solutions that are realistic, adapted to the local circumstances of the different groups of students. This puts heavy demands on the teachers. They need support. For the curriculum work on the national level we need new ideas and perspectives. For the teachers, responsible for something to happen in the school and in the classrooms, it is even more important.

Final Remarks

This is an important conference. And as a respresentative of the Swedish National Agency for Education and as the one in the agency who is responsible for the agency's programme for development of education, I expect much of this meeting, as the results of discussions, workshops, seminars, and so forth. We know that much is still to be done. We know that boys and girls are made differently and that there are different criteria for evaluating their results. We know that the gaps between the poor achievers and high achievers, between the good schools and the bad schools, are too great. We know that interaction between teacher and student is gender bound, that it is difficult for teachers to see that they behave differently towards boys and girls, and that it is difficult for them to see what to do. Gender and education concerns and includes both the teachers, the girls, and the boys. It is not a "girl-problem." It should not be a problem only for conscientious female teachers.

The basic, important questions have already been formulated in the discussion document for the conference. Some of the questions are, in a way, not possible to answer – only to discuss and try to cope with. Others can be answered now or later. If some of the questions will be given an answer during these days and others will be documented in some more elaborated form, you have taken an important step forward and we will be able to bring some stimulating ideas into our own work – hopefully also into the discussion going on much closer to the classroom level – thereby initiating a true reform process.



Kerstin Mattson *Photo: Lennart Jonson ©*

MATHEMATICS, GENDER AND RESEARCH

Elizabeth Fennema
University of Wisconsin-Madison

Introduction

Research! What is it? Is it important? What knowledge about mathematics and gender does research contribute? And perhaps most importantly, can research provide new and different insights into the complex relationships between gender and mathematics? If we achieve this understanding, can we help females achieve equity in mathematics? Or are issues related to gender so value-laden and embedded within our environment that an understanding of gender and mathematics can be attained only in relation to personal experience? Can change in mathematics education vis-a-vis gender only occur when society changes? The answers to these questions are complex and probably they cannot be answered at all. However, I firmly believe that research can contribute to understanding gender and mathematics and that this understanding will help in achieving equity. The purpose of this paper is to report some of what we have learned from research and to speculate about what research can help us to learn in the future about the complexity of the interactions between gender and mathematics.

My own value positions influence strongly what I am going to say. This is nothing new. My entire professional career has been predicated on the belief that women deserve equity with men in all walks of life, and that belief has informed a significant part of my scholarly activities, particularly in the area of gender and mathematics. I have always believed that I can learn how to better facilitate the learning of mathematics by females through research.

I also believe that there are routes other than research for gaining knowledge about how to help females achieve equity. Many others have studied gender and mathematics and/or developed interventions designed to assist females in learning mathematics. Their work has greatly enriched my work and I am indebted to them. However, this paper is focused on what I know best; that portion of my professional work concerned with mathematics and gender.

Research, Mathematics and Gender

In 1974, my first article about gender was published in the *Journal of Research in Mathematics Education* (Fennema, 1974). In this article, which was a

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

review of extant work that had been done on sex differences in mathematics, I concluded that while many articles had been poorly analyzed and/or included sexist interpretations, there was evidence to support the idea that there were differences between girls' and boys' learning of mathematics, particularly in items that required complex reasoning; that the differences increased at about the onset of adolescence; and that these differences were recognized by many leading mathematics educators. As an aside, it was really the writing of that 1974 article that turned me into an active feminist, compelling me to recognize the bias that existed toward females, which was exemplified by the recognition and acceptance by the mathematics education community at large of gender differences in mathematics as legitimate.

The Fennema-Sherman studies (Fennema & Sherman, 1977; 1978; Sherman & Fennema, 1977), sponsored by the National Science Foundation and published in the mid-1970s, documented sex-related differences in achievement and participation in Grades 6-12. Although there were many subtle results, these results basically agreed with those of my original review with respect to gender differences in learning. In addition, Sherman and I found differences in the election of advanced level mathematics courses by males and females. When we coupled the achievement differences with the differential course enrollment, we hypothesized that if we could encourage females to participate in advanced mathematics classes at the same rate that males did, gender differences would disappear. Many things are learned as one does research, and from the stating of this hypothesis I learned that what you write and say can stay with you a long time. This hypothesis, labeled as the **differential course-taking hypothesis**, became a point of attack by Julian Stanley and Camilla Benbow (Stanley & Benbow, 1980), who used their interpretations of some of their studies as a refutation of our hypothesis. They then used their work as evidence that gender differences in mathematics are genetic. Although widely attacked and disproved, the publication of their claims in the public media did have unfortunate repercussions (Jacobs & Eccles, 1985).

Affective or attitudinal variables were also examined in the Fennema-Sherman studies. Identified as critical were beliefs about Usefulness of mathematics and Confidence in learning mathematics, with males providing evidence that they were more confident about learning mathematics than were females, and males believing that mathematics was and would be more useful to them than did females. It also became clear that while young men did not strongly stereotype mathematics as a male domain, they did believe much more strongly than did young women that mathematics was more appropriate for males than for females. The importance of these variables, their long-term influence, and their differential impact on females and males was reconfirmed in many of our later studies, as well as by the work of many others (Leder, 1992).

One cognitive variable also studied in the Fennema-Sherman studies was spatial skills or spatial visualization, which I continued to investigate in a three-year longitudinal study in collaboration with Lindsay Tartre (Fennema & Tartre, 1985). Differences between females and males in spatial skills, particularly spatial visualization or the ability to visualize movements of geometric figures in one's mind, have long been reported. (Maccoby & Jacklin, 1974). Since items that measure spatial visualization are so logically related to mathematics, it has always appeared reasonable to believe that spatial skills contributed to gender differences in mathematics. We found that while spatial visualization is positively correlated with mathematics achievement (which does not indicate causation), not all girls are handicapped by inadequate spatial skills, but perhaps only those girls who score very low on spatial skills tests.

The Fennema-Sherman studies have had a major impact, although they were not particularly innovative, nor did they offer insights that others were not suggesting. However, they were published in highly accessible journals just when the concern with gender and mathematics was growing internationally. Partly because the studies were accessible and not generally controversial and because they employed fairly traditional methodology, their findings have been accepted by the community at large, and many have used them as guidelines for planning interventions and other research. The studies have been identified by two independent groups (Walberg & Haertel, 1992; Anon., in progress) as among the most quoted social science and educational research studies during the last two decades. Each week, I still receive at least one request for information about the Fennema-Sherman Mathematics Attitude Scales, which were developed for those studies. The research reported in these studies, in conjunction with the research of others, has had a major impact. The problems of gender and mathematics were defined and documented in terms of the study of advanced mathematics courses, the learning of mathematics, and certain related variables that appeared relevant both to students' election of courses and learning of mathematics.

After completing the Fennema-Sherman studies, with the indispensable aid of many others (Laurie Reyes Hart, Peter Kloosterman, Mary Koehler, Margaret Meyer, Penelope Peterson, and Lindsay Tartre), I broadened my area of investigation to include other educational variables, particularly teachers, classrooms, and classroom organizations. We studied teacher-student interactions, teacher and student behaviors, and characteristics of classrooms and teaching behaviors which had been believed to facilitate females' learning of mathematics.

The series of studies dealing with educational variables, reported and summarized in the book Gilah Leder and I edited (Fennema & Leder, 1990), suggested that it is relatively easy to identify differential teacher interactions with girls and boys: in particular, teachers interact more with boys than with girls, praise and scold boys more than girls, and call on boys more than girls. However, the impact of this differential treatment is unclear and difficult to ascertain. The data that resulted from the studies do not support the premise that differential teacher treatment of boys and girls causes gender differences in mathematics. This conclusion has also been reached by others (Koehler, 1990; Leder, 1982; Eccles & Bluemenfeld, 1985). In 1993, there still is not sufficient evidence to allow us to conclude that interacting more or differently with girls and boys is a major contributor to the development of gender differences in mathematics.

Many intervention programs have been designed to help teachers recognize how they treat boys and girls differently. Unfortunately, such programs do not appear to have been successful in achieving the elimination of gender differences in mathematics. I believe that differential teacher treatment of boys and girls is merely a symptom of many other causes of gender differences in mathematics and that, as in medical practice, treating the symptom is not sufficient to change the underlying cause.

Identifying behaviors in classrooms that influence gender differences in learning and patterns in how students elect to study mathematics has been difficult. Factors that many believed to be self-evident have not been shown to be particularly important. Consider sexist behaviors, such as those indicating that mathematics is more important for boys than for girls. No one would deny that such behaviors exist. However, Peterson and I (Peterson & Fennema, 1985; Fennema & Peterson, 1986) did not find major examples of overall sexist behaviors on the part of teachers, but rather small differences in teacher behavior, which, when combined with the organization of instruction, made up a pattern of classroom organization that appeared to favor males. We also found patterns of teacher behavior and classroom organization that influenced boys and girls differently. For example, competitive activities encouraged boys' learning and had a negative influence on girls' learning, while the opposite was true with cooperative learning. Since competitive activities were much more prevalent than cooperative activities, it appeared that classrooms we studied were more often favorable to boys' learning than to girls' learning.

In connection with this series of studies, Peterson and I proposed the Autonomous Learning Behaviors model, which suggested that because of societal influences (of which teachers and classrooms were main components) and personal belief systems (lowered confidence, attributional style, belief in usefulness), females do not participate in learning activities that enable them to become independent learners of mathematics (Fennema & Peterson, 1985). This model still appears valid, although my understanding of what independence is has grown and I believe that independence in mathematical thinking may be learned through working in cooperation to solve mathematical problems.

Intervention Studies.

By about 1980, there were some rather consistent findings from research on gender and mathematics. We knew that in the United States when studying mathematics became optional in the secondary schools, fewer females than males were electing to study mathematics; young women did not believe that mathematics was particularly useful and tended to have less confidence in themselves as learners of mathematics. We had strong evidence that boys stereotyped mathematics as a male domain and we had identified many societal influences that suggested that mathematics learning was not particularly appropriate for girls. Based on these research findings, with the help of three others (Joan Daniels Pedro, Patricia Wolleat, and Ann Becker DeVaney), I developed an intervention program called Multiplying Options and Subtracting Bias (Fennema, Wolleat, Becker, & Pedro, 1980). This approximately one-hour, school-based program, composed of video tapes and a workshop guide, was extensively evaluated, particularly with regard to its effectiveness in increasing females' participation in advanced secondary school mathematics classes and its impact on confidence and perceived usefulness. We reported that this short intervention, which helped girls and boys recognize the importance of mathematics and the stereotyping of mathematics that was prevalent, resulted in more girls, and more boys, electing to take mathematics courses. A more negative finding, however, was that pointing out the sexism that exists in classrooms and their environment increased the girls' anxiety about mathematics.

Another educational finding, which has not been systematically studied by me, or anyone else that I know of, is the variation in gender differences in mathematics across schools and across teachers. Casserly (1980) reported that some schools were much more successful in attracting females to the most advanced mathematics classes than were other schools. The Fennema-Sherman studies reported variations in the size of the differences between schools. And one large urban school system in the U.S. reported substantial discrepancies in gender difference scores by high school on the SAT, a college entrance examination that is recognized as being a good test of mathematical reasoning (Personal Communication, 1985). Confirming this variation between schools and teachers, recent unpublished work of mine has identified certain teachers whose classes consistently show greater gender differences in favor of males than do the classes of other teachers.

My next set of studies was conducted with Janet Hyde. For these studies, we did a series of meta-analyses of extant work on gender differences reported in the U.S., Australia, and Canada (Hyde, Fennema, Lamon, 1990; Hyde, Fennema, Ryan, & Frost, 1990). The results indicated that while gender differences in mathematics achievement might be decreasing, they still existed in tasks that required functioning at high cognitive levels. It also seemed that the more nearly tests measured problem solving of the most complex cognitive level, the more tendency there was to find gender differences in mathematics in favor of males. The international assessment reported by Gila Hanna (Hanna, 1989) reported results that basically confirmed this.

My work has not been the sole chain of inquiry that has occurred during the last two decades. The work of Jacquelynne Eccles, Gilah Leder, and others (Leder, 1992) has been conducted independently and closely paralleled the major themes I addressed. An overly simplistic and not inclusive summary suggests that during this time scholars documented that differential achievement and participation of females and males existed; next, some related educational and psychological variables were identified; explanatory models were then proposed; and finally (or concurrently in some cases) interventions, based on the identified variables, were designed to alleviate the documented differences.

One line of inquiry that I have not pursued, but which has added a significant dimension and more complexity to the study of gender and mathematics, is the work that has divided the universe of females into smaller groups. In particular, the work of the High School and Beyond Project (a large multiyear project that documented gender differences in mathematics as well as many other areas) as interpreted by Secada (1992), and the work of Reyes (Hart) and Stanic (1988) has investigated how socioeconomic status and ethnicity interacts with gender to influence mathematics learning. The U.S., as well as many other countries, is a highly heterogenous society, made up of many layers, divisions, and cultures. The pattern of female differences in mathematics varies across these layers and must be considered.

For a number of years, because I was asked so often to speak about gender and mathematics, I compiled a list of what I had concluded research, my own and others, had shown. Following is a portion of the list I made in 1990.

Gender Differences In Mathematics: 1990

1. Gender differences in mathematics may be decreasing.
2. Gender differences in mathematics still exist in:
   - Learning of complex mathematics.
   - Personal beliefs in mathematics.
   - Career choice that involves mathematics.
3. Gender differences in mathematics vary:
   - By socio-economic status and ethnicity.
   - By school.
   - By teacher.
4. Teachers tend to structure their classrooms to favor male learning.
5. Interventions can achieve equity in mathematics.

On the basis of an examination of the five items on this list that still reflect my thinking, it is clear that in the two decades following my original review in 1974, my understanding of gender and mathematics has grown as far as related variables are concerned, but the same gender differences, albeit perhaps smaller, still exist. I now can describe the problem more precisely. I know that large variations between groups of females exist; I know that there are differences among schools and teachers with respect to gender and mathematics issues; I know that females and males differ with respect to personal beliefs about mathematics; and I know that interventions can make a difference. I understand that the issue of gender and mathematics is extremely complex. And I accept without question the basic premise of the International Commission on Mathematical Instruction Study Conference on Gender and Mathematics (1992) – i.e., that "there is no physical or intellectual barrier to the participation of women in mathematics." But in spite of all the work done by many dedicated educators, mathematicians, and others, the "problem" still exists in much the same form that it did in 1974.

Now before I sound too pessimistic, it should be noted that there are many females who are achieving in mathematics and are pursuing mathematics-related careers. However, let me reiterate that in spite of some indications that achievement differences are becoming smaller, and they were never very large anyway, they still exist in those areas involving the most complex mathematical tasks, particularly as students progress to middle and secondary schools. There are also major differences in participation in mathematics-related careers. Many women, capable of learning the mathematics required, choose to limit their options by not learning mathematics. And while I have no direct data, I strongly suspect that the learning and participation of many women, who might be in the lower two thirds of the achievement distribution, have not progressed at all. I must conclude that many of the differences that were reported in the 1970s, while smaller overall than they were then, still exist in 1993.

New Scholarship on Gender and Mathematics

Much of what we know about gender and mathematics has been derived from scholarship that has been conducted using a traditional social science research perspective – i.e., a positivist approach that has looked basically at overt behaviors such as answers on a mathematics test, the amount one agrees with an item that is part of a Confidence scale, interactions between a teacher and a student, or the career decisions that students make. Usually, in positivist educational research, studies are done by a researcher deciding which overt behaviors are important to study, figuring out a way to count or measure the behaviors, and then studying the counts in some way. Often groups are studied – e.g., all females, all black females, or all females who study physics at a university. Measures of central tendencies (averages) and variability between groups are compared, or relationships between the variables are examined. The purpose is usually to describe the groups with respect to a behavior, examine relationships between behaviors, or to use the results that are found to predict the behavior of others. For example, when the Fennema-Sherman studies reported differential achievement, it was based on comparing the mean responses and standard deviations on a standardized achievement test by a well-defined group of females with the mean responses and standard deviations of a group of similar males. Certain findings of differences were identified as statistically significant, which can be interpreted as meaning that the findings did not occur by chance but probably existed in the universe of similar males and females in the same proportion as in the population studied.

Studies conducted from a positivist perspective have provided powerful and rich information about gender and mathematics. Positivist scholarship should continue, particularly in order to continue the documentation of gender differences in participation and achievement in mathematics. However, an understanding of gender and mathematics based on studies done from this perspective are limited. Perhaps it is evidence of narrow vision, but I do not believe that we shall understand gender and mathematics until scholarly efforts conducted in a positivist framework are complemented with scholarly efforts that utilize other perspectives. Many educational researchers are using new perspectives and their work is beginning to provide important insights into teaching, learning, and schooling. I believe that research conducted within these new perspectives would also provide important insights into the issues involving gender and mathematics.

Although there are many directions that scholarship on gender and mathematics could take, I would like to discuss and provide examples of research from two perspectives: cognitive science perspectives, which emphasize the irrelevance of female-male differences, and feminist perspectives, which emphasize that female-male differences are critical to the learning of mathematics.

Cognitive Science Perspective

Brown and Borko (1992) define cognitive psychology as "the scientific study of mental events, primarily concerned with the contents of the human mind (knowledge, beliefs) and the mental processes in which people engage." Central to this perspective is the idea that much of behavior is guided by mental activity or cognitions. Since it is difficult to get at mental processes, data collection techniques and interpretation are very different from that used in positivist research. Usually the sample size is small and individual interviews provide much of the data. In most such research related to schooling, researchers request that the subject report about his/her mental processes, either asking subjects to think aloud during problem solving, or stimulating recall where subjects might view a video tape of their actions as they are asked to report what they were thinking. Researchers examine the personal reports and look for universals that apply to all people. Much of current research in the mainstream of mathematics education is being conducted utilizing a cognitive science perspective. Many studies of teachers' knowledge and beliefs, as well as most of the work on learners' thinking within specific mathematical domains, are examples of this approach.

Consider one of the more robust lines of inquiry in mathematics education, the work on addition and subtraction done with young children (Carpenter, Moser, & Romberg, 1982). Addition and subtraction were defined precisely as semantically different word problems that can be solved with addition/subtraction; children were asked in individual interviews to solve the problems; and the interviewer probed, either during or following problem solution, until s/he understood how the problems were solved. The researchers looked for and identified patterns of problem-solving behavior that seemed to reflect the mental activities of the children.

Out of this work, universals were identified with respect to how young children, both females and males, come to understand basic arithmetic ideas. Counting and modeling solution strategies, which are developed intuitively in order to make sense of one's environment, were among those universals. These universals are found in many cultures – both in those cultures where schooling is at a minimum and in highly schooled societies (Adetula, 1989; Olivier, Murray, & Human, 1990; Secada, 1991). These modeling and counting strategies provide the foundation for the child's development of more sophisticated understanding.

While most researchers working within this paradigm have not specifically investigated gender differences, for the last ten years Tom Carpenter and I have examined gender differences as we investigated the application to instruction of the universals identified by the work in addition and subtraction (Carpenter and Fennema, 1992). We have noted some differences in maturity between young girls and boys with respect to problem solving, but no differences in how boys and girls solve arithmetic problems. Thus, at our present level of knowledge, we believe that the processes used by females and males to make sense of arithmetic are essentially the same. And, when such findings are used by expert teachers, gender differences in mathematics appear to be irrelevant. Because such teachers can understand each individual's mental processing, instructional decisions do not appear to be influenced by teachers' beliefs related to gender. They teach to the cognitive level of their students and are not affected by superfluous issues.

Cognitive science research also has provided insights into teachers' behaviors, knowledge, and beliefs, although little has been done related to teachers' cognitions about gender. Such studies may lead to deeper understanding of gender differences in mathematics as understanding is gained about the mental life of students, teachers, and others, and how it influences daily decisions about learning mathematics. Unfortunately, there are not many studies related to gender that have been done using this perspective. Once again, I turn to my own and my colleagues' work. Our last study on gender and mathematics concerned teachers' knowledge of and beliefs about boys' and girls' successes in mathematics (Fennema, Peterson, Carpenter, & Lubinski, 1990). Although teachers thought the attributes of girls and boys who succeeded in mathematics were basically similar, teachers' knowledge about which boys were successful was more accurate than teachers' knowledge about which girls were successful; and teachers attributed the boys' successes more to ability and girls' successes more to effort. Linda Weisbeck's (1992) results add some interesting dimensions to our knowledge of teachers' cognitions. During stimulated recall interviews, teachers reported that they thought more about boys than about girls during instruction. However, the characteristics they used to describe girls and boys were very similar.

It appears that teachers are very aware of whether the child they are interacting with is a boy or a girl. However, they don't think that there are important differences between girls and boys that should be attended to as they make instructional decisions. Boys just appear to be more salient in the teachers' minds: Teachers appear to react to pressure from students, and they get more pressure from boys. Interventions designed with this finding in mind would be very different from interventions that assume that teachers are sexist.

Another student, Carolyn Hopp (1994), who is currently finishing her dissertation, is concerned with what happens in cooperative small groups that influences the learning of mathematics, particularly the learning of complex mathematics like problem solving. While this work is still preliminary, it appears that boys and girls engage in different mental activities during cooperative problem solving, and the impact of working in cooperative groups on their learning may be quite different depending on what mental activity is engaged in during the cooperative activity. Just working in small groups does not ensure that girls will learn mathematics. It depends upon what goes on as the groups engage in cooperative activity.

Thus, while research conducted from a cognitive science perspective is still in its infancy as far as gender and mathematics are concerned, such studies can provide knowledge that will help us understand the underlying mechanisms that have resulted in gender differences in mathematics. Consider the case of the relationship between confidence in learning mathematics and the actual learning of mathematics. It has been assumed for at least two decades that lower confidence contributes to gender differences in mathematics. (In self-defense, if you read my writing carefully or listen to what I have said, I have never said that. In fact, I have often said that we do not know how confidence influences learning.) Perhaps, a careful study of males' and females' perception of what has influenced their development of confidence in doing mathematics, and how their confidence has impacted on their study and learning of mathematics, might give us better insight into the relationship of the two. Or, studying the impact that teachers' perception of the confidence of their students has on decisions that teachers make during mathematics instruction might provide deeper insight into teacher-student interactions.

Knowledge derived from a cognitive science perspective has enabled some teachers to eliminate gender differences in mathematics. Carpenter and I have been investigating how knowledge of the universals of children's thinking about whole-number arithmetic could be used in classrooms and whether this knowledge would make a difference in what teachers did and how children learned (Carpenter & Fennema, 1992). We called the project Cognitively Guided Instruction (CGI) and we continue to investigate it today. Basically, we shared with teachers what we knew about the universals of children's learning, enabled them to become secure in that knowledge, and supported them as they applied the knowledge in their primary classrooms. Briefly, we found that teachers could acquire this knowledge of universals and use it in classrooms to make instructional decisions about individual children. CGI teachers' beliefs about children changed and children in CGI classrooms have learned mathematics in excess of anything we expected.

At the beginning of our first study, before teachers had learned about children's thinking, we found that first-grade boys were better problem solvers than first-grade girls. In succeeding studies of children in Grades 1-3, who have spent a year with teachers who know and understand children's thinking, we have found variable gender differences. Often, no differences exist; sometimes they are in boys' favor, and at other times they are in girls' favor. It appears that when teachers make instructional decisions based on their knowledge of individual children, overall gender differences are not found. It also appears that among certain teachers, although they are few in number, gender differences in favor of boys usually exist across classrooms and years; and among even fewer teachers differences in favor of girls are found across years. We are just beginning to try to ascertain whether we can identify components of their classrooms or cognitions that encourage the development of these gender differences in certain teachers' classrooms.

Thus, it appears that research utilizing a cognitive science perspective can be helpful in gaining an understanding of gender and mathematics. It enables us to go beyond surface knowledge and overt behavior to develop an understanding of underlying mechanisms. With this understanding, future educational directions can be identified.

Feminist Perspectives

Included in the broad group of approaches to research that I am calling feminist are perspectives that have been defined as feminist methodologies, feminist science, feminist epistemologies, and feminist empiricism. I am not an expert on these, and thus will not try to provide a thorough discussion. (For that, I refer you to Bleier, 1984; Campbell & Greenberg, 1993; Harding, 1987; and Shakeshaft, 1987). While within the work of scholars included in this tradition there are marked differences that go beyond the purview of this paper, they do share a commonality. Without exception, they focus on interpreting the world and its components from a feminine point of view, and the resulting interpretations are dramatically different from what exists today.

Feminist scholars argue very convincingly that most of our beliefs, perceptions, and scholarship, including most of our scientific methodologies and findings, are dominated by male perspectives or interpreted through masculine eyes. According to feminist scholars, because females have been omitted, the view of the world as interpreted through masculine perspectives is incomplete at best, and often wrong. If women's actions and points of view had been considered over the last few centuries, according to many of these feminist scholars, our perceptions of life would be much different today.

A basic assumption of feminist work is that there are basic differences between females and males that are more prevalent than the obvious biological ones and that results in males and females interpreting the world differently. Although many of these scholars present convincing arguments about how the world influences males and females differently, most feminist writers that I have read are basically uninterested in whether or not such differences are genetic or related to socialization. It is enough for them that the differences exist. These differences influence one's entire world and life. For those who are just thinking about this idea for the first time, I recommend that you find a little book called *The Yellow Wallpaper* (Gilman, 1973). Written about 100 years ago, it gives a picture of one woman's view of her world and, at the same time, the picture of that same world from her husband's viewpoint. Both views impress the reader with their accuracy – and they are dramatically different.

These scholars work in several areas, almost all of them are outside mathematics education. Some are trying to interpret a basic discipline of concern (such as biology or history) from a female, rather than a male, point of view. They argue that almost all scholarship, including the development of what is called science and mathematics, has been done by men and from a masculine view point, utilizing values that are shared by men, but not by women. Those major bodies of knowledge that appear to be value-free and to report universal truth are in reality based on masculine values and perceptions. Since males' roles and spheres in the world has been so different from females' roles and spheres (Greene, 1984), these bodies of knowledge do not reflect 50% of human beings and thus are incomplete and inaccurate. Jim Schuerich (1992) has suggested that a feminist science is better than a value-free science. To support this, he draws from Charol Shakeshaft's (1987) work on educational administration and Carol Gilligan's (1982) work on the development of moral judgment. Each of them has demonstrated quite conclusively that research on male-only populations has produced results that were not only incomplete, but were wrong.

The idea of masculine-based interpretations in areas like history or literature, and even in medical science, is not too difficult to illustrate, nor even to accept. Many conclusions in medical research have been based solely on male subjects; their inaccuracy is easy to illustrate. History has been presented as if most of our ancestors were male and as if important things happened predominately to males in the public arena. The use of male names by female writers in order that their writing be accepted, or even published, is commonly known. Does the prevalence of this attitude apply to mathematics and if so, how? Can mathematics be seen as masculine or feminine? Is not mathematics a logical, value-free field? The idea of a masculine or a feminine mathematics is difficult to accept and to understand, even to many who have been concerned about gender and mathematics. A few people are working to explicate what a gendered mathematics might be – in particular, Suzanne Damarin (in press), Zelda Issacson (1986), and Judith Jacobs (in press), who are struggling to define what a feminist approach to the study of mathematics education might be.

One way to approach the problem of a gendered mathematics is not to look at the subject, but to examine the way that people think and learn within the subject. This has been done in other disciplines: The work of Belenky and her colleagues (Belenky, Clinchy, Goldberger, & Tarule, 1986) in identifying women's ways of thinking and knowing has been provocative as we consider this question. Many within the field of gender and mathematics studies have interpreted this kind of basic research as necessary if we are to identify what female-friendly instruction might be – e.g., the greater inclusion of cooperation rather than of competition in classrooms. Others have argued for single-sex schools oriented to the mathematics instruction of females. Running through these suggestions, it seems to me, is a basic belief that females learn differently and perform differently in mathematics than do males.

Another theme that informs many of the feminist perspectives is the necessity for women's voices to be heard (Campbell & Greenberg, 1993). To these scholars, it is not enough that researchers identify important questions, which are then studied objectively using a positivist approach. Females must have a hand in identification of the questions; females' life experiences become critical, so that the world can be interpreted from a female perspective. So we see subjects as co-investigators, women reporting their own experiences, and women as the main subjects under investigation helping to interpret results. There are not many of these studies available currently in mathematics education, but I predict that we shall see increasing numbers of them as the importance of female voices is recognized.

It is too early to be able to assess the impact that studies using feminist methodologies will have on our understanding of gender and mathematics, both the identification of the problem and its solutions. It appears logical to me that as I try to interpret the problem from a feminist standpoint, it is different from what I focussed on earlier. Instead of interpreting the challenges related to gender and mathematics as involving problems associated with females and mathematics, I begin to look at how a male view of mathematics has been destructive to both males and females. I begin to articulate a problem that lies in our current views of mathematics and its teaching. I am coming to believe that females have recognized that mathematics, as currently taught and learned, restricts their lives rather than enriches them.

Whatever our own value position about feminism and mathematics, I believe that we need to carefully examine how feminist perspectives can add enriched understanding to our knowledge of mathematics education. And, indeed, we should be open to the possibility that we have been so enculturated by the masculine dominated society we live in that our belief about the neutrality of mathematics as a discipline may be wrong, or at the very least, incomplete. Perhaps we have been asking the wrong questions as we have studied gender and mathematics. Could there be a better set of questions, studied from feminist perspectives, that would help us understand gender issues in mathematics? What would a feminist mathematics look like? Is there a female way of thinking about mathematics? Would mathematics education, organized from a feminist perspective, be different from the mathematics education we currently have? Suzanne Damarin (in press) has stated that we need to "create a radical reorganization of the ways that we think about and interpret issues and studies of gender and mathematics." Many scholars believe that only as we do this will we be fully be able to understand gender issues in mathematics. Perhaps my beginning to believe that the decision by females not to learn mathematics nor enter mathematics-related careers because mathematics has not offered them a life they wish to lead is an indication that my old view about learning and teaching mathematics, as well as about gender and mathematics, was immature and incomplete. I am beginning to believe that an examination of what the female voices in the new research are saying will help me – and perhaps others – to understand teaching, learning, gender, and mathematics better.

New Research Perspectives: Some Contrasts

Cognitive science and feminist perspectives, while sharing surface similarities, are based on dramatically different assumptions about females and males. These assumptions dictate the questions that are addressed, how studies are designed, and how evidence is interpreted. They are assumptions that are more far-reaching than issues of scholarship; they influence how we view the entire issue of gender and mathematics. What is the magnitude and impact of differences between females and males? Are males and females fundamentally different, so that all decisions about mathematics and understanding gender and mathematics need to be made on the basis of these differences? Or are males and females fundamentally the same, with the exception of their biological differences, and are these differences irrelevant with respect to mathematics? Cognitive science research, as it identifies universals, would suggest that looking at the world through either feminine or masculine eyes does not make sense. Feminist perspectives suggest just the opposite: Female/male differences permeate the entirety of life and must be considered whenever scholarship is planned.

The implications of these assumptions are dramatic, in both doing and understanding research, as well as work in the field of gender and mathematics. Each individual should think deeply about his or her own beliefs and re-interpret knowledge about gender and mathematics in relation to these beliefs or assumptions.

In Conclusion

Research has provided rich documentation of and knowledge about variables that are related to gender and mathematics, and moderate change in what has happened to females within mathematics education has occurred partly because of this scholarship. We need to continue research that documents the status of gender differences as they exist. However, research, as we know it, must be supplemented with new types of scholarship focussed on new questions and carried out with new methodologies. Such scholarship will help in the identification of important emphases for further new research; it will also ensure that women's voices will become a major part of all educational scholarship.

While I have chosen to focus this paper on research as I perceive it, there have been other forces at work during the same time that our research knowledge has been accumulating. Many innovative interventions have been developed, based on the intuitive knowledge of concerned individuals. Mathematicians, educators, teachers, and parents have become aware of the issues related to gender and mathematics. We have come a long way. We have a long way to go to accomplish equity in mathematics education.

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RECRUITING AND RETAINING GRADUATE STUDENTS IN THE MATHEMATICAL SCIENCES AND IMPROVING THEIR CHANCES FOR SUBSEQUENT SUCCESS

Mary W. Gray
American University, Washington D.C.

Although the percentage of PhDs in mathematical sciences in the United States going to women has risen over the past 30 years from 6% to 22%, the increase is far less dramatic than comparable results for law schools and medical schools, where, starting from even smaller percentages, the representation has grown to nearly 50%. Moreover, much of the percentage increase, until very recently, was due less to more women than to fewer men. Many of the reasons why there are still relatively few women doctorates in mathematics are simply the same difficulties that face anyone seeking to earn a PhD in the field, but these reasons are magnified for women. Chief among these is uncertainty. A doctorate in any field can take a long time, but the period of time itself is not a critical factor – medical training can last for many years. The difference here is that, once on the track, the end is certain and attainable, subject only to the level of intelligence and diligence. Predicting the amount of time required to complete a dissertation can be difficult, particularly in a field such as mathematics, where the creative effort is qualitatively different than what went before. It is like asking for a novel from a doctoral candidate in literature.

The additional obstacle facing academics in any field is the probationary period before the tenure decision; for women, this long period of training and apprenticeship coincides with prime child-bearing years in a way that inevitably presents problems. Moreover, in mathematics there remains substantial prejudice; the belief that women cannot do mathematics – or at least first-rate mathematics – is now rarely expressed openly, but it has not disappeared among those doing the hiring and tenuring in mathematics departments in research institutions.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

Opening up the Pipeline

It used to be thought that the primary choke point in the pipeline to mathematical careers for women was the junior year in high school, where females differentially dropped out of the further study of mathematics; that disparity has largely vanished. For example, in 1989 at the University of Michigan, 76% of the first-year women students had four years of high school mathematics, compared to 90% of the men, whereas 20 years ago the proportion of women was about one third that of the men. Now we see two other points where the dropout rate for women is substantially greater than for men. The first is after the baccalaureate; nearly 50% of the Bachelor's degrees in mathematics go to women, compared with about 35% of the Master's and 22% of the doctorates. Women graduating from college do not see graduate school as a desirable option as do men, for several reasons. One is their different undergraduate experiences; another is the uncertainty discussed above; still another is the lack of role models – if one sees few women on the faculties of the PhD granting institutions, it does not appear as a reasonable option for oneself. Still another is the differential record of support; women who do go to graduate school are more likely than their male classmates to have teaching assistantships requiring substantial time away from their own work rather than fellowships or research assistantships. For example, in competition for prestigious National Science Foundation fellowships a male applicant in mathematics is three times more likely to be successful than a female applicant. This might be explained by the fact that women have lower average Graduate Record Examination scores (but, on the other hand, women applicants have higher grade point averages) and are more likely to come from liberal arts colleges than from research universities. However, the same is true in all fields in which the NSF fellowships are awarded, but in all other fields except chemistry the success rate for women is roughly the same as for men (in chemistry it is twice as high for men as for women). Those who have served on selection committees have explained the differential in mathematics on the basis of the far more positive letters of recommendation for the men.

The question of letters of recommendation may reflect the difference in undergraduate experiences mentioned above. Generally, nearly half of the women who major in math and science come to college with another major in mind; the late start may be because of less depth and breadth of mathematical training, less of a close relationship with an advisor, less encouragement to consider graduate school. As research indicates that women are more sensitive to indicators of how well they are performing, these last two factors are especially critical. Women need encouragement: they need information about opportunities; they need to be taken seriously by teachers and counselors; they need to have a comfortable relationship with an advisor. There is also some indication that women do better in discovery-based classes and in a more cooperative atmosphere, where they see mathematics in terms of problem solving. Women may thrive on competition, but competition of a different kind; it is important that they not be overlooked for research internships and other opportunities.

The other choke point in the pipeline comes in employment. The difficulty is no longer primarily in getting a first job; in 1992 for the first time women were hired at the top PhD granting institutions in the same proportion as they were represented among the PhDs granted by those same institutions. However, even though the percentage of doctorates in mathematics going to women has hovered around 20% for over 10 years, women constitute only 5% of the full-professors in PhD-granting institutions. Thus, even after they are hired, women drop out differentially, largely for the same reasons they drop out of graduate school – lack of support and discrimination.

Strategies for Improvement

What, then, can be done about recruiting and retaining women (and those from other underrepresented groups) in graduate school? Many of these ideas work generally for any students; it is just that additional effort may be required.

There is no magic formula, no easy route to success; as is the case with any worthwhile goal, it takes a certain amount of dedication and hard work. There are three parts to nurturing the nature that is there if one looks for it.

1) Recruitment. To recruit it is necessary to go to where they are, that is, to women's colleges and to other liberal arts colleges and state colleges and universities that have larger proportions of women among their mathematics majors than do most research institutions. These students may not have had as much mathematics as have many Ivy League graduates, but there are plenty of very talented students. Funds for faculty to visit these institutions or for potential students to visit the graduate institutions are helpful but by no means essential. Using mail and e-mail and talking to faculty at meetings can be very effective if real commitment to having women in a program is conveyed. Regional mathematics conferences, particularly of the Mathematics Association of America, are a good place to meet faculty and students from smaller schools. Having current students come to these to give talks serves a number of purposes – experience and exposure for the speakers and a visible sign to those in the audience of the effectiveness of the program. Another source of promising students are those who did not go straight to graduate school or who dropped out with only a Master's degree. They may be found teaching in high schools, in community colleges or four-year colleges; they may be in government or industry. These older students sometimes need more support services, but they bring experience that can be profitably shared with other students. They can be sought out, made to feel wanted, and convinced that success is possible.

Recruitment may also have to include convincing the prospective students that they want a career in mathematics. Demonstrating that mathematicians are not necessarily isolated or arrogant and that the work can be cooperative, useful, and even socially relevant may persuade mathematically talented students that they might change the world – or at least have a personally satisfying life – as mathematicians just as effectively as they could as physicians or lawyers.

Above all, it is important to keep in mind that a student stands a better chance of success if she is not isolated, not one of a kind. The commitment to recruit needs to extend to creating a critical mass of students, not mere tokenism. Moreover, the commitment to attracting women needs to extend to hiring faculty as well. Potential women faculty need to be treated as desirable colleagues mathematically, not as someone sought to fill a quota. There are two obvious things that need to be done: first, the idea that candidates can be linearly ordered needs to be abandoned – who has not in the past made a hiring mistake, particularly when hiring junior faculty on the basis of promise more than performance? Second, too narrow a focus on a particular field or subfield limits the pool to such a great extent that it will be difficult to find women; departments need to ask themselves what real and lasting harm would ensue to the department if someone with a different specialty were to be hired.

2) Support. Financial support is crucial, for women students are less likely to be able to call on family resources – either from parents or spouses – to supplement their stipends or to cover emergencies. Even institutions with limited resources can often be persuaded to provide special funding. External sources of financial support also are possible; nationally there are programs such as the U.S. Department of Education Patricia Roberts Harris fellowships; local businesses, industries, and foundations may be willing to provide some supplemental funding. A loan fund for crises, good summer jobs, opportunities to tutor, consult, or teach to increase income are all important – but it is essential to keep in mind that the students are primarily just that, and to help them limit their other responsibilities. Women, both teaching assistants and new faculty, may end up spending more time on teaching than is good for their research progress; they may take on more counseling chores or more committee assignments – as the token woman or on groups dealing with concerns like daycare or affirmative action that should be the business of everyone but frequently are not.

If there is a co-op education or internship program on campus, they can help with summer or academic-year placements. The practical experience can be as valuable as the money. Other support services, such as assisting students in forming groups to reduce housing costs and, in the case of single parents, to share babysitting chores or costs, can help.

A particularly useful program that American University has devised is the creation of two-thirds-time instructorships, at two-thirds of a reasonable salary, for advanced PhD students. In addition to being helpful on the money side, the instructorships look good on the students' resumes.

But support needs to take other forms as well. In particular, one cannot let students jump in at too high a level. If what is needed are some advanced undergraduate courses, encouragement to start there (and perhaps extra years of support) needs to be forthcoming. Much of what is required is simply whatever most graduate students need, but maybe with some extra effort to see that the students you have recruited receive it. A friendly department, with a feeling of inclusiveness, mathematically and socially, provides extra support for all students. Something as simple as a quick turnaround time on dissertation drafts can make the difference between a student's sticking with it or dropping out. In general, in any department there are likely to be faculty known to be effective dissertation supervisors and those whose students seem to get degrees in spite of their advisers, if at all. If possible, one can steer women students to those with whom they have the best chance for success. This can go far to reduce the uncertainty factor cited above as a deterrent to careers in mathematics.

3) Persistence. Who has survived graduate school without ups and downs of spirit? Who has not had moments of self-doubt when the creative juices seemed unbearably sluggish? Faculty, but most of all other students or recent graduates, need to be there to share triumphs and setbacks. Getting students through a PhD program is often a cooperative effort.

But persistence is needed not only through receipt of the degree. Making sure the newly minted PhD has an appropriate job, has an idea for the **next** paper, has been clued into what makes for success in her new department, will assure that mathematics does not lose a carefully nurtured new doctorate. There comes the time when the graduates learn not only that they can make it on their own, but that with success goes the obligation to help along the next generation – to return the mentoring from which they have benefited. A mark of success is not only that the graduates are doing good mathematics and effective teaching, but that they are themselves running programs to increase the participation of women in mathematics.

What Next?

There are many areas where there is much research to be done. Although there have been many intervention programs, for the most part they have not been accompanied by any long-term studies of effect; in particular, very few programs use random assignment so that their effect could be reliably assessed. Moreover, all too frequently intervention programs do not address the real issues – they focus on women and girls at the wrong stage or they are not focused on strategies for real improvement. There has been little cross-institutional research and almost no longitudinal studies, even though the choice to do mathematics is made over time. There is a critical need for carefully designed observational studies in which the impact of classroom dynamics can be studied in depth.

Specifically research might include

1) exit interviews of those not going on to graduate school, those who drop out in graduate school, and those who leave professional employment after completing the doctorate;
2) reviews of the academic environment – faculty-student interaction, peer interactions and support, competitiveness, physical environment, counseling, faculty expectations, discrimination, and harassment;
3) studies of possible differential effects of instructional methods and course content;
4) investigation of the selection of subspecialities to determine whether gender differences exist and, if so, why;
5) analyses of financial support patterns for men and women;
6) evaluations of publication practices, including acceptance rates by sex and the effect of "blind" refereeing;
7) examinations of sex effects in other indicia of professional success – citations, invited talks, positions in professional societies;
8) surveys of hiring and tenuring policies, practices, and results;
9) assessments of means of encouraging and rewarding faculty and administrators who promote the advancement of women and mathematics.

It is also important to publicize research that has already been done, in particular recent research on cognitive differences. There remains a perception of inherent male-female differences that has infected society at large and many in the mathematical community, partly due to a possible publication bias that favors studies highlighting purported differences.

A GENDER PERSPECTIVE ON MATHEMATICS AND PHYSICS EDUCATION: SIMILARITIES AND DIFFERENCES

Karin Beyer
Roskilde University

Introduction – My Background

I graduated in pure physics with mathematics as second subject and have been engaged in research and developmental work in physics and science education for many years. Among other things, I have been involved in the founding and further development of Roskilde University, the programmes of which are 50% project organized. Since 1983 I have been engaged in research projects on the gender polarizing effect of physics in the upper secondary school in Denmark. My work has benefitted very much from international cooperation and networking primarily based on contacts made at the biennial international conferences of the GASAT (Gender And Science And Technology) Association.[1]

THE GASAT ASSOCIATION
an association of those concerned with issues arising from interactions between Gender And Science And Technology

OBJECTIVES

- To encourage research into all aspects of gender differentiation in science and technology education and employment
- To foster gender equity in science and technology education
- To facilitate the entry of women into employment in the fields of science and technology and their progress within such employment
- To foster socially responsible and gender-inclusive science and technology
- To provide a forum for dissemination and discussion of research findings and experiences of those working in the field
- To provide a support network working towards the objectives outlined above

As can be seen, GASAT has a sphere of interest much broader than gender and science education; this breadth of interest in most respects is highly valuable. To be able to exchange research results, intervention designs, and so forth in a more

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

focussed way, several formal or informal networks have been formed in relation to the GASAT Association.

Most science educators are engaged in the education of pupils in primary or lower secondary school, whereas my main interest is with students in upper secondary school, ages 16 to 19, and at university level, where conceptual knowledge and theory structures play a more dominant role than for younger students. Therefore it is very important for me to benefit from your and others' work in mathematics education.

It is my goal to improve education for males as well as for females, but it is my sincere conviction that to accomplish this it is necessary to include the *gender perspective* in the theoretical framework of teaching and learning. As a result of my research in gender and physics, I find increasingly that

it is significant to acknowledge the complexity of learning processes as result of the interaction of many different factors.

Further, my "position" in the area of gender and education can be summarized in the following statements:

No single factor can explain the observed gender differences; on the contrary the explanation is likely to be found in the gender-specific way the interaction of the many factors takes place.

These and other statements will be developed further in this paper and related to the concept of gender differences in *learning style*. Right from the beginning, I would like to emphasize, that

regarding gender polarization, the situation is NOT the same in mathematics, science, and technology.

On this basis I hope to encourage and contribute to a discussion of the similarities and differences of the gender aspect of math and physics education. I will present some personal views, but I am certainly not able to give a complete treatment of the item framed by the title of the paper.

Gender and Math, Science, and Technology

Very often, and at this conference as well, gender inequities in mathematics, science, and technology are referred to in the same breath, as if it were one homogeneous problem. This is not the case. The gender situation is not the same in biology as in physics, in chemical engineering as in electronic engineering, in science teaching as in science research, in mathematics as in physics. There are differences among the subjects with regard to gender patterns of recruitment and students' interests and achievement. It is very important to be aware of these differences. Otherwise, we risk ignoring important factors relating to the various subjects, their role in the school system and other aspects of society and culture, and we could be trapped into seeking explanations in (postulated) deficiencies of girls and women.

In the sections to follow, I will deal with the significance of the educational structure and the character or nature of the various subjects and their teaching tradition. In particular, I will compare the situation in mathematics with that in physics.

The Significance of the Educational System

Among young school children, girls are nearly as interested in physics topics as are boys. Gender differences increase with age. (Sörensen, 1990; Lie & Sjöberg, 1984; and several contributions to GASAT conferences). In mathematics the same pattern is seen (Hanna, 1993a, p.29; Hanna 1993b; and references herein), but with gender differences that are significantly smaller than in physics .

At the school level, where math and/or the various sciences become optional, relatively more girls than boys drop out of these subjects or choose the less advanced courses (Beyer, Blegaa, Olsen, Reich, & Vedelsby, 1988; Skog, 1983; Sjöberg & Imsen, 1988). Generally, students' interest in physics decreases during secondary school. As access to further education involving mathematics and science usually requires the highest level of science and math in secondary school, the gender specific choices in school cause the well-known gender differences in further education.

These phenomena have been partially explained by reference to gender differences in values, motivation, and internal belief systems (Fennema & Peterson, 1985; Kloosterman, 1990). Our research (Beyer et al., 1988) confirms that these factors play a very important role, as will be discussed later in this paper. In addition we find it important to stress the significance of the educational structure, the intrinsic character of the subjects, and the traditional way of teaching and assessment. These points will be discussed below, and I could emphasize the first aspect this way:

The gender specific pattern of educational choice is a product of the educational structure.

Research has shown that the greater the number of electives and the earlier the choices have to be made, the more profound are the patterns of gender differences to be found (Skog, 1983; Beyer et al., 1988; Sjöberg & Imsen, 1988, p.246).

Generally speaking, the tendency since the late 1970s has been that many researchers and feminists in the U.S. have been concerned about "gender and math-ematics," whereas the focus in Northwest Europe has been on gender differences in physics.

Although one should be very cautious in characterizing the big and educationally very diverse country, the U.S., I think it is right to say that physics in high schools is usually avoided entirely, leaving mathematics as the "hard" subject. (The notion of "hard" subjects is further discussed in the next paragraph.) At the same time mathematics becomes optional in early adolescence in many American schools, and this subject serves as the critical filter towards further education in the math-science-technology area. In most European countries, on the other hand, mathematics was and is a compulsory subject until the age of 16 or later, and only minor gender differences have been observed in comparison with the situation in physics, which is therefore considered as the main critical filter subject in these countries.

Historical Case 1

Until about 1960 Danish school pupils had no mathematics but only *arithmetic* as a subject in the lower grades; the "real" subjects, *physics* and *mathematics* (involving algebra and geometry), did not appear until adolescence at age 13 (grade 6 or 7). At that time it was expected and thought quite natural that boys would show a higher interest and outperform girls in mathematics as well as in physics. Nowadays mathematics is a compulsory subject from preschool to about age 18, with a choice between levels only from age 16. In general, girls like mathematics and perform well. So mathematics is *not* perceived as a gender polarizing subject, but physics is!

In upper secondary school (the "Gymnasium," ages 16-19) students have to choose either the language-line or the math-line. In both cases they will have the common core subjects such as history and Danish literature, as well as a certain amount of English and a second foreign language. Since the mid-1980s, 45% of the students in the math-line have been female. Prior to the reform of 1988, the lines were further divided in a number of tracks after the first year. To obtain mathematics at the highest level, the students had to choose the math/physics track. This track was chosen by about 50% of the boys and 20-25 % of the girls at the math-line, whereas the majority of the girls chose the biology track with math at a lower level. After the latest reform of the "Gymnasium" the tracks have been replaced by a system of electives. Students now have to choose two (or three) subjects at an advanced level, and it is now possible to choose *math at the highest level without having to take physics as well.* About 80% of the male students and 70% of the female students in the math-line choose high-level mathematics, whereas high-level physics is chosen by only about 30% of the males and 10% of the fe-males.

These facts indicate that students' choices are indeed influenced by the educational structure and the system of curricular options. This has been further supported by research projects among students in the Danish Gymnasium (Beyer et al., 1988; Vedelsby, 1991). It was demonstrated that students' choice of line and track in the upper secondary school was determined partly by students' positive interests and partly by their wish to opt away from physics (i.e., to choose physics at the lowest possible level).

The choice of electives or branches or tracks is influenced by the image of various subjects or general educational tracks. Very often branches with a high degree of math and science are perceived as more prestigious and in better accord with boys' and men's careers than with those of girls and women. To choose a math/physics dominated branch is equivalent to claiming that one is rather smart in these subjects. It is generally more difficult for girls than for boys to declare that they possess such categorical competence and confidence (Beyer et al., 1988; Sjöberg & Imsen, 1988).

The mere existence of soft and hard parallel options creates a gender polarization.

Therefore educational policy should remove such barriers and strive for educational systems with

as few and as late choices as possible. And the different streams (lines or branches) should be as balanced as possible with respect to the image of being hard/soft and masculine/feminine.

Historical Case 2

At the end of the 19th century in Denmark, the situation of women in higher education was very similar to the situation in a number of other countries. Relatively few females in Denmark attended a (private) upper secondary school and took the entrance examination that allowed them to go on studying at universities. Females were not yet allowed in the public "Latin schools" but had to prepare themselves for the final examinations at private schools for girls. Since 1871 the secondary schools (Latin schools) had a line with mathematics and physics parallel to the traditional classical line with the ancient languages, Latin and Greek. The classical line was required for most university education leading to jobs as higher officials, public servants, and so forth, and it was chosen by the vast majority of the males. Actually, around the turn of the century, relatively more females than males chose the math-science line.

As a consequence of a major school reform in 1903, the upper secondary school changed so that (a) girls were allowed in the new "gymnasium" schools that replaced the Latin schools, and (b) the classical line was complemented by a line with modern languages and history, as well as one with mathematics and science. These changes had consequences for the examinations from 1910 onward, and in the next few years a change in the pattern of choices took place, resulting in the well-known picture of the majority of the males choosing the math-science line and the majority of females choosing the modern language line.

This example demonstrates again that the gendered pattern of educational choices is greatly influenced by the educational structure as well as by the prestige and competence associated with the various subjects or tracks.

Interlude on the Inevitability of Gender Polarization

It has been implicit in the discussion above that gender polarization in physics or mathematics is a thing to be avoided or opposed. I certainly see no reason why educational success in these subject should in any way be gendered. Nevertheless, it is my conviction, that gender polarization as such is inevitable. Women and men are different and gender-specific roles and expectations will always exist as a social expression of the relations between males and females. However, it is important to emphasize that gender roles are not invariable. On the contrary, they are a result of factors in society and culture in general, including social class and the power relations between men and women. So the concepts of gender, masculinity, and femininity are dynamic and a result of a historical process, as can be seen by comparison of the situation in different cultures and historical periods. See for example Kahn (1994) and several other interesting contributions in Owens (1994).

Each society or subculture invokes methods to ensure the reproduction of gender roles. In modern Western societies with a relatively high degree of equity in public life, with both wives and husbands active as breadwinners, the mechanisms repressing either gender can be rather disguised.

Each historical-cultural period must evaluate the prevailing gender roles (and related myths) and judge whether the restrictions put on men and women are acceptable or not.

What Characterizes a Hard Subject?

By a hard subject I mean a subject that is usually perceived as a difficult subject (in a certain educational and cultural context). The concept of "hardness" is not very precise, but in most cases it is used to mean that hard school subjects are characterized by high cognitive demands.

When achievement is measured – at school examinations or in special achievement research projects such as APU (Beyer et al., 1988; Johnson & Murphy, 1986) – physics is seen to be the hardest subject among the sciences. Why?

A hard subject is not necessarily a masculine subject in the sense that more males than females are attracted by it. However, in science it seems to be the case that the hardest subject, physics, is also the most gender-polarizing subject. Why?

It is often said that physics is a difficult subject because of its high content of theoretical concepts and the use of mathematics. However, many students – including many girls – like mathematics and perform well in this subject.

Another characteristic of physics is experimental work. Could this be the reason for the hardness and also for the masculinity of the subject? It is unlikely, for biology and chemistry are also experimental and, in fact, experimental work is rather popular with both girls and boys. In Denmark chemistry is chosen by a higher proportion of females than physics, and in biology the majority of students are female. A similar situation exists in a number of countries, including the U.S.

Physics is the hardest subject among the sciences because of (a) its hierarchical theoretical structure in a mathematical formulation, and (b) because of the fundamental (inherent) interplay between theory and reality.

Experimental results seldom yield obvious data that automatically give new knowledge about the "real world." Experimental data have to be interpreted in the light of models or theories. This is difficult and requires understanding at a relatively high cognitive level. In the same way, problem solving, where one is required to use scientific concepts in a new context, is very difficult. Real understanding of concepts such as *acceleration, power, and moment of inertia* requires more than just a mathematically expressed definition, and the concepts of *energy, force,* and *heat* can hardly be introduced solely by a formula.

Teachers, I presume, generally observe that students have difficulty applying even simple mathematics if it appears in a context of a physical problem. The fact is, that *it is more difficult* to identify and solve the mathematical part of the physical problem, than it is to solve the pure mathematical counterpart. Regarding this difference, Mallow (1986) characterizes math as being taught in comprehensible clusters of concepts, the test of which is relatively straightforward; for example, "Integrate the function $\sin^2(x)$ from 0 to 2," whereas the same question in physics immediately requires synthesis of various concepts and operations; for example, "What is the energy dissipation over one cycle of an alternating current through a resistor?"

Many school subjects involve to a certain degree the use of scientific or other formal concepts. But often the conceptual structure is less hierarchical and theory bound than in physics (and mathematics) and often it is only a part of the subject content that can be characterized as "hard." For example, we have found that in biology in the Danish "gymnasium" (upper secondary school, ages 16-19), certain areas such as ecology (requiring systems analysis) are perceived as harder than other areas of the subject.

The Teaching Tradition

It is very important to stress that the above discussion of "hard" or "soft" subjects does not necessarily characterize a property of the subject as such, but may be very much a consequence of the traditional way in which it has been taught. For example, in the Danish "gymnasium," chemistry is taught as a harder subject than biology, and physics as a harder subject than chemistry and mathematics. This is an effect of the teaching tradition. Although I cannot document it, it is my impression that the difference in "hardness" between chemistry and biology is less pronounced or nonexistent in England. Further, one can easily imagine mathematics taught as harder than or equally as hard as physics, with a combination of theory, applications, and interpretations. I assume that a focus on modeling, as a process involving understanding of the "real" system that is to be modeled, could very easily end up as a hard subject. It would also be a very important subject in my opinion.

Of course, physics could also be taught in a relatively soft version, with orientation about phenomena in nature, elementary observations, and practical work. This is often the case in science courses for younger children. However, successful courses of this kind, giving both boys and girls a positive experience and useful skills, is no guarantee that they will like or succeed in a "harder" physics course during turbulent adolescence (See Sörensen, 1990, 1991).

Languages, economy, sociology, and philosophy are all subjects that *partly* utilize hierarchical conceptual structures, systemic approaches, theory-based interpretations, and so forth. There is some evidence that whenever these subjects (or their subdisciplines) are taught as "hard" subjects, they are perceived as really difficult and more boys and men are attracted than girls and women. Generally, at least in Denmark, these subjects are introduced in a softened way at school level, the "hardest" being German, French, or Latin grammar. However, teaching in foreign language is never exclusively grammar or other formal aspects, so other areas of competence are valued. The second historical case above indicated that the function as critical filter changed over from Latin and Greek to mathematics and physics in the beginning of the 20th century.

Gender Differences in Learning Style (1)

When student achievement is assessed through tests or examinations it is, not surprisingly, found that problems or tasks relating to the "hard" areas of subjects are the most difficult ones. At the same time it is found that "hard areas," for example, tasks requiring application of theoretical physical concepts, are the areas resulting in the greatest gender differences (e.g., Johnson & Murphy, 1986). Why is it so, that the difficult aspects of science seem to be even more difficult for girls than for boys?

In an attempt to answer this question I will refer to gender differences in "learning style" (a concept I will develop later in this paper). However, it is important to stress that there is no single-factor explanation of the phenomenon of gender differences in performance. On the contrary, the research on gender differences in science education repeatedly emphasizes that learning is a very complex process involving a multifaceted interplay among many different factors. So "learning style" is more a label than an explanation in itself.

I will discuss my concept of learning style and gender later in this paper as I try to pull together the different strands of this discussion. At this point, I will simply state that I do not find any adequate explanation relating to gender-based differences in "intelligence" or any other fundamental difference in the way females and males think. In fact, most available data show the opposite (see, e.g., Brush 1991 and references therein). I find it much more fruitful to look for students' affective responses to learning and assessment. So the core of my concept of "learning style" is related to the important *interplay between cognitive and affective factors*. The significance of affective factors is in accord with the recognition that learning is *context dependent.*

Context, Applications, and Perspectives

Interviews among students in the Danish gymnasium (age 17) have shown that it is important for girls that they perceive physics as relevant to themselves and others, or to important problems in the environment or society (Beyer et al., 1988). So it is important that physics be illustrated by applications relevant to everyday experience and treated in a broader historical or philosophical context or in a science-technology-society perspective. See also Head (1985) and Grant and Harding (1987).

For boys the context and perspectives seem to be less important. They are often motivated by an (unconditional) fascination with technology or by "the fun of pure physics." Further, they sense that they *have to* learn the subject, that physics *is* important for their future, one way or the other. Girls do not share these beliefs. They are much more dependent on an explicit demonstration of the relevance of the subject. Learning is a serious endeavour and to engage themselves fully in it, girls must know that it is important. They tend not to work hard "just for fun." Further, although some girls are really interested in physics and/or technology, their fascination is seldom unconditional; they want to develop a more qualified view of the different areas of physics and technology (Grant & Harding, 1987; Solomon, 1989). Of course, we also met girls who had already turned their backs on the subject, but they were not in the majority. We interviewed only students in the math-line of the gymnasium (chosen by 80% of the boys and 50% of the girls in the gymnasium). Students in the language-line are known to hold less positive attitudes towards science in general and physics in particular (Beyer et al., 1988). See also the investigation of preferences and values of Danish students by Mallow (1993).

Now we know what will be a motivating and relevant curriculum but, unfortunately, it is not that easy! One often runs into severe dilemmas, some of which I will just outline:

(1) Presenting physics in an everyday context will often add to motivation, but students tend to give an everyday explanation in everyday language without the physical reasoning that is really required (Blegaa & Reich, 1987; Beyer & Reich, 1987; Haggerty, 1987; Murphy, 1994). A pure physics problem disguised in an everyday context is more difficult than in the traditional version. It is especially difficult to change between the two different codes of scientific and everyday language (Solomon, 1994; Murphy, 1994).

(2)Taking the STS perspective seriously by treating the interplay between science, technology, and society in a theoretical framework will make the subject very difficult (in the sense that sociology and history can be very "hard" subjects). Even in a "softer" version,

> this curriculum might put some students off, for example, some "tinkering boys." Further, it must be admitted, that adding STS as part of the curriculum (which I find very important) does not in itself imply improvement in students' learning/understanding of the basic physical concepts and theories.

The ambition should really be to induce "double-qualifications": comprehension of the character of physics and good knowledge of some "hard" parts of this subject, combined with awareness and understanding of historical and philosophical as well as STS-perspectives.

Gender, Context, and Assessment

Another aspect of science education where the significance of context has been increasingly acknowledged is assessment. At the latest International GASAT Conference a number of papers on gender and assessment were presented, all stressing the influence of context (e.g., Hildebrand & Allard, 1993; Hegarty-Hazel, Logan, & Gallagher, 1993; Murphy, 1993). Murphy has been able to draw on the extended quantitative data from the science project of the Assessment of Performance Unit (APU) in the U.K., which makes her results especially interesting.

A key contribution of the APU project was the development of an *educational assessment* in opposition to traditional psychometric assessment (Murphy 1994). The APU science framework (Murphy & Gott, 1984) reflected a view of science education as an experimental, problem-solving activity involving a complex interaction of demands. In this view science understanding is a product of scientific conceptual and procedural understanding (Murphy, 1994). The APU science results were based on a broad definition of scientific achievement using tasks involving an extensive range of methods of presentation, operation, and response. Murphy (1994) further explains:

> A characteristic of the results was the variability in individual performance across the subject. This variability in performance was due to *sub-effects* related to the tasks and their administration and students' interaction with them. The collection of background information concerning students' interests, attitudes and experience – both outside and inside school – made it clear that the sub-effects arose in part from *affective* responses of students. The APU data provided influential evidence of the interdependence of students' affective and cognitive responses. (p. 62)

The APU results have made it very clear that students' performance on tests is not only dependent on the "cognitive level" of the task, but also very much on the affective response to the content and context of the problem. This is something that we would expect to matter in the teaching and learning situation but typically assessors assume that content is irrelevant. The results from the whole APU science question banks showed that irrespective of what criterion was being assessed questions that involved such content as health, reproduction, nutrition, and domestic situations were generally answered by more girls than boys across the ages 11, 13, and 15. The girls also tended to achieve higher scores on these questions. In questions with a more "overtly" masculine content, for example, building sites, racing tracks, or anything with an electrical content the converse was true.

> For example, girls and boys at ages 11 and 13 are equally able to use tables and graphs. Yet if the graph is about the day in the life of a secretary some boys do not respond whereas girls will tackle the question with confidence and so overall obtain a higher score. If on the other hand the graph is about traffic flow through a town the reverse happens. For some students the questions are not about graphing skills but about their experience of the content. (Murphy, 1994, pp. 70-71)

These performance effects arise from the combination of avoidance by some students and heightened confidence by others. The questions therefore measure students' affective rather than cognitive responses.

Presenting a task in a certain context will give different cues to different students, depending on their learned views of what is relevant and of what they have come to value, that is, depending on characteristics they bring to school and classroom situations.

> In one investigation pupils were asked to find out which of two test materials would keep them warmer when stranded up a mountainside in cold, dry, windy weather. The expectation was that students would lug cans of hot water with the test materials and measure the rate of cooling of the water. A hairdryer was provided if they wanted to speed up the process. Assessors observed girls dipping the test materials in water, blowing air through them, cutting prototype jackets and lugging cans with the wet test materials and then determining the rate of cooling. These behaviours were observed for some girls, in all sorts of schools but not for boys. The context of the mountainside was meant to provide a purpose for the task, not to cue the students in any particular way. APU research has shown that girls as a group tend to value the circumstances that the activities are presented in. Boys conversely judge the context and the content of many activities to be idiosyncratic and irrelevant, as such are more in tune with assessors' intentions. The girls mentioned earlier were concerned to find out if the effectiveness of the materials was affected by their porosity and waterproofness and whether indeed the materials were suitable for making the proposed jacket, hence the prototype jackets. When girls did set up an investigation it was often to explore multivariable effects and was therefore more complex than the assessment intended. (Murphy, 1994, p.72)

These effects were found across the APU science investigations indicating that

> the problems girls and boys perceive and the solutions they judge to be appropriate are often different given the same set of circumstances (Murphy, 1994).

It is of utmost importance that results regarding gender differences in performance are not perceived as inevitable facts about gender specific aptitudes. The outcomes must be interpreted in light of the significance of content, context, social and practical setting, type of activity and response required, and the interaction with characteristics of the student. In this way assessment can provide teachers with invaluable insights.

> Without information about the different ways students perceive themselves and their capabilities teachers cannot develop effective strategies to help them. It was suggested that questions measure students' alienation rather than achievement; nevertheless, alienation ultimately leads to *relative* underachievement as girls and boys fail to engage with certain learning opportunities. It is therefore essential that content and context effects are considered in assessment situations and that students themselves are involved in this. (Murphy, 1994, p. 71)

So assessment should serve, not as a discriminative tool, but as a means to improve students' learning and to facilitate curriculum planning and evaluation. Educational assessment is seen as an integrated part of the teaching and learning processes.

That I have based this section on assessment so heavily on the APU project, and on the work of Murphy in particular, is because I find it both enlightening and encouraging that she and her co-workers through the years, and on the background of very solid and to a great deal quantitative research, have developed an understanding of the problem complex of gender and science education that accentuates the important interplay between affective and cognitive factors and the context dependency of assessment and learning. This corresponds very well with my concept of gender differences in learning style, which is built on more modest and partly qualitative data. In accord with this view, I find it inadequate to assess the outcome of learning processes independent of the context and educational conditions in general.

Gender Differences in Learning Style (2)

Learning is a very complex phenomenon in which many factors are involved and interact. The way in which persons engage themselves in learning processes and the outcome of this is dependent on various personal characteristics as well as the subject content, the context, the educational setting and the teaching style, the personal relations in the classroom, and the classroom climate in general. Learning and teaching must be seen in a broader perspective than just the interaction of an individual student, the teacher, and a text or task. Further, processes and outcome of classroom practice are influenced to a great deal by culture and society. The subject content, the context, the cues of problems or tasks, as well as the social/human interactions in the classroom are perceived and interpreted by the individual student (and teacher) in a way that is greatly influenced by the self-concept and the image of the subject formed outside school and by social norms, gender roles, and expectations. A few essential elements in this complex will be discussed below.

Meaningful Learning and Motivation: From Interest to Engagement.

Every teacher knows that it is very important for learning that students be motivated. Much research has aimed at revealing students' interests and preferences. However, even a teaching sequence in accord with such results is not always successful and, conversely, another sequence might engage the class totally, although the topic was originally seen as of little interest.

What is really significant to learning outcome is not interest or curiosity in a vague sense, but the stronger meaning of motivation related to students' engagement and commitment.

Ausubel and Novak emphasize the importance of *students' will to learn meaningfully* (Ausubel, 1968; Novak, 1977), and Fennema and Peterson (1985) describe the Autonomous Learning Behaviour (ALB) that is characteric of students who insist on developing their own understanding and finding their own procedures for prob-lem solving. In meaningful learning and ALB, students take the responsibility for their own learning.

Meaningful learning is acquired through differentiation of concepts and the relationship of them to prior concepts, that is, by integrating them in "the cognitive structure" of the learner. The student gains understanding of a new concept (i.e., the concept becomes meaningful to the student) by employment accompanied by persistent reflection. So, learning is hard work, and requires that the student be really committed and pursue true understanding.

To achieve meaningful learning, students often have to revise or even abandon earlier acquired knowledge. Such processes are often associated with insecurity and anxiety. As described in "confluent pedagogy" (Grenstad, 1983), learning is like taking off on a great expedition, one has to leave the well known and secure, without knowing the risks involved, to be able to gain the unknown award. So the learner must possess some self-confidence and willingness to take risks. It is not only a matter of "intelligence" and prior knowledge, or of interest and motivation. Even bright and interested students might have severe difficulties enduring the anxiety and therefore are (or become) blocked towards progress in a certain subject area, for example, physics (Mallow, 1986). In this sense, a kind of *"negative motivation"* has to be added to the usual positive one (Ormerod, 1983), which might influence the "resultant drive" significantly.

The way students encounter this challenge of "exploration" and pursue meaningful learning and ALB is dependent on a number of affective factors, among which are their self-concept and their attributions of success and failure. Fennema and Peterson (1985) have suggested an explanation of the observed gender differences in achievment in high-cognitive-level mathematics involving confidence, perception of the usefulness of mathematics, the sex-role congruency of mathema-tics, fear of success, and attributional style. These variables interact with and influence each other; in combination they form an internal belief system, which interacts with external/societal factors and influences the development of autonomous learning behaviour. In my opinion, these factors play an even more important role in physics than in mathematics. Mathematics is perceived as generally more relevant for students' future education, job, or life than physics, and also interest and success in mathematics is less offensive to gender-role expectations than success in physics – at least in the 1990s and in Denmark.

Gender Differences in Attributional Style.

Fennema and Peterson (1985) and others (see Fennema & Leder, 1990) found certain differences in the internal belief system of male and female students respectively. For example, more females showed a tendency toward "learned helplessness." I think it is important to emphasize that

gender differences in fear of success and attributional style (1) is a result of traditional gender-specific socialization and education, and (2) is context dependent.

The limited self-confidence and aspects of "learned helplessness" in physics education are neither personal character nor general traits of the female student. On the contrary, they seem to be a result of gender-specific teacher feedback and a teaching style with little attention to students' learning habits and attributions. Little self-confidence (or even anxiety) in one subject, for example, physics, does not imply the same problem in other subjects, e.g. mathematics (Mallow, 1986).

Too often unsuccessful students in physics, for example, believe that their more successful peers gain their understanding by a kind of "revelation," whereas they themselves are not born with the aptitude needed in physics. They experience that learning strategies that worked well in low-cognitive-level subject-areas are not sufficient in "hard subjects." Students who had success early in school are not always prepared for the frustrations and difficulties that must be overcome when the cognitive level of some school subjects suddenly increases during adolescence. Very often girls suffer more severely from this effect than boys do.

Gender and the Significance of Metacognition.

Therefore, it is very important that learning strategies are conceived as part of the curriculum and that teachers are conscious of students' attributions and try to hinder the evolution of *learned helplessness*. As stressed by Baird and Mitchell (1986), Baird, Fensham, Gunstone, Penna, and White (1991), and Baird, Fensham, Gunstone, and White (1991), students should reflect consciously on their results and learning processes to "learn how to learn." The goal should be development of *metacognition.*

Metacognition is related to responsibility of learning, and training has three aspects: increasing students' knowledge of what learning is and how it works; enhancing students' awareness of learning progress and outcome; improving students' control of learning through more purposeful decision making.

Science teaching striving for development of students' responsibility and metacognition might first be perceived as increase in the demands, but in the long run it will lead to enhanced learning outcome for most students. Some boys develop metacognition by themselves (despite the teaching?), but other boys and many girls do not. So, teachers should explicitly treat this aspect of learning. Female students tend to demand a more profound understanding of concepts and theory to be able to go on, than do many male students (Sjöberg & Imsen, 1988; Beyer et al., 1988). These "perfectionist" students would profit from a teaching style that demonstrates the fact that a thorough understanding of theoretical concepts is seldom gained in one step, but has to be developed along time by application of the concept in different contexts and careful reflections. A teaching climate that encourages students to try out their understanding, for example, by formulation of reflective questions, without pretending that they are super bright, will support the development of metacognition for all students.

Students must perceive a *true challenge*, and this again presupposes a balance between cognitive and affective factors. The demands must be neither too high nor too low. Self-confidence, the climate of teaching, and personal relations in the class are important factors, and so are the kind and amount of work required, its perceived relevance, the possibilities to get personally involved in and have control of learning and assessment (Baird, Fensham, Gunstone, Penna, & White,1991). Sörensen (1993) has emphasized the special significance for girls of being involved in certain aspects of choice and planning of their work. Surprisingly, Baird, Bensham, Gunstone, Penna, and White do not explicitly include the gender perspective in their theory, research design, or interpretations, although most of the factors they stress as significant for students' active involvement in reflective learning are also emphasized in research on gender and science education.

Conclusions

The discussion above has demonstrated that it is very important to acknowledge the complexity of learning phenomena and the significance of the intricate interplay of cognitive and affective factors. All the factors mentioned are active for male students as well as for female, but the interplay, that is, the way factors enforce or counterbalance each other, seems to a certain degree to be gender specific. There is no single-factor explanation of the observed gender differences in science (or math) education, but some factors, among them several affective ones, tend to be more crucial for females than for males. Therefore, it *is most inadequate to conceive science learning as a purely cognitive and mainly individual process, as many science educators tend to.* I will go as far as to label it *an unforgivable reductionism!*

Physics and mathematics differ in a number of ways essential to the problem of gender-polarization. Generally, physics is conceived of as a "harder" and more masculine subject than mathematics is, and with a closer relationship to technology and to powerful positions. Not only the image of physics, but also the teaching tradition, is harder than that of mathematics. Further, physics is primarily seen as important for future experts, whereas mathematics is perceived as relatively more useful for general life. In many countries physics holds the position of a critical filter subject in the educational system.

These factors interact with students' personal beliefs and external gender-specific expectations, resulting in less interest and positive motivation for physics in comparison with mathematics among girls and women. Additionally, early socialization and schooling tend to develop gender-specific styles of learning, which might lead to female resistance to the processes resulting in meaningful learning and autonomous learning behaviour.

A teaching style that ensures a warm and supportive climate that focusses on learning strategies and the development of metacognition will *enhance learning for many boys and most girls*. Teachers should be aware of students' attributions of failure and success and prevent the development of learned helplessness. The teacher also has to be aware of the gender-specific (cognitive and affective) responses of students to various contexts and perspectives. Further, students' perceptions of challenge are influenced by the possibility of being personally involved and partly responsible for planning and work processes. Having an influence on the work seems to be even more important to girls than to boys.

So, gender differences are not only related to subjects and curriculum, but also to classroom climate and teaching style in a more general sense. As physics and mathematics are often perceived of as purely objective and impersonal endeavours, the uncertain learner can easily feel isolated and helpless. Supportive human relations are even more important in "hard" and cognitively demanding subject areas and also more important to females than to males.

From my discussion of the gender-specific aspects of learning it should be evident that there is no such thing as "a gender-neutral physics or mathematics education." On the contrary, gender is always present in classroom and in learning processes. What we can hope for is to create an education that is supportive and challenging for both male and female students, a *gender-fair* or, even better, a *gender-inclusive math and physics education.*

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[1]Contact address: Alison Baignée, GASAT Executive Secretary, Canadian Council of Professional Engineers, 401 - 116 Albert Street, Otowa, Ontario, K1P 5G3 CANADA.

IS GENDER A RELEVANT VARIABLE FOR MATHEMATICS EDUCATION? THE CASE OF THE FRENCH SITUATION

Josette Adda
Université Lumière Lyon 2
Université Paris 7

Some figures published by the French Ministry of Education are of interest to our ICMI Study. Although they show a particular national situation, we can make the hypothesis that the situation is not very different in other parts of the world by comparing analogous information from some countries.

The published official French statistics present two aspects of the relation of mathematics education with gender: (a) **performance** and (b) **orientation and choice**. As we shall see, it is plain that gender is not a relevant variable for the study of the first aspect, whereas it is a relevant variable for the second aspect. It is important to interpret and explain the imbalance between these two facts.

Is Gender a Relevant Variable for Performance?

National assessments recently organized by the Education Ministry give a better understanding of the French school system. I shall quote here some of the results that were published in DEP-1991.

First are the results of the September 1990 national assessment at entrance into CE2 (3rd class of primary school with children normally 8 years old) with 88 items in French language and 71 items in mathematics, and at entrance to the 6th (the first year of secondary school, with children normally 11 years old), with 107 items in French language and 97 in mathematics. The two tables of Figure 1 show that at both levels the difference between results of boys and girls in mathematics is very little and positive or negative depending of the fields of items, whereas it appears that gender has a great influence on the results in French language, with girls doing much better than boys!

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

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TABLEAU 3 : MOYENNE DES ÉCARTS DES SCORES ENTRE FILLES ET GARÇONS TOUS AGES CONFONDUS AU CE2			
Disciplines	Filles	Garçons	Écarts F/G (en %)
Français /88	60,5	55,5	+ 9,0
– compréhension /25	15,9	14,6	+ 9,1
– connaissance du code /49	34,4	31,4	+ 9,5
– production de texte /14	10,1	9,5	+ 7,1
Mathématiques /71	57,1	57,2	- 0,3
– géométrie /10	6,3	6,4	- 1,4
– mesures /14	7,9	7,8	+ 0,5
– travaux numériques /47	31,9	32,0	- 0,3

TABLEAU 7 : MOYENNE DES ÉCARTS DES SCORES ENTRE FILLES ET GARÇONS TOUS AGES CONFONDUS EN 6ème			
Disciplines	Filles	Garçons	Écarts F/G (en %)
Français / 107	59,5	53,5	+ 11,4
compréhension / 15	10,7	10,3	+ 3,6
unités constitutives du mot /7	3,9	3,2	+ 20,1
lexique / 17	9,0	8,5	+ 5,6
morphosyntaxe / 18	9,1	7,6	+ 19,2
structure phrase texte /22	9,7	8,3	+ 16,7
production-expression /28	17,3	15,5	+ 11,4
Mathématiques / 97	71,0	70,5	+ 0,7
numération et nbres déci. /26	18,8	18,9	- 0,8
techniques opératoires /9	6,8	6,4	+ 5,8
sens des opérations / 13	9,1	9,1	- 0,4
figures géométriques / 10	8,1	8,1	+ 0,1
réception-trait-production /39	28,2	27,9	+ 1,1

Figure 1

The three tables of Figure 2 below show the situation in relationship to the socio-economic status of the father for CE2, the 6th, and the 3rd (at the end of college, with children normally 15 years old) (source DEP-1991) for students who are "on time" or who are "late" (which means that they have repeated some classes).

TABLEAU 7: SCORE MOYEN OBTENU AUX ÉPREUVES DE MATHÉMATIQUES DU CE2 SELON L'AGE, LE SEXE ET LA CATÉGORIE SOCIALE DU PÈRE				
(sur 71) Catégorie sociale du père	Garçons à l'heure	Filles à l'heure	Garçons en retard	Filles en retard
Agriculteurs	53,1	49,9	33,1	42,2
Indépendants	43,9	46,9	42,8	41,8
Cadres supérieurs	53,8	53,1	40,2	42,6
Cadres moyens	50,7	49,5	44,4	41,3
Employés	47,2	47,1	41,4	40,8
Ouvriers	43,0	43,0	38,6	37,3
Ensemble	47,5	47,1	39,9	38,6

TABLEAU 8: SCORE MOYEN OBTENU AUX ÉPREUVES DE MATHÉMATIQUES EN SIXIÈME SELON L'AGE, LE SEXE ET LA CATÉGORIE SOCIALE DU PÈRE				
(sur 97) Catégorie sociale du père	Garçons à l'heure	Filles à l'heure	Garçons en retard	Filles en retard
Agriculteurs	73,2	75,1	61,6	62,6
Indépendants	74,7	74,1	64,8	65,0
Cadres supérieurs	79,8	78,7	69,4	67,6
Cadres moyens	78,1	77,0	63,8	65,9
Employés	73,7	73,7	65,5	65,6
Ouvriers	72,0	70,3	62,5	62,5
Ensemble	75,4	74,5	63,6	63,8

TABLEAU 10: SCORE MOYEN OBTENU AUX ÉPREUVES DE MATHÉMATIQUES EN TROISIÈME SELON L'AGE, LE SEXE ET LA CATÉGORIE SOCIALE DU PÈRE				
(sur 200) Catégorie sociale du père	Garçons à l'heure	Filles à l'heure	Garçons en retard	Filles en retard
Agriculteurs	94,1	113,3	96,0	59,9
Indépendants	108,8	115,7	86,0	83,0
Cadres supérieurs	136,0	131,0	105,5	90,6
Cadres moyens	126,4	117,9	96,7	83,4
Employés	126,3	112,4	94,6	85,1
Ouvriers	103,1	100,4	85,9	74,9
Ensemble	121,9	116,2	92,4	81,1

Figure 2

It appears that the influence of socioeconomic background is much more important than gender for the mathematical performances of students, as shown in Figure 3 below. Figure 4 shows the variance according to gender, age, and social category. The relevant variables are age and socioeconomic status (many studies have shown the links between those two variables), rather than gender.

TABLEAU 11 : ÉCARTS MOYENS DES SCORES OBTENUS EN FRANÇAIS ET EN MATHÉMATIQUES PAR LES FILLES ET LES GARÇONS TOUTES CATÉGORIES SOCIALES CONFONDUES		
	Français	Mathématiques
CE2	5,6	- 0,2
Sixième	6,0	0,5
Troisième	4,5	- 4,3

Les écarts sont rapportés à des scores totaux tous également ramenés à 100.

Figure 3

TABLEAU 1 : UN PRÉJUGÉ TENACE NE RÉSISTE PAS À LA RIGUEUR DE L'ANALYSE DE LA VARIANCE
Pourcentage de variance expliquée (R^2) par le sexe, l'âge et la catégorie sociale du père de l'élève

	Français	*Mathématiques*
CE2	*18,7*	*13,1*
Sixième	*28,0*	*19,5*
Troisième	*9,1*	*18,6*

Part relative du sexe, de l'age et de la catégorie sociale du père de l'élève dans le pourcentage de variance expliquée

		Français	*Mathématiques*
Le sexe	*au CE2*	*13,7*	*5,7*
	en Sixième	*11,3*	*0,2*
	en Troisième	*35,1*	*3,6*
L'âge	*au CE2*	*45,7*	*46,4*
	en Sixième	*67,3*	*78,4*
	en Troisième	*50,2*	*73,5*
L'origine sociale	*au CE2*	*40,6*	*53,6*
	en Sixième	*21,4*	*21,4*
	en Troisième	*14,7*	*22,9*

Figure 4

Is Gender a Relevant Variable for Orientation and Choices?

There is a shocking contrast between the figures quoted above, showing the abilities of girls in mathematics before the levels of orientation, and the figures below showing the reality of the scientific choices after orientation.

Figure 5 gives the distribution of students according to gender in the sections of the terminal class of secondary school (preparing the "baccalaureat") in 1990-91 (published in DEP-1992). To understand the situation it is useful to know that the sections with most mathematics are C and E. D is scientific but especially for biology, A and B are the literary sections, and F, G , H the technical ones:

Le second cycle général et technologique par section

[1] RÉPARTITION DES ÉLÈVES PAR SEXE ET SECTION – 1990-1991
(France métropolitaine)

	Public			Privé			
Section	Garçons	Filles	Total	Garçons	Filles	Total	Total
Terminale :							
A	12 998	58 392	71 390	4 567	15 299	19 866	91 256
B	27 087	44 218	71 305	10 127	12 047	22 174	93 479
C	37 517	21 688	59 205	8 701	4 515	13 216	72 421
D	29 775	30 917	60 692	10 695	8 748	19 443	80 135
E	9 970	639	10 609	674	57	731	11 340
F	36 228	13 732	49 960	4 888	5 478	10 366	60 326
G	27 103	55 101	82 204	7 820	12 666	20 486	102 690
H	139	65	204	196	50	246	450
Brevet de technicien	7 712	2 181	9 893	979	940	1 919	11 812
Total	188 529	226 933	415 462	48 647	59 800	108 447	523 909

Figure 5

It is interesting to represent these numbers graphically in two ways: the distribution of 100 students of each gender in the final year by section (Figure 6), and the distribution of 100 students in each section by gender (Figure 7). (See Adda, 1993.)

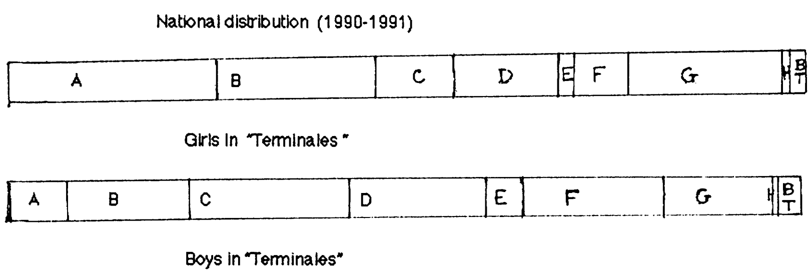

Figure 6

Interpretation

In our modern world, it is clear that these orientations are not equivalent, the possibilities for the best jobs and good appointments are linked with the scientific orientations and so the scales of Figure 6 are very similar to the social scale; that "choice" looks very difficult to understand after having remarked the equalities of the possibilities. How can we explain that distortion? I think that it is the consequence of the conjunction of **prejudices.**

Prejudices of the parents: French parents seem to feel a good job is less necessary for girls than for boys (they think that girls will be taken care of by their husbands) and when they have to make financial choices (for example, to pay for additional mathematics lessons), they select brothers over sisters.

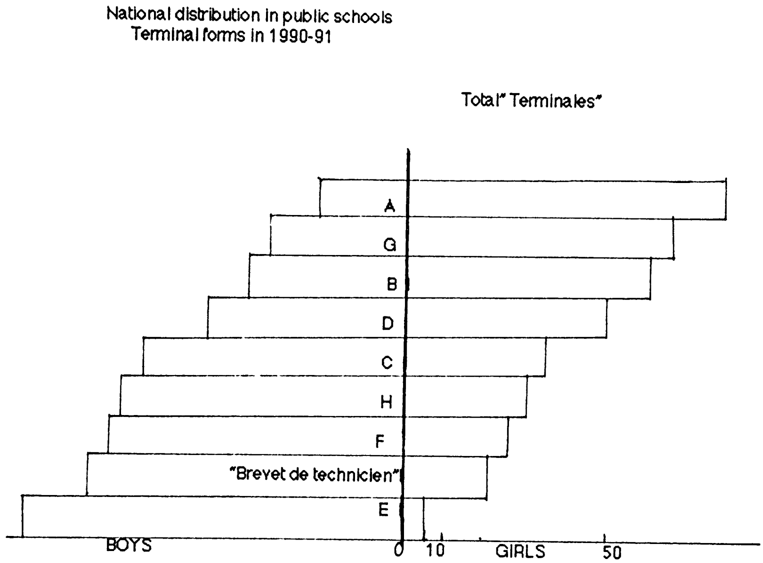

Figure 7

Prejudices of the teachers: During the school years, orientations are the result of teachers' decisions, and it appears that teachers (both male and female) require more from girls than from boys for scientific orientation: the best proof of this is that the proportion of success in baccalaureat C is often higher for girls than for boys. Gasquet and Ruffieux (1990) note that in some places the girls of section C are so overselected that they have a 100% success rate!

Prejudices of employers: The inequality of judgment of teachers is not completely absurd: when applying for a job, a female has to be better than a male to obtain the job – if they have equal qualifications, it is well known that the employer will choose the male (there is always the argument about absences for maternity leave, and now in some places in France it appears that to obtain a job women have to swear that they will not become pregnant in the first two or three years; that is, of course, completely illegal).

Prejudices of students: All the prejudices above are integrated by the students and they combine to push the girls to renounce scientific studies.

There also exist some specific prejudices well known in the community of mathematics educators, especially the image of femininity, the "fear of success" and other psycho-sociological bridles studied by Fennema and Leder (1990). In fact, all those prejudices contribute to the constitution of gender as a "social condition," as was said by Fennema in her lecture at the present conference.

Is it possible to **change** the situation? All prejudices can be destroyed by information and knowledge. The book *Allez les filles*, written by the two sociologists Baudelot and Establet (1991), has had public success. The French association "Femmes et Mathématiques" takes opportunities to present our problem in newspapers and particularly in women's magazines. This association, in conjunction with the Association of Mathematics Teachers (APMEP), organizes information meetings for mathematics teachers and secondary students (see Adda, Goldstein, & Schneps, 1994) . The French government has mounted a national campaign of information: in schools there are posters and documents (papers, booklets, and even a video) especially in orientation offices, telling girls "c'est technique, c'est pour elle" ("it is technical, it is for her") and showing girls happily doing technical and scientific jobs. A national scholarship was created in 1986 to help girls going into scientific or technical studies; for the past two years it has been a "Prix de la Vocation," awarded to 40 girls in each region among students in the final year of secondary school who want to continue their studies in the scientific or technical domain. As a member of the jury for that prize, it is interesting to read the essays of these girls – it is clear that they are aware that to succeed in science study is not enough for a girl; they are very conscious that they will also have to fight against many social constraints and prejudices.

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DEVELOPMENT AND EVALUATION OF A GENDER INCLUSIVE CALCULUS

Mary Barnes
University of Sydney

Introduction

In 1988 the Australian Government set up a program to encourage the increased participation of young women in mathematics and science. It sponsored a number of curriculum development projects, one of which is described here.[1]

In Australia, calculus is included in mathematics courses offered to the more capable students during their last two years of high school, that is, when they are between 16 and 18 years of age. Alternative courses, which involve no calculus, are provided for students of lower attainment. The Introductory Calculus Project was asked to develop materials aimed at making calculus accessible to a wider range of students in Australian high schools, especially girls, and helping them to find it enjoyable and rewarding. The materials were designed to be used in a flexible way, so that teachers in different states could easily adapt them to meet the requirements of local syllabuses.

During 1990, draft materials were tried out in schools in four states, and the results used to revise the draft before publication. The outcome was a series of 10 booklets and 2 teacher's guides, entitled *Investigating Change: An introduction to Calculus for Australian Schools* (Barnes, 1991, 1993).

Teaching and Learning Calculus

Why calculus was chosen. A main objective of the project was to provide an exemplar of gender-inclusive curriculum at the senior school level, to show that senior high school mathematics courses can be made more widely accessible than the traditional courses and still be challenging and interesting for all students. Calculus was selected as the topic mainly because of its role as a "gatekeeper." Many employment and other opportunities are barred to people who have not studied it, and familiarity with its ideas and techniques is required by a wide range of tertiary courses. Despite recent changes in schools that encourage girls' participation in courses that include calculus, they are still chosen by a smaller proportion of girls than of boys.

Teachers' views about calculus. At the beginning of the trial, teachers were asked about students' reactions to calculus courses in the past. A common reply

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was that students have preconceived ideas that calculus is going to be difficult, which cause them to give up too easily. "They expect it to be difficult. So even if it's not, they are already thinking 'It's going to be difficult,' so they defeat the purpose before they've even begun, I guess." (MT, N3).[2]

Teachers thought this attitude resulted from the level of abstraction, the perceived lack of relevance, and the fact that there is usually too much content to cover and too little time available. "You tend to be more abstract – maths for maths' sake – and teach now, understand later when the applications come." (MT, V1) "It never seemed very relevant with a lot of the kids ... There was always so much content to get through, that you could never stop and explain to them why." (FT, A)

The teachers felt that many students never understand what calculus is about, and learned little more than how to carry out routine calculations and do "number crunching." Learning calculus appears to be a pointless exercise for many students, undertaken only to gain university entrance qualifications, and not out of any real interest in the subject.

Students' attitudes to calculus. Before the trial began, we asked the participating students what they had heard about calculus. Many had heard nothing at all about it. A very few, more of whom were boys than girls, were aware that it was useful in "engineering, science, astronomy, that sort of thing." Others thought of calculus as "the real maths" and said they felt "grown up, intellectual, mature" now that they were studying it.

A substantial minority, mainly girls, said they had heard that calculus was "pretty hard." This information had been passed on to them by siblings, senior students, friends at university, and parents. One girl said her parents kept saying, "Wait till you do calculus" (F, V2). Another told us "Mum said she felt sorry for me" (F, N3). We were also told that "Students who have heard about calculus think 'Help!' and freak out." (F, W3) Another girl said, "I've been told by a lot of people that I'll fail calculus, it's really hard." (F, V2)

This suggests that there is a mystique associated with calculus. It is "the real maths" – difficult, abstract, irrelevant, something that ordinary people cannot be expected to understand, especially if they are female. This helps to explain why less confident students, many of whom are girls, tend to drop out of calculus courses.

What We Set Out To Do – Breaking Down the Mystique

The main aim of the curriculum we were planning was to break down the mystique associated with calculus; to show that it can be meaningful, useful, and enjoyable; and, above all, that it is a subject of value to girls as well as boys, and that they can be successful in it. Sue Willis (1989) has argued that a curriculum inclusive of girls should

> expect to embed mathematics in real-world social concerns and people-oriented contexts; presenting it as making 'humansense', non-arbitrary, non-absolute and also fallible; and presenting a social-historical perspective to help students become aware of the 'person-made' quality of mathematics. Further, we would emphasise the aesthetic and cultural values of mathematics as well as the instrumental values, rejecting narrowly instrumental interpretations of mathematics while demonstrating that its instrumental uses are very broad and not restricted to a few scientific careers. (p 38)

This was the kind of curriculum we set out to create. Some of the special features of the student and teacher materials produced are described below. A more detailed description can be found in Barnes (in press).

Investigative activities. We wanted students to gain a sense of achievement and to feel "ownership" of the mathematics they are learning. With this in mind, we planned a curriculum based on open-ended problems and carefully structured investigative activities, in the belief that if students can discover mathematical relationships for themselves, they will experience the satisfaction of creating their own mathematics and may begin to appreciate the nature of the mathematical process.

The series begins with a set of problems set in "real world" contexts. Students work on these in groups, drawing on their existing knowledge and understanding to devise ways of tackling them. They are encouraged to experiment, make estimates, form and test conjectures, and try to find explanations. Here is one of these introductory problems:

> *Logging the forest.* You have obtained aerial photographs of a section of bushland where logging has taken place recently. From the photographs it is possible to make out which parts have been logged and which are undisturbed, but the logged areas have very jagged boundaries. The logging companies claim that less than 10% has been logged, but conservationists say it is much more. How could you check these statements? If someone claims that your answer is not worth paying any attention to because it is only an approximation, how would you reply? How could you get a more accurate answer?

A photograph showing a region of logged forest is provided, for students to try out their ideas. This activity lays foundations for work on areas, which leads eventually to integration. Other introductory problems prepare for work on functions and variables, rates of change, and maxima and minima.

Many of the activities in the series use computer graphing utilities. Students compare graphs of many different functions, explore their properties, look for patterns and similarities, form generalisations and use the computer to test them on other functions. Computers help students to form powerful and memorable visual images of new concepts.

Collaborative work. Throughout the course, students are asked to work together in small groups, to verbalise their ideas and, through discussion and negotiation, to develop shared meaning. Many activities use a sequence such as:

- think about the question by yourself and try to visualise the situation;
- describe it to others in your group and say what you think will happen;

- discuss any differences in your answers and try to reach agreement;
- write down what your group decides;
- test your ideas on a computer or calculator or in some other way;
- and then try to explain or justify your answer.

Activities like this promote the development of visualisation and communication skills that are important for success in mathematics.

Encouraging reflection. Students are encouraged, alone or in a group, to reflect on the work they have done and the conclusions they have reached, and to evaluate their own efforts and their understanding of the material. David Clarke, Max Stephens, and Andrew Waywood (1989) have pointed out the value of reflective activities, both in encouraging students to construct meanings and make connections and in facilitating communication between teachers and students. At the end of each section is a series of "looking back" activities, involving working with others to prepare summaries of what has been learned in the unit. Students are asked, for example, to pick out the "big ideas" and explain how they are related to one another and why they are important, and to explain meanings and give examples of key terms and notations.

Real world contexts. As described above, the curriculum is problem-driven, developing mathematical ideas out of practical contexts, and dealing wherever possible with students' own experiences or with social issues that are of concern to many young people today. We hope these approaches will make calculus seem less abstract and more relevant to students' interests. In particular, we want to get away from the emphasis in most calculus courses on applications to physics and engineering, usually more familiar to boys than to girls.

Doing mathematics in practical contexts involves modeling. The text discusses the process of setting up a mathematical model, asking what variables are relevant, what factors can be ignored and what simplifications need to be made to obtain a model simple enough to handle. It emphasises the need to use one's judgement to make decisions. Later, students are encouraged to question the assumptions, to think about how to improve the model, and to investigate alternative models.

Mathematics as a human endeavour. Mathematics is often presented as a completely culture-free body of knowledge – absolute truth, eternal and unchanging. Many women have reported that it appeared to them to be an impersonal subject, in which imagination and creativity had no place, and they felt they could not relate to it (Buerk, 1985; Isaacson, 1990). To change these perceptions, we want students to realise that mathematics is growing and changing, that it is created by real people, and that its development is influenced by values and problems derived from the broader society. One way of dealing with some of these issues is to look at the modeling process, as described above.

Another approach is to discuss the historical development of mathematics.

Gary Brown has described how he has used the history of mathematics in teaching mathematics courses from a humanistic and cultural viewpoint, and Patricia Perkins reports that she has found the use of history to be successful in a girls' school (Russ et al., 1991).

The text contains a number of historical sections that aim to show how calculus developed. For example, it discusses some of the problems in which people were interested at the time calculus was invented and that led to its creation. There are stories about Newton and Leibniz and others that try to show how they thought about calculus and to explain some of their ideas and their notation, and there are outlines of the way people's thinking about some topics changed over time. One section deals with the development of the function concept and another discusses how people worked out areas of regions with curved boundaries at different periods in history.

Accessibility. The text is written in a friendly style, using informal language at the beginning, and carefully explaining new terms and notations before using them. We wanted students to be able to read it for themselves, so they could become more independent and not always need to rely on their teachers for explanations.

The materials have been produced as a series of slim booklets, easy to carry around and use, instead of as one large heavy text. An attractive layout was designed, with interesting illustrations and wide margins that can be used for additional notes. We did not want students to find the books off-putting at first glance, like many mathematics texts.

The Trial

Ten schools, 14 teachers and over 250 students were involved in the trial. Nine of the teachers were male and 5 were female. The teachers and selected students from each class were interviewed before the trial began and on two subsequent occasions. In addition, all students in the classes concerned completed short questionnaires at the beginning and end of the trial. Although the main purpose of this was to gather ideas about how to improve the text materials before publication, the trial process also gave valuable insight into the responses of students and teachers to this innovative curriculum, and into their attitudes to the teaching and learning of calculus. The rest of this paper reports some of these results.

Investigation and Open-Ended Questions

For most of the students who took part in the trial, learning mathematics by means of investigative activities was a new experience. There were some initial misgivings: "We didn't really have much of an idea what it was leading to or what it actually was.... I couldn't actually sit down and say 'OK this is calculus, this is what you do' I couldn't say it to someone else." (F, N3)

In general, however, students appear to have overcome these misgivings, and to have derived a great deal of satisfaction from investigative activities. They liked the idea of finding out things for themselves and thought this would mean that they remembered them better. They enjoyed the challenge and felt that it gave them a much better understanding of important concepts, and so increased their confidence in their mathematical ability. Some seemed to experience a sense of increased power from realising that they could work out things for themselves if they needed to. Girls expressed such feelings more strongly than boys, but this could be because girls were much more responsive than boys to the interview questions generally. Typical examples of what they said include: "It makes me think ... I like to have a problem and sort it out. It just feels good when you've broken through it all and you finally get the answer ... I think that's very satisfying." (F, W2) "You feel like you've conquered something and you know how you get it and then you use the formula and it's really good." (F, N4)

> It is like you're searching and searching for something but you don't know what it is – it's like you're blindly feeling around – but also when you learn a new topic it does give you a lot more confidence because ... you don't just get a rule, you know why this rule works, and you know how this rule has been found. (F, W2)

Students seemed to experience "knowing why this rule works and how this rule has been found" as something new in mathematics. On the other hand, some of them could not see the point of investigative activities. They preferred to learn mathematics by a more traditional approach, what one called "hard-core teaching," in which "you are given a formula, it's there, you go and do it." This is what they were used to, and most of them had been successful at it. They measured success by the number of textbook exercises they completed. "We ... like to get the formula and just get on with it, but a lot of girls ... liked it because they saw what it was all about before you got onto it." (F, N1)

> I like to be shown as well, so the conventional teaching method appeals to me more. If I can't work something out I get frustrated, but if someone shows me then I can go back and understand it. By myself I may just leave it, so I learn more if I'm told. (F, W1)

Most of the teachers were concerned that it would take them too long to cover the syllabus by this method, although some of them realised there was a payoff.

> I have been so overwhelmed by the insight that these kids have about graphs and the derivative function and what the derivative does, that I can't say it was a waste of time because they are just so much more powerful than ... the previous group [students in the year ahead]. Their skills are just so different. Their ability to justify things like the chain rule, well I found that truly breathtaking. (MT, W2)

> Well probably the weaknesses in the long term are that it takes time to get through this sort of approach, and people will say "Well where will I get the time from?" But then you have to weigh that against "What will the kids know at the end of it, and how useful will it be to them?" and I think probably the usefulness and what they understand is going to outweigh the time factor. (FT, W3)

> So I think that the most enjoyable part was watching the kids discover quite a few things

> that we would normally give them a bundle of rules for, just have told them. ... It has changed my way ... of teaching calculus in that I will let the kids discover things rather than just going and telling them "Here's the rule". (MT, W3)

I find it a devastating criticism of traditional mathematics teaching that many students expect to be given "a bundle of rules" without explanations, and many teachers see it as normal to do this.

Working Collaboratively

Although some teachers were already experienced in using collaborative work in their classes, many were not. A few were very uncertain about whether it would lead to effective learning, and worried that the classroom would be too noisy or that students might get off-task. One or two made very little use of group activities, preferring to have students work individually from the booklets, supplementing this with chalk-and-talk when they judged it necessary. A girl in one of these classes said, "I think we'd get more out of it if our teacher let us do it in groups more." (F, W1)

Most students, too, had done little formal group work in mathematics lessons at the time the project began, although in some classes informal cooperation among groups of friends was common and was encouraged.

Many students were initially doubtful about how much they would learn. They were concerned that they might get into a group in which some people were not interested in working, and made remarks about people who "bludge" or "don't pull their weight." In most cases, these doubts were dispelled by their experience. "Because there's a limited amount of computers people actually have to work together. And some people don't like working with other people but ... when they do work together they actually enjoy it." (M, W3)

However, very many students qualified their comments by remarking: "It all depends on who you have in your group." Some felt it was better not to be with your best friends because you were more likely to gossip instead of concentrating on the task; others said they felt more comfortable working with people they knew well. "Some people you work better with and you are more open. ... because if you say something silly you don't feel as bad." (F, N1)

Both females and males appreciated the support of fellow students in their group, pointing out that working with friends had the effect of increasing their participation and building their confidence. "You are not so worried, so you do participate more." (M, N2) "We're relying on each other for confidence." (F, W2) A boy said that when working in a group "you can feel free to be wrong." (M, N2)

There are, however, some students who will always prefer to work alone. This preference may be related to a view of learning mathematics that involves doing a large number of exercises rather than, or as a way of, developing understanding.

A few were concerned that discussion can lead to confusion. They were looking for certainty, clear-cut rules and a single right answer, and did not enjoy "argument, confrontations, 'Are we right? Are we wrong?' " (M, N2) One girl recognised the importance of verbalising your thoughts in order to clarify them. "If you can say to someone else what you think it is, then you know that you actually do think that, 'Ah, so that's what I think!' " (F, W1)

Other students took a little time to work out how to function most effectively when working in groups, but they showed responsibility and maturity in the way they thought about these issues. "Whatever it is the group are doing, I think everyone should produce something of their own, and then get to compare and see the differences." (F, N1)

Generally the majority of students preferred working collaboratively, especially the girls. They liked being able to share ideas and obtain different points of view on a question. They realised that several people working together could solve problems none of them could have managed alone, and that they could help one another to understand.

Using Technology

Most students loved the computer activities. Indeed, many reported that this was what they liked best about the whole course. "I actually looked forward to a maths lesson for once." (F, N1) "It's a fun way of doing maths." (F, V1)

Before the project began, a number of students who had little prior experience with computers expressed helplessness when they were told that the course would involve some computer work. This group included males as well as females; the issue here does not appear to be gender, but access to computers. "I don't know anything about computers. I don't know how to use them. I just stay right away from them." (F, V1) "I don't like it when we use computers, 'cos like, you've got kids in that class that can do it with their eyes closed and write out programs, and I don't even know how to type. It's too hard, computing." (M, N2)

Once they had actually begun using computers in the course, these attitudes were largely dispelled. Teachers did not report any gender differences in the responses of girls and boys to computer activities, although one noted, "It is good to see a couple of the girls who have never used a computer before and are quite confident with this one." (MT, N3)

A few students failed to understand the point of the computer activities. They saw the computer as simply a way of drawing graphs more quickly and more neatly than by hand, and were unimpressed. "It did it all for you and you just got to see what it looked like. You didn't really have to think about it." (F, N3)

The majority, however, found the use of computers motivating and recognised that the computer activities led to better understanding. "You're not using your time for petty things that the computer can do in a few seconds. You can spend more time actually working out problems." (F, W2) By using computer graphing utilities, students were able to build up visual images, for example of functions and their derivatives. They found they were able to use these images in solving problems when they were away from the computers. Some students commented: "When we were doing the exercises we could sort of visualise." (F, N1). "Well, you get to see, you actually get to see – we are all working on graphs, and you actually get to see what they look like, rather than just a whole heap of algebra." (F, N1) "It was good using the computers, you could actually see everything happening and when you're working out the gradient functions and that, you could actually see the lines ... creating tangents and that ... With a computer you can actually see that they're not lying to you." (M, W3)

The value of computer activities in a course such as this seems to be threefold. Firstly, it provides a context for a great variety of investigative activities that are interesting and motivating for both females and males. Secondly, having a group of two or three students working at one computer is an effective way of introducing students to collaborative work. They are focussing on the computer, rather than on one another, and this may be an advantage while they are still trying to find ways of working together effectively. Thirdly, computer graphing activities can strengthen students' abilities to visualise graphs of functions, and make it easier for them to think visually when they are away from the computer. In view of reported gender differences in spatial visualisation skills (Connor & Serbin, 1985; Tartre, 1990), we can expect girls to be advantaged by any techniques that promote improved visualisation skills.

Reflection

One of our objectives was to encourage students to reflect, both on the content they were learning and on the learning process, but this may not have been made sufficiently clear in the draft materials, to either students or teachers.

It appeared that many of the teachers did not appreciate the importance of reflection in learning mathematics, or did not think that the looking back activities suggested would be effective, and so omitted these activities. One teacher referred to them as "busy work." As a result, students were often not very sure about what they had learned, because they had done nothing to pull it all together. In some cases, teachers prepared summaries themselves and gave them to students, which appears to be what they frequently do when using the standard textbook. However, having students make their own summaries helps to develop important learning skills, and students who avoid this activity may be less well prepared to take responsibility for their own learning.

Some of the girls interviewed showed a great deal of thoughtfulness and responsibility about the way they worked and had found techniques that led to effective learning. One pair of girls described how they realised the importance of reflecting on what they had done, and made an effort to pull it together in a summary.

> We hadn't put it all together and we hadn't understood it fully. We just thought about it, we worked it out in the end. The crunch was we didn't understand it – we couldn't derive the formula from the graph ... what we did – we worked out a summary like that and that's how we remembered it – we got all that – but putting it all together was complicated at first. (F, W3)

Another group explained that at the start one person acted as recorder for the group, and wrote down their answers to the questions posed in the text. Then they realised that when they did that, only one person ended up with the piece of paper with the answers, which made it difficult for the others to work at home or to revise, so they decided that they would all write down the conclusions. These same girls had also thought about how to get the most out of the work with computers, realising that passively watching the computer draw graphs was ineffective.

> You have to make sure on the computer that you just don't plug it in and draw the graph – you have to actually say in the group, "What is going to happen?" and make sure everyone in the group understands why – and then do it. We just plugged it in for a while and then we didn't learn anything. But then at the end we did realise it was a whole lot easier. (F, W3)

Few boys reported thinking deeply about what they were learning or how they learn best. There is no way of knowing whether this indicates a failure to reflect on these matters or simply a failure to communicate their thoughts.

Real World Problems

The use of problems drawn from real world contexts clearly added to the appeal of the text for nearly all students. "It was good how it was made interesting with real life situations, rather than just applying numbers." (M, W1)

Working on these problems does seem to have changed the perception of the relevance of calculus for many students, especially girls. "The whole book makes maths seem so much more real ... you're sort of solving real problems. We are now. Falling bricks and throwing balls in the air." (F, W2) "The thing that's best about them is how they're related to life. Most of the problems were practical. It makes it easier." (F, V2)

Teachers, too, appreciated this aspect of the materials. "Calculus has traditionally used examples that made it a boys' domain. The girls are doing better because the examples are more interesting to them." (MT, V1)

As some students recognised, problems dealing with familiar situations are not only more interesting, but can also be easier – students can use their intuition about the situation to help make sense of what they are doing and to judge the reasonableness of their answers. However, there is a reverse side to this. If the context is unfamiliar, the problem becomes much more difficult, because clues that could help students to make sense of the mathematics are missing. Instead, there is the added difficulty of a new situation and possibly unfamiliar language.

One way of dealing with this is to bring in practical activities that will provide students with experiences needed to make the situation real for them and, where it was feasible, such activities were suggested in the text.

One objective of the course was to raise students' awareness of the wide range of applications of calculus. Inevitably, this meant that some problems dealt with unfamiliar topics. For example, applications to economics caused consternation in several schools because they involved ideas that were new to both teachers and students.

Using practical applications also involves developing awareness of the process of mathematical modeling. This was quite new to some students. They were used to doing standard types of exercises that were largely a matter of applying the correct formula. Some liked the open-endedness of some of the problems and the fact that at times they were expected to use their judgement to decide what was appropriate. "It's good in a way because you have to pull out the information you need." (F, A) Others found this worrying. "You had to think about it a lot more. There were no real formulas." (M, N2)

A few students felt insecure when they were presented with questions that did not have a single correct answer, or problems that were incompletely defined. "There was one about cardboard being cut out to make boxes. You didn't know how much cardboard and there wasn't enough explanation." (M, N3) Students are not used to having to make decisions like this about how to interpret a question, nor are they accustomed to having to read questions carefully to decide which of the information given is relevant. And they need to learn to think about the assumptions that underlie the model they are applying, to decide whether they are reasonable, and to state them clearly. This sometimes led to considerable discussion. "I thought 'Well, hang on a minute, you have to make your assumption clear.' ... They have been encouraged to sort of dispute me, because sometimes this involves my own interpretation." (MT, N2) This is very different from what most students have been accustomed to in mathematics.

Using History

A few students, all girls, indicated that, as a result of the historical sections, they had begun to realise that mathematics was a meaningful human activity, that "real people actually discovered these things." (F, W1) Another said that it was "better to know the roots of everything, where everything developed from." (F, N1)

> They [the sections about history] were good because it showed where it came from. We weren't just jumping into something where we didn't know what was going on. It showed us that there was actually meaning behind it and it was thought out. I enjoyed it. (F, V2)

Many students said that the historical sections provided a pleasant change from doing mathematics. "It does make maths more interesting. It makes you think a bit more about maths." (M, W1) The most common response to these sections of the text, however, was to say that they were interesting, but irrelevant. One student described them as "trivia." Another said, "I didn't think it was all that relevant – I'm more interested in the work we have to do." (M, V1) Some students went much further, and said they thought it was a complete waste of time. "It's nothing that you need to know about. ... When would you ever use it?" (F, W2) and "I thought that that was a bit of a waste of time really." (M, W3)

Two classes who were given assignments on aspects of the history of calculus expressed great resentment.

> It was all right to have the questions there, 'cos I suppose if you really wanted to you could think about it, but the fact that we had to do them, go home and do them for homework and actually hand them in – lots of people weren't used to doing that in maths. ... It didn't seem fair to do something that had no point towards our exams, because we knew we weren't going to be tested on it. We just felt like it was a waste of time. (F, W3)

These attitudes are clearly a consequence of assessment requirements. Students are aware of the kind of questions that will be set in external examinations, and learning how to answer these is the only thing that matters to them in mathematics. The whole mathematics course during the last two years of high school is therefore dominated and distorted by assessment requirements. This seems to have the greatest effect on the highest achieving and most ambitious students, who have the most to gain from success in examinations.

Making Mathematics More Friendly

We set out to produce a text that both male and female students would find "user-friendly," and tried to achieve this by the use of informal language and style and applications dealing with people rather than machines. We also tried to make the text less authoritarian, to move away from the idea that there is a single correct answer for every question, and to avoid telling students anything that they could work out for themselves. Most students appreciated these differences from the style of their usual textbooks, saying, for example, "It didn't intimidate us, not like a lot of maths textbooks do." (F, N1)

We took care to keep the language as simple as possible, introducing new terms gradually, and explaining the meaning of new notation. Students noticed this and appreciated it, saying, for example, "Some text books are too confusing. This book is presented in our language." (F, W1) and "It doesn't use terms and signs you've never seen before. It's related to real life problems and they run through each one clearly. It's a lot easier to understand for a beginner." (M, V2) Teachers reported that students who missed a lesson were able to catch up by themselves. "It's made my teaching much easier as it involves less explanation from me which would normally take up a fair amount of my time.... It means I have more time to get around to assist students individually." (MT, V1)

There was, however, disagreement among students about the length of the explanations and the wordiness of some of the problems. Some of the higher achieving students felt that there were too many words and that it was too repetitive. Others felt that they needed all the explanation they could get and did not think that there should be any cuts. A partial solution to this difficulty may be to encourage able students to learn to skim read judiciously.

Students really liked the slim booklets, which were light and easy to carry around. They also liked the illustrations and the wide margins with additional comments in them. The different appearance of the books, however, led to concern on the part of one or two teachers. One was worried that "because it doesn't look like their usual textbook, the boys won't think it's serious mathematics, and won't take it seriously." (MT, N2) Teachers seemed to feel a need to convince their students that they were learning "real mathematics." One tried to do this by showing that, when they had completed work on one of the units, they were able to solve problems from their normal textbook.

Gender Differences

Working with others. Several teachers observed that girls worked better than boys in group situations and were better in discussions. "Kids who didn't want to share their ideas were boys. Girls seem more ready to share and tend to talk more freely." (MT, V1) Some teachers thought boys tended to waste more time and were more disorganised.

There was indirect evidence confirming this from interviews with the students. Although both girls and boys enjoyed the group activities, many of the girls had clearly given a great deal of thought to ways of organising their groups so as to gain the most benefit from the activities.

Wanting to know why. Some teachers reported that the girls in their classes were more concerned than the boys to understand the material and to know why rules worked. "The girls probably have a tendency to want to know why. The boys are happy just to do it and come up with the answer and think that's great." (MT, V2) Another said: "[Girls] ask 'Why?' They always want to know, 'Why, why?' Whereas boys, on the whole, if you tell them that something is something, they'll say 'Fair enough,' and they'll go on with the work, but the girls need to have ... a reason as to why something is so." (MT, N3)

These claims need further investigation. My own experience tends to confirm them, and similar observations have been made in the past (see, e.g., Department of Education and Science, 1989), but it is not clear how generally they apply. There were both girls and boys in the current study who told us that they preferred the traditional approach, of being given a formula, shown how to do exercises, and then getting on with plenty of practice. If it is indeed true that girls place more emphasis than boys on understanding, this would go a long way towards explaining why girls are more likely to drop out of traditional mathematics courses. Wanting to know why indicates a more mature approach to study and a better understanding of what mathematics is about. The approach adopted in these materials should suit these girls well, because it emphasises careful development of new concepts. Students are given the task of constructing the mathematics for themselves, and searching for explanations until they are satisfied that they make sense. The "memorise and practice" approach, however, may well lead to better results in traditional tests and exams.

Teachers' perceptions of assertiveness and ability. Some teachers thought boys were more assertive in class, but others said the same of girls. Sometimes assertiveness or aggressiveness was linked with creativity, but it seems that this only happened when it was the boys who were being assertive. "The good girls were good routine workers, let's say. The aggressive or creative students were, in my limited experience in this area, the boys." (MT, W2)

This is reminiscent of Valerie Walkerdine's (1989) observation that teachers often devalued girls' achievements by saying that they are "good routine workers," whereas boys' more aggressive, rule-breaking behaviour is associated with creativity. In contrast, another teacher reported, "I would say on the whole, the boys are better. But then again I've got to be careful there because the vocalists in the room are girls, they tend to speak out more and the others sort of follow." (MT, N3) Here, although the girls are being assertive, the more quietly behaved boys are judged to be "better."

Looking Back on the Trial

At the final interview, students and teachers were asked to reflect on the whole of the course. One teacher described the satisfaction her students experienced when they had come full circle and were able to use calculus to solve the problems discussed at the very beginning. A girl thought that the course was "a gentle lead into calculus." Others felt a little disappointed that calculus had failed to live up to its fearsome reputation.

The reaction from some of them was "You mean that's all calculus is?" and I said "Well that's part of what calculus is. We've looked at one little part of it." "But everybody said that it was hard." and I said "Well, yeah, it's a hard idea." But then they start to say to you "But it's not" and they had this let down feeling. (FT, W3)

Many students were extremely positive; for example: "I've learnt more in this semester than I've learnt practically in the whole of my maths career." (F, A) We need to think about how to ensure that many more students, both girls and boys, will have similar experiences, and find mathematics interesting and rewarding.

Other students, both male and female, thought the whole exercise had been "a total waste of time" and said we should give up the project and go back to the old way of teaching calculus.

Conclusions

Effectiveness of the materials. We set out to create a curriculum that girls, as well as boys, would find interesting, motivating, and enjoyable and that would therefore encourage their increased participation in mathematics. We also wanted to make it empowering – we wanted students to experience a sense of achievement as a result of the mathematics they were creating, to feel in control of their learning and to realise that it could help them to make sense of the world around them. They should be able, not just to do mathematics, but also to talk about it and justify to others what they have done.

Some of the strategies adopted seem to have been effective in helping to achieve these objectives. The investigative activities, especially those using computers, received positive responses from almost all students. A majority also enjoyed working collaboratively and found that this helped them to solve problems and make sense of the mathematics they were doing. Efforts to make the text accessible were effective with most students, but less so with some high achievers, who thought the explanations too long-winded and apparently directed towards students of lower ability.

Other strategies were less successful. There was little evidence that including sections about the history of calculus did much to awake interest in mathematics or to change students' perceptions of the nature of the mathematical process. And so few classes used the reflective activities that they can hardly have been said to have been given a fair trial.

Implications for curriculum change. A text such as *Investigating Change* can support a teacher who wants to introduce a more open, exploratory, collaborative, and accessible style of teaching, and it can provide more autonomy to students by giving them more responsibility for their own learning. They can read the text for themselves, instead of depending on explanations by their teacher, and they are encouraged to evaluate their own work and develop ways of judging whether their solutions make sense. But it is important to be aware of the limitations to what can be achieved by written materials. The teacher's role in determining how to implement these ideas in the classroom is crucial.

Most of the teachers in the study found that using the materials in the spirit intended involved a change in their style of teaching, and many of them found this difficult. One or two felt uneasy about the fact that they were no longer the sole source of knowledge and authority in the classroom. Some feared a loss of control when students were working in groups. Others worried about how best to organise activities so that students could discover results for themselves, especially within the constraints imposed by the timetable.

> I find it often quite difficult to help students to discover things for themselves. By all means it's the best approach and kids get a wonderful sensation when they are able to discover for themselves and that little light bulb appears above their heads, but often while they are grappling for the solution, you're beside them trying to hold back too many

> prompts and the kids are begging you to give them the answer. I find that sometimes very difficult and I also find that finding the time within the class to develop this idea and reach a solution in the last ten minutes. I find that it sometimes just doesn't work out that way. Sometimes the kids need a lot more time and because ... it is less structured it does become a difficult thing to handle in a class. (MT, V1)

If innovative curricula, such as *Investigating Change*, are to be more widely implemented, teachers need support in making the necessary changes in teaching style and in working out how to organise their teaching so as to make the time for exploratory activities.

The influence of external examinations, and the assessment practices in use within some of the schools, caused problems during the trial. They placed severe time constraints on the teachers, and had a major influence on what students thought was important in mathematics and on how they thought they should set about studying it. Many students, for example, thought that investigative activities were a waste of time that could be better spent on drill and practice. It also appears that the inclusion of historical material in the text did little to change the attitudes of most students to mathematics and their beliefs about the nature of mathematics. Without change in assessment procedures, it will be difficult to make major changes in teaching practice.

Implications for research. Some students in the study thought that many of the activities in the text were not "proper mathematics," because they did not involve applying formulas and rules, and they lacked the clarity and certainty of the single correct answer. Some thought that the exploratory approach was a less efficient way of learning mathematics than the one they were used to – rote learning accompanied by plenty of drill and practice. There is a need for research on the culture of the mathematics classroom, in particular to look at what students regard as "proper mathematics," and what expectations they have about how they should be taught and how they think they can learn mathematics most effectively. We need to investigate how these views are formed and how they might be changed, and find out more about how assessment policies and procedures affect students' views about learning mathematics.

Two of the teachers claimed that girls tend to place more emphasis than boys on understanding what they are learning. This assertion needs further research because, if this is the case, it provides strong justification for courses such as the one described here.

This study was undertaken as part of a formative evaluation of the Introductory Calculus project, and the materials have since been modified. A summative study, carried out over a longer period, is needed, to look at the effects on participation, achievement in external assessment, understanding of calculus concepts, problem solving and modelling skills, approaches to studying mathematics, and perception of the nature of mathematics.

Notes

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2. Each of the quotations in the article has been assigned two symbols to indicate its source. The first gives the gender of the person quoted and whether (s)he was a student or a teacher, according to the following code:

F = Female student M = Male student FT = Female teacher MT = Male teacher

The second symbol indicates the school and the state. Details of the schools are given below. All students involved in the study were in Grade 11.

Symbol	State	Type of school		No of teachers & classes in study
A	Australian Cap Ter	Co-ed college,	Grades 11 &12	2 female teachers, 2 classes
N1	New South Wales	Girls' High School,	Grades 7 to 12	1 female teacher, 1 class
N2	New South Wales	Boys' High School,	Grades 7 to 12	1 male teacher, 1 class
N3	New South Wales	Co-ed High School,	Grades 7 to 12	1 male teacher, 1 class
N4	New South Wales	Co-ed High School,	Grades 7 to 12	1 female teacher, 1 class
V1	Victoria	Co-ed High School,	Grades 7 to 12	2 male teachers, 2 classes
V2	Victoria	Co-ed High School,	Grades 7 to 12	2 male teachers, 2 classes
W1	Western Australia	Co-ed High School,	Grades 7 to 12	1 male teacher, 1 class
W2	Western Australia	Co-ed High School,	Grades 7 to 12	1 male teacher, 2 classes
W3	Western Australia	Co-ed High School,	Grades 7 to 12	1 female, 1 male teacher, 2 classes

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CHOICE, SUCCESS, AND CONTEXT IN QUESTIONS IN UNIVERSITY STATISTICS EXAMINATION FOR MALE AND FEMALE STUDENTS IN NEW ZEALAND

Thora Blithe and Megan Clark
Victoria University of Wellington

Background

There have been many interdependent changes in the education system in New Zealand in the past decade, much of which has only recently impinged on the tertiary sector. Many more jobs than before require some mathematical or technical skills, and the function of Forms Six and Seven (the last two years of secondary schooling, both non-compulsory) is no longer solely to prepare students for university. There have been large increases in the proportion of the age cohort going on to senior secondary education, as shown in Table 1 (Clark, 1992).

Table 1 Secondary School retention rates by Form level

	Form 3 - 7
1980	14.6
1991	39.9

This has led to a much greater diversity in the skills, backgrounds, and abilities of the senior student population. There is also an increase in the proportion of young women taking mathematics at sixth form, from 55% to 76% over the period 1974 to 1991 (private communiction, Ministry of Education, 1991). This increase is in part due to social pressure to continue with mathematics and in part due to recent efforts by schools to make mathematics as accessible and as relevant as possible, to as many pupils as possible. The proportion of the female cohort going on with mathematics in the senior school is expected to continue to rise. These days many more subjects are available at school than before, and it is just as likely that a university mathematics student will have studied economics as physics at school (private communiction, Ministry of Education, 1991). An issue or universities (and schools) is whether they can adequately respond to the needs of other disciplines and this broader range of student ability.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

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A new mathematics curriculum for ages 5 to 18 has come into effect in New Zealand in 1993. It is a curriculum that emphasizes process before knowledge and the importance of grounding the pupils' learning in their own world. The universities still concentrate their mathematics applications in physics, engineering and allied disciplines.

There is much research (Howson & Mellin-Olsen, 1986; Stipek, 1984; Barnes, Plaister, & Thomas, 1987) to show that girls habitually attribute their successes to hard work and luck rather than ability and their failures to lack of ability, whereas boys view it the other way around. In facing advanced work, girls have a decreased expectation of success. The attrition rate between undergraduate and graduate studies for women is high. They need to acquire greater confidence (Fennema, 1979). Many university staff are aware of these findings but some are not aware of the pervasive nature of these effects among young women. The language of many prescribed texts leaves a great deal to be desired. In the first chapter of Feller (1968), that popular text in probability theory, we read: "The magic of statistics embraces all phases of life to the extent that young girls watch the statistics of their chances to get married."

Apart from a few projects, most university assessment in mathematics is still end-of-course examination based. It is questionable whether this is adequate as a record of authentic achievement, and Anthony (1991) suggests that more questions relating to process should be used. It is against this background that the following study was made.

Introduction

At the end of the 1991 and 1992 academic years, examination results for a number of first- and second-year mathematics and statistics courses at Victoria University were analysed for gender differences, differences due to ethnicity, and for the effect (if any) of taking more than one paper. Because of small numbers of students, third-year and higher level courses were not analysed in this way. In Pure Mathematics, differences were minimal, but two first-year statistics courses exhibited some striking differences with regard to gender. The two courses were:

STAT 131 *Probability and Data Analysis*: A course for students majoring in the physical sciences, engineering, computing, and mathematics. This course requires seventh form calculus.

STAT 193 *Statistics for the Natural and Social Sciences*: This course does not require seventh form calculus, although it is desirable that the student has taken the seventh form paper Mathematics with Statistics. This is the recommended course for those majoring in biological sciences, social sciences, and medicine and it is a prerequisite for the highly sought after Psychology major. This course leads to second-year statistics only if a pass in first-year calculus is also attained.

Gender Differences in Examination Performance and Question Choice

Gender differences found in examination performance in these courses are summarised in Table 2.

Table 2 Gender Differences

Course	Number in Class		% Female		Significant differences in question choise		Significant differences in performance	
	1991	1992	1991	1992	1991	1992	1991	1992
STAT 131	185	182	28	29	*		one question	2 questions
STAT 193	261	282	54	64		*	4 questions & overall (in favour of females)	4 questions & overall as 1992

* denotes the presence of a statistically significant difference at the 5% level.

In STAT 131, the more technical and abstract of the two first-year courses, females prefer to answer standard questions and do better than males in some of these questions, whereas males often perform better overall in this course. This better male performance was most marked in abstract questions. In STAT 193, females consistently perform better overall. They perform best in questions involving environmental issues but which are nevertheless standard problems. In part these differences are a consequence of choices that students make at school. From table 2 it can be seen that the percentage of females in STAT 193 is approximately double that in STAT 131, reflecting the fact that one and a half times as many females as males study biology in the final year of secondary school while two and a half times as many males as females study physics. STAT 193 is a prerequisite for Psychology majors, a discipline that typically attracts a high proportion of female students.

Typical graphs of question choice and performance are shown in Figure 1. Detailed results for these two courses follow.

STAT 193 *Statistics for the Natural and Social Sciences*

1991 In this course females performed better in four questions: a t-test on chemical residues in brown pelicans; an analysis of variance on I.Q. levels; a contingency table on spouse abuse; and a regression involving cave art and animal remains. There were no significant differences in proportions choosing to answer particular questions. Females performed better overall (female mean 61.7%, male mean 56.6%).

1992 Again females performed better overall (female mean 59.5%, male mean 55.9%) and females performed better in three questions: a confidence interval on foetal heart rates; a confidence interval on parasites in mosquitos; and an

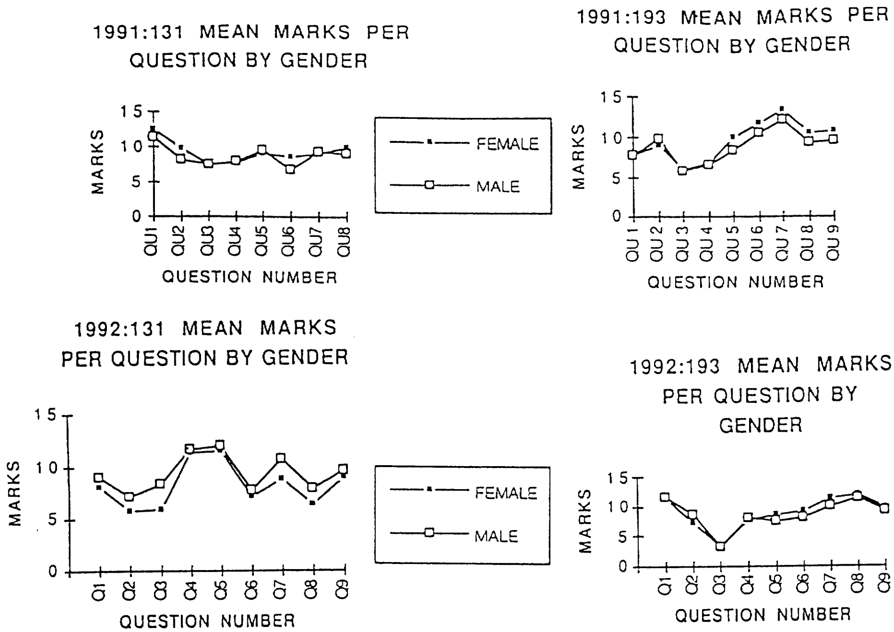

Figure 1 Typical Graphs for Statistics Courses

analysis of variance on the abilities of mentally retarded people to learn basic life skills. Males performed better in one question on urns/balls and binomial probabilities relating to payment of hospital bills. One question had a significantly higher proportion of males attempting it (males 66%, females 49%). This was a question in which part one was a confidence interval of a mean chemical concentration while part two was a test relating to a proportion of vaccinated infants in the population.

STAT 131 *Probability and Data Analysis*

1991 Females preferred a question in this examination involving a piece of standard theory (31% of males, 45% of females attempted this question); performance was not significantly different. Females performed better in one question, which was on a multiple-choice test, and the mean area of fencing, that is, involving familiar algorithms.

1992 Males performed better on all questions and overall, although this difference was only significant on two questions, one of which concerned permutations, combinations, and the geometric distribution applied to political preferences; and the other, theoretical properties of a distribution function plus a simple normal computation.

Differences In The Courses

These two first-year statistics courses have been analysed in greater detail over a longer time period (Clark, 1992). Results for the past few years are shown in Table 3.

Table 3 Participation and Success in First-year Statistics Courses

	% female		female % of A grades	
Year	STAT 131	STAT 193	STAT 131	STAT 193
1986	23	45	21	59
1987	20	44	26	62
1988	30	36	27	48
1989	21	41	25	59
1990	37	52	37	57
1991	28	54	38	60
1992	29	64	20	73

Notes:
1. The jump in percent female in STAT 131 in 1988 is mainly due to the removal of the previous requirement that all first-year engineering students must take STAT 131. Males still make up the bulk of engineering students in New Zealand.
2. A deliberate effort was made to advise and encourage females into STAT 131 in 1990. It is of concern that the proportion of females in STAT 131 has only with particular effort approached that of females in general mathematics courses, i.e., algebra, geometry and calculus for which the proportion is usually 30 to 35%.
3. A small group of students take both courses and for such students their grades in both courses are remarkably close, usually within 3%, so there is no reason to suspect that one of the courses is markedly easier than the other.

In general, it is easier for females to succeed in the STAT 193 course in the sense that they consistently take a larger proportion of the A grades than their proportion of the class, whereas the converse is sometimes true for STAT 131.

As noted earlier, in STAT 131 male performance was often better overall and this was most marked in the abstract questions. Of the non-abstract questions a large proportion (36-50%) involved material likely to be more familiar to males than to females (e.g., computing, machinery) and the rest were judged to be equally accessible. None of the question parts came from what we might term the "world of women." In the STAT 193 results, when there are differences they are in favour of females. Each year there were some very abstract questions, which were shunned by male and female students alike and in which performance was poor. The non-abstract questions were predominantly "people" or "animal" questions and, infrequently (12%), relating to manufacturing. Males and females chose the same questions.

In general, male students in STAT 193 are not males doing standard mathematics courses. The majority are doing no other mathematics and may not even have done mathematics as far as the seventh form. These males are not studying in technical or mathematical areas and it seems clear that the problems that appeal to the females in the class are, by and large, the problems that appeal to this group of males who have mostly rejected, or not been successful, in mainstream mathematics.

Success in the examination, however, is not entirely due to the content of the examination. The courses themselves have major differences. One is in the problems set in weekly assignments, which may have contributed to the confidence of the women students in STAT 193. All assignment problems for three years were categorised into three groups: firstly, abstract and traditional (urns, P(AUB)...); secondly, problems about people and animals (blood pressure, bacteria...); and a third category of non-traditional and non-abstract problems whose content could be seen as belonging to that sphere of activity that is traditionally thought to be appropriate to males (Morse code, stock market...). Over the three years STAT 131 has had up to a quarter of its assigned problems from the "boy's world" and close to a quarter framed in terms of people or animals. STAT 193, however, has had less than one eighth of it's problems from the "boy's world" and between 50% and 80% have a people or animal context, that is, there is far more emphasis on problems involving people in STAT 193 throughout the course.

This difference in context suggests some reasons why female students might prefer STAT 193 as there is much research to show that "the most influential factor in girls, attitudes to study is whether the subject is perceived as a 'male' or 'female' subject" (Royal Society, 1986). Another suggestive difference is in the staffing of these courses. STAT 131 has no female lecturers while STAT 193 does have one. This raises the possibility that this is the source of some of the success of women students in STAT 193. Results of an exploratory study conducted in 1993 indicate that the bulk of the students (89%) do not believe this to be a factor. Whatever the truth of the matter, we cannot instantaneously find more female staff but perhaps we should be making an effort to offset this imbalance in the gender of staff by taking particular care with assignment questions.

In 1991, in one of the STAT 193 assignments, students were given a randomised choice of contexts for what was essentially the same problem: scores on a psychological test of hostility or strengths of different concrete mixes. By and large, students answered the first option available but women were more likely to take the second option if the first was about concrete. Overall, 73% of females and 53% of the males took the "psychologist" option. In 1992 results were very similar on a choice of contexts, one concerned with trucks and one with the delivery of health services. Overall, 60% of females and 39% of the males took the "health" option, and women were more likely to take the second option if the first was "trucks." These results support the idea that specific example material is effective with particular subsets of the population.The need to cater for a much wider group of people has already been established.These results are consistent with results found by Purser and Wily (1992) in their work with secondary school students in

New Zealand. In their study it seemed that "students would prefer to see the connection between the maths they do and the lives they live."

Assessment

Not a great deal of work has been done so far with student preference and performance in different assessment modes. Until this year the STAT 131 course has had no internally assessed component. However, the STAT 193 course has a large internally assessed component: the Data Assignments. These are five linked assignments given throughout the year, all performed on a single data set. These data sets are specific to an individual student and each consists of data on a set of hypothetical mothers and newborns from a region in a year and includes information on gender, weights of mothers and babies, mothers' smoking habits, and socio-economic group. Throughout the year the students investigate their data sets. Although some specific questions must be answered, these assignments are much more open-ended than a traditional assignment, the students have much more control over what they decide to do, the material of the assignments is ostensibly within a relevant social context, and collaboration on the work is possible and appropriate up to a certain point. Furthermore, the results of these investigations have to be recorded clearly. These assignments can be seen as not as competitive as the final examination questions and much less threatening. If a student gets a very low score in one of them, there are four more in which to recover a respectable grade. These are all qualities that the literature (National Research Council, 1989) describes as "girl friendly" and this proves to be the case. In STAT 193 females do significantly better than males in both the traditional timed examination questions and in these Data Assignments, but they do even better in this investigational, open-ended setting, as shown in Figure 2.

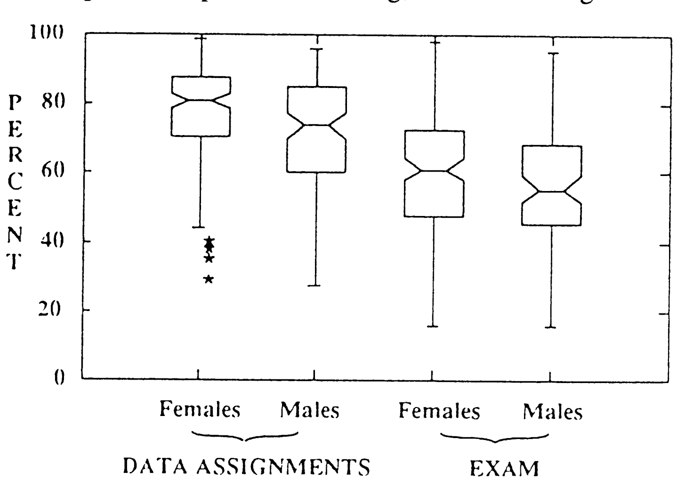

Figure 2

The Exploratory Study

In 1993 a sample of 102 class members participated in an exploratory study which consisted of 10 pairs of statistical questions. For each pair, students were asked to specify which of the two options they felt they would prefer to answer if they were required to, or if they had no preference. They were also asked if they could say why they chose particular options.

This study clearly indicated that in some areas students had very definite preferences (70% or more of the group) for particular contexts over others. In particular, problems involving money, the stock market, investments, and any allied topic, along with engine parts and other machinery oriented problems, were soundly rejected by this group of students. This in part reflects the bias of these particular students who are not taking commerce courses or engineering but the strength of the reaction was unexpected. There were further indications that, no matter how relevant a question seemed on animal or botanical issues, if the question was in competition with a question involving people then the people question would be preferred. This was highlighted when a choice between a question on soil types and one on blood glucose levels in diabetics was offered. Of those students who had a clear preference, twice as many opted for the question involving the diabetics. This was further illustrated by a question in which they had to choose between the contexts of a political opinion poll and the incidence of sheep intestinal cancer in New Zealand. These are both current and relevant issues. Forty-three of the respondents would rather attempt the opinion poll question, 43 were neutral, and only 16 opted for the intestinal cancer. These results cannot be entirely explained by the relative proportions of social science to biology majors in the group (2:1). For those students able to explain their preference the criteria were clear and very similar. Those preferring the sheep cancer stated: "it's more interesting", "I'm interested in biology not politics," and "I'm not going to vote Alliance". Reasons given for preferring the opinion poll option were: " I can relate to it", "I can vote so it affects me", "it's a current event", and "option B is gross." This also serves as a useful reminder that "relevance" must not be the only consideration. Topics such as the destruction of the ozone layer may add relevance to our classes but we should be sparing in the use of such material so that statistics (and mathematics) doesn't become irretrievably associated with the unpleasant (Clark, 1990).

This study also confirmed a result found by Purser and Wily (1992) with high school students, that is, no matter how interesting an example is, if it is in competition with a standard, recognisable problem then the standard problem is preferred. Some of the reasons given for preferring the standard problem presented by the exploratory study were quite revealing: "I like the problem, it reminds me of junior school", "seems simpler", "more like what's been taught", "more concise", "clearer", "easier", "less problem solving", "looks familiar", "tree diagram process is much more familiar than following the highway and road diagram", "clichéd." This was not a simple situation however. It might be argued that the alternative topic of vehicle routes (along with a male driver) would deter many female students but the proportions of males and females choosing this option were not significantly different. The non-standard problem was presented with a diagram, which to a few (four) seemed helpful but to some (four) other students in this class it was clearly perceived as a hindrance.

However, the problems that students prefer as test items are not necessarily the same as the illustrative examples that they would prefer in class or the ones that they actually opt to attempt, as was seen in the discussion of student choice in the final examination.

Purser and Wily (1992) in their work with high school students comment on "how much the students, particularly girls, were influenced by the gender orientation of the context of the question." We should probably proceed on the assumption that the situation is no different for university students. The exploratory study suggests that the young women are less likely to have no preference among the options than young males, tending to confirm the suggestion that context matters more to them ($t_{67} = 1.988$).

Concluding Remarks

Obviously it is desirable that students gain the ability to answer non-standard and abstract questions and become successful problem solvers, but for students to be successful problem solvers they need to develop attitudes to risk-taking such as confidence, open-mindedness, and perseverance. Lorcher (1988) describes how economically and ethnically disadvantaged girls were less inclined to attempt questions than boys and middle-class white girls. The avoidance of risk-taking appears to be taken to extreme lengths in this group of students. It is necessary therefore to make the situation appear less risky, something not done conspicuously well at the moment. One way to do this is to make examination questions more comfortable and less threatening. The indications are that in New Zealand, girls at secondary school are doing better at problems that have "meaning and significance" for them (Howson & Mellin-Olsen, 1986; Clark, 1993). The situation is likely to be similar for university students. This implies that we should present problems that make all the groups not previously well served by the mathematics curriculum, in particular female, Pacific Island, and Maori pupils, feel more at home in their mathematics courses. It seems reasonable that there are contexts for questions that are significant and meaningful to various groups and that enable them to demonstrate their knowledge.

This paper suggests that to be fair to female students (and other subgroups) there must be a choice of question types in examinations so that there are a reasonable number of questions that females might be expected to be relaxed and confident about tackling. This means that word problems should have contexts other than trains and boats and planes, ballistics, machinery, and manufacturing. It is useful to remember that many young women see being good at mathematics as being unfeminine (Sherman, 1987). The way we currently ask questions may be implicitly asking them to opt out of belonging to the group of feminine girls, an unreasonable demand for any educator to make. It becomes important, then, to set work that implies that mathematics is a subject that is feminine, Maori, and otherwise inclusive.

Negative attitudes to study are consistently related to poor academic performance (Watkins, 1982). Possibly we need more questions set by women. If students knew this to be the case, they might well feel that the odds were stacked less strongly against them, ensuring a more confident approach to the examination. University study in mathematics needs to be a more comfortable and equitable experience for all students before the standard of achievement can rise.

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Conference opening. Kerstin Mattson and Miguel de Guzman

Photo: Lennart Jonson ©

ENHANCING FEMALE PARTICIPATION IN MATHEMATICS THROUGH PRE-PROGRAM INTERVENTION

Laura Slater Cavallo
Tracy A. Bibelnieks
University of Minnesota

Introduction

Self-confidence regarding mathematics ability is a distinguishing characteristic that separates male and female students at all levels. The literature reflects that quality of instruction, role models, feedback, and reassurance of accomplishments are integral to the pursuance and success of women in mathematics. To ensure participation and retention of females in mathematics, this encouragement must occur at critical junctures from parents and instructors.

Prior to intervention activities, female students in the University of Minnesota Talented Youth Mathematics Program (UMTYMP) were primarily chosen to participate in the Program qualifying examination by school identification. Program statistics indicated that the schools did not do as well at identifying mathematically talented female students as they did with male students. Even when a female student qualified, she was more likely to turn down admission than her male counterpart. Once in the Program, given equal ability and equal grades, female persistence was lower. Finally, the overall performance of female students was below the overall performance of male students. A self-study of the Program pointed to a serious need to address the issue of equity. The Program responded by designing intervention activities specifically for females, underrepresented, and economically disadvantaged students at the 6th to 12th grade level.

The results of the intervention efforts have had a major effect on increasing the participation of the targeted groups of students in UMTYMP, as well as creating significant improvements in the Program as a whole. This paper will discuss the pre-program interventions and the successes as they relate to females.

Design, Goals, and Content of UMTYMP

In this section, we provide a broad overview of the design and structure of UMTYMP. This description deals mainly with general aspects rather than specific details. These details can be found in Keynes (1991).

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The University of Minnesota Talented Youth Mathematics Program (UMTYMP) is a statewide program aimed at providing an alternative educational experience for Minnesota's most mathematically talented students. The Program currently enrolls over 500 students from the Twin Cities (Minneapolis/St. Paul) metropolitan area as well as small towns in rural Minnesota. The Program and home schools identify students in Grades 5–8 who have scored above the 95th percentile on any standardized math test and invite them to take part in a qualifying examination. From approximately 1400 students who test annually, about 140–150 are invited to participate.

The Program provides an intense academic environment and a different culture of mathematics via a sequence of specially designed accelerated mathematics courses. These students attend a two-hour class one afternoon each week, after school, for 30 weeks from September through May. The students average 8–10 hours of weekly homework to be completed in school during release time from their regular mathematics program or at home. In the first two years of the Program (the high school component), students study a customized program in Algebra (equivalent to Algebra I and Algebra II) in the first year, and Geometry/Mathematical Analysis in the second year. The high school component is taught by outstanding certified high school mathematics teachers and undergraduate teaching assistants, many of whom graduated from the Program. As mandated by a 1984 Minnesota State law, two full years of high school mathematics credit in Algebra I/II is to be granted for completion of the first year UMTYMP Algebra and one year's credit each in Geometry and Math Analysis for the second year UMTYMP course.

Following the high school component, the students study calculus of one and several variables, linear algebra, and differential equations in the college component. This component consists of special university honors courses created for the UMTYMP students by the School of Mathematics at the University of Minnesota. As most students ultimately pursue careers in widely varying areas of mathematics, science, engineering, and technology, the courses provide carefully monitored problem-solving curricula, which are taught by senior university faculty from the University of Minnesota and from local colleges at the outreach sites. In the first year, differential and integral calculus are studied. In the second, sequences, series, and linear algebra are covered. A third year explores linear analysis – a course in multi-variable calculus using linear algebra and geometry. For students who are still in high school, a fourth year topics course in advanced undergraduate mathematics is offered. Topics have included Topology, Combinatorics and Probability, and Algebraic Geometry.

Because students graduate from high school at several points in the Program, the courses in the college component are designed so that at the end of each year students may easily move into regular university courses. A student earns 8–10

University of Minnesota undergraduate honors mathematics credits for successful completion of each year of the UMTYMP college component. These credits are regularly transferred to other colleges and universities.

History and Philosophy of Program Intervention Activities

There are several articles written on gender equity that address concerns about gender equity in the classroom – in particular, the mathematics classroom. Several of these issues are addressed in the AAUW report: *How Schools Shortchange Girls* (1992). Noteworthy concerns include:

- differences between girls and boys in math achievement are small and declining,yet in high school girls are still less likely than boys to take the most advanced courses and be in the top-scoring math groups;
- the gender gap in science, however, is not decreasing and may, in fact, be increasing;
- even girls who are highly competent in math and science are much less likely to pursue scientific or technological careers than are their male classmates.

With financial support provided by The Bush Foundation of Minnesota, a variety of interventions were started in 1988 to address these concerns. The overall thrust of the Bush project was to maximize the curricula and experiences developed for females in mathematics and science, improve family and educational support, and improve female's attitudes towards mathematics and sciences. With this grant, UMTYMP examined activities that would encourage girls to test, enroll, and remain in UMTYMP. The Program identified the following five goals with regard to females:

- increase female interest in testing for UMTYMP – 40% female testing pool;
- increase the quality of the total female application pool – females should account for 40% of qualified students;
- increase female participation in UMTYMP – entering class at least 30% female;
- increase female retention in UMTYMP – retention rate of females should equal that of males;
- improve female achievement within UMTYMP – comparable performance and grade distribution for females and males.

A major thrust of the UMTYMP intervention project focused on identifying females who nearly qualified for the Program and providing them with activities that would maintain their interest in retesting for UMTYMP. A very significant factor influencing female UMTYMP participation appeared to be a willingness to retest in the following year. Historically, the Program noted that a significant percentage of girls made large gains in score upon retaking the UMTYMP qualifying examination. Thus, UMTYMP created a near-qualifier population.

In 1988, the first group of females that scored within 5 points of qualifying for UMTYMP was identified. Experimental early interventions with females began that same year (1988-89). Initial interventions focused on areas of peer support, older female role models and mentors, special study sessions, and extracurricular social and academic activities that would encourage, support, and develop systems assisting girls. Seven target schools were chosen for a tryout of intensive support programs for female candidates for UMTYMP and for current UMTYMP female participants. Other interventions included extracurricular activities such as pizza and bowling parties. This intervention proved to be so successful that the near-qualifier group was expanded to include females within 10 points of qualifying. A special spring examination was created, and a three-week summer enrichment institute was developed. The data on success upon retest for the near-qualifiers in the Bush interventions and beyond is shown in Table 1.

Table 1 Retest data for near-qualifying females.

Females within 5 points of qualifying:					
1st Test	**Retest**	**#Tested**	**#Qualified**	**%Qualified**	**Ave. Gain (points)**
Fall '89	Spring '90	16	10	63	6.1
Fall '90	Spring '91	8	5	63	6.0
Fall '91	Spring '92	6	4	67	6.3
Fall '92	Spring & Summer '93	29	18	62	6.6
Females within 6-10 points of qualifying:					
1st Test	**Retest**	**#Tested**	**#Qualified**	**%Qualified**	**Ave.Gain (points)**
Fall '89	Spring '90	36	11	31	9.8
Fall '90	Spring '91	41	13	32	9.7
Fall '91	Spring '92	29	9	31	11.1
Fall '92	Spring & Summer '93	46	15	33	9.3

The success of the near-qualifier population was also realized in an increase in female enrollment in the entering UMTYMP Algebra I class. Female enrollment went from 22% in 1987 to over 40% in 1990.

The original Bush Foundation grant covered a three-year period, September 1, 1988 - September 1, 1991. This period covered a full year (1988-89) of special interventions to encourage female students to participate in UMTYMP, and two years (1989-90, 1990-91) of support activities, social programs, and summer activities for girls who qualified and enrolled in UMTYMP. The near qualifier population allowed the Program to achieve the first three goals that had been set forth at the start of the Bush intervention. The overall focus of the interventions then became to encourage female students to persist once in the Program, and to achieve similar grade distributions and performance levels to those of the male students. These goals were quickly achieved in the high school component (Algebra and Geometry/Math Analysis). Moreover, the Program identified, tested, and modified a series of activities and interventions that could be replicated in a wider context. For example, several interventions were modified to include both male and female participants and were expanded to encourage girls to remain in the Program through the college component. An important goal of the Bush Intervention Program became to retain the same percentage of females testing for and participating in Calculus I who had successfully finished Geometry/Math Analysis.

At the end of 1990-91 academic year, the Program did a careful review of the impact of the intervention activities. It was noted that for any given activity only 10–15% of the UMTYMP girls were attending. This was in spite of the increased retention rates and performance. At the same time, the Program noticed a decline in the number of females testing and enrolling in the Program (from 40% enrolling in 1990-91 to 36% enrolling in 1991-92). Program administrators concluded that once a significant number of females were enrolled in the Program they found their own support network and that the emphasis on intervention projects had to be returned to pre-program intervention activities to ensure continued participation of females.

In 1991, the Program was awarded funding from the National Science Foundation Young Scholars Program's Early Alert Initiative (EAI) to develop academically oriented enrichment activities as opposed to the mainly social events of the Bush Project. In 1993, the Program was awarded a Pre-Freshman Enrichment Program (PREP) grant from the Department of Energy to continue these efforts.

The primary goal of these more recent grants is to provide underrepresented students, including females, the opportunities to learn and enjoy mathematics and become aware of its applications and roles in society. A secondary goal of these efforts is to achieve full participation of these students in UMTYMP. To achieve these goals, the Program carefully reviewed the Bush Intervention Program and determined that a specially designated set of interventions with a broad set of objectives should be available to any group of near qualifiers. The objectives set forth are to:

- provide a comfortable, interesting setting for the target students to learn and enjoy mathematics, to become more aware of its application, and to understand its role in society;
- provide opportunities for targeted students to become familiar with mathematical problem-solving in a stimulating, small-group environment;
- expose students to innovative mathematics and research methodology;
- create a mathematical environment facilitating one-on-one interaction between students and highly successful faculty, business people, and researchers;
- help students become aware of communication problems in mathematics and improve their communication skills;
- provide information on the nature of mathematics to enable students to better understand career opportunities in mathematics and related areas;
- address social and cultural issues facing talented female students, students of color, and economically disadvantaged students;
- expose students to a variety of careers and disciplines that rely upon mathematics, e.g., engineering, chemistry, physics;
- enhance UMTYMP information dissemination highlighting the value of mathematics programs that provide enrichment in mathematics and its applications
- prepare and encourage target students to test and participate in UMTYMP.

These objectives are achieved through an academic-year program that culminates in a four-week summer institute. This program provides a broad enrichment experience in mathematics and science, as well as helping to prepare students for qualification in UMTYMP.

Pre-Program Intervention Curriculum

The academic-year program consists of a once-a-month program for targeted mathematically talented, near-qualifying students in 5th, 6th, and 7th grades. Some current UMTYMP algebra students participate to provide peer tutoring, a social context, and motivation for participation in UMTYMP. The social and family context for learning is extraordinarily important. Parents are encouraged to attend activities and are asked to help their children carry out their commitment to the Program. Participating students are required to attend at least six out of nine events, with the September orientation program being mandatory for both students and their parents.

The initial September activity is social in content and includes an orientation for the students and their families to inform them of the Program and its expectations and goals. It is designed to introduce parents and students to the Program and to the context of mathematics to be covered throughout the Program. Subsequent to the orientation, the enrichment events are structured to include a directed mathematical group activity, a demonstration or tour, time for lunch or dinner, and group interaction. The activities are designed to include: close interactions with senior research mathematics faculty, graduate and undergraduate students; innovative content material and instructional approaches; small-group problem solving and group interactions; hands-on computer and calculator activities; introduction to research approaches and methodology; career exploration with professionals, including discussions on cultural and social barriers; and college options.

The majority of the activities are held at the School of Mathematics on the University of Minnesota, Minneapolis campus. The Minneapolis campus is the central office of UMTYMP, and Special Projects have access to a large mathematics faculty, faculty in diverse areas, microcomputer labs, mainframe computers, and high-tech facilities. Activities are held in various classroom/labs with excellent facilities for computation and research.

Because students have a wide variety of learning styles, it is important to present mathematics in its full context, with opportunities to understand ideas from a variety of viewpoints. Mathematics faculty of the University of Minnesota and current UMTYMP high school teachers work together to enhance and select specially tailored mathematical activities. UMTYMP teachers supervise group mathematical activities and present innovative problem-solving curricula. Geometric and formal intuition have been combined as a foundation for introducing algebraic concepts, geometric and graphical connections, and instances of mathematics in the real world. Many mathematical subjects that are not typically taught to students of this age are also incorporated into the mathematics activities. Some topics covered include:

- modular Arithmetic;
- group theory;
- probability – population sampling, combinatorics;
- statistics – mean, median, and mode; stem-and-leaf plots; histograms; linear regression;
- geometry – constructions, calculations, and measurements of angles; polyhedra; platonic and Archimedean solids; tilings; tessellations; curvature and angle defect;
- visualization – relating hypercubes to shadows to visualize the 4th dimension in 3-dimensional space; hyperbolic space;
- and fractals.

Technology and visualization is an important component in the enrichment curriculum. Students are provided with TI-81 Graphing Calculators to use during the enrichment activities. They are encouraged to explore a large number of examples and connections between geometric visualization and algebraic manipulation. Other software that is used for instruction and discovery is *Mathematica* and *Geometer's Sketchpad.*

As described, students work primarily with mathematicians and mathematics teachers during the academic year. High interest in mathematics and its role in science is critical if talented youngsters are to take full advantage of future opportunities, and students should be exposed early to important ideas of mathematics, both for intrinsic interest and for connections with the world in which they live. Therefore, scientists and engineers from the university and business communities are invited to work with the students in workshops, demonstrations, and structured discussions, in order to present the broad range of applicability of mathematics and to stress the many career paths open to students of mathematics. Students also are given the opportunity to go on site to such places as university and industrial laboratories, private and/or state colleges, The Science Museum of Minnesota, The Minnesota Zoo, and major Minnesota high-technology corporations located in the Twin Cities. The NSF-supported Institute for Mathematics and Its Applications provides speakers and programs on applications of mathematics within its facilities and the NSF-funded Geometry Center contributes opportunities for unusual visualization and computer graphics tours, speakers, and programs. The students see the workplace of scientists and engineers firsthand. Students carry out hands-on activities illustrating the applicability and character of the mathematics they are learning. They interact with people whose livelihood is the application of mathematics to science and engineering. A sampling of the speakers, demonstrations, and presentations offered in the enrichment program includes:

- a demonstration on holograms – how they are made and the physics/mathematics involved in their creation;
- an activity sponsored by the University of Minnesota Geometry Center that involved hands-on computer applications;
- an activity sponsored by the Science Museum of Minnesota on solving crimes and forensic engineering;

- an activity at the Minnesota Zoo which focused on probability and statistics of animal populations;
- a demonstration by the University of Minnesota Chemistry outreach program;
- the UMTYMP annual mathematics fun fair with presentations and demonstrations from students, university mathematics and science faculty, and local high-technology corporations;
- a speaker from the area of Operations Research;
- an activity sponsored by the Aerospace Engineering Department of the University of Minnesota on the design and construction of a wind tunnel and a water channel;
- an activity sponsored by the Architecture Department of the University of Minnesota on the computer design of 3-D architectural models;
- an activity on the mathematics and physics of amusement park rides that included a visit to a local amusement park;
- a speaker who talked about problem solving as an engineer and how engineers do research to improve industrial products.

Future curriculum and problem settings will be based upon successful materials developed and tested at major curriculum improvement sites. New activities will incorporate visualizations developed in the Geometry Center, applications developed in the Institute for Mathematics and Its Applications, and materials developed nationally through NSF and other efforts. NSF efforts of particular value include the MathFINDER CD-ROM, The Math and the Mind's Eye materials, the HIMAP materials, the Geometry and Visualization materials, the Interactive Mathematics Project materials, and other projects developing comprehensive middle school mathematics materials.

Students who have attended at least six of the nine events during the academic year are eligible to participate in a 4-week commuter summer enrichment institute in late June through early July. The summer enrichment institute continues the rich curriculum of the academic-year activities and further emphasizes mathematics and related topics as a culture and method of scientific inquiry, mathematics as a career enabler, and mathematical thinking as a challenging but exciting opportunity for each participant. An exciting final week of hands-on computer visualization activities at the research-oriented, NSF-supported Geometry Center is currently under development.

Progress Report on Intervention Activities

From the beginning of the Bush Intervention Program, UMTYMP has conducted an ongoing evaluation of the intervention activities. Project activities are assessed through participant evaluations after each activity. These evaluations are reviewed along with Program staff comments and other anecdotal information after each activity to enable the Program to learn what types of programs are most effective in attracting and exciting the students. Attendance and other numerical data are combined with the affective evaluations to alert the Program to possible modifications in future activities and summer institutes. The Program also collects long-term performance data, which allows for longitudinal statistics on performance from first Program contact through completion of undergraduate and graduate studies.

Each year, the UMTYMP database is enhanced with new measures aimed at the performance of specific demographic groups. Serious collection of enhanced gender specific data began in 1988. Two years ago additional data collection was begun with underrepresented and economically disadvantaged students. Through its normal data collection processes these recently identified groups will be followed for many years to come. Data on female interventions has enabled UMTYMP to specifically validate assumptions made by other researchers in the education of females. These data clearly show the importance of social interaction and environment on the acquisition of knowledge and the future choices made by bright female students in the area of science and mathematics.

The data collected since 1988 on the five goals for the Bush Intervention Project indicate some significant progress. In each of these years, the percentage of females testing for UMTYMP varied between 42% and 46%, and exceeded the Program's goals. The quality of the female applicant pool, measured by the percentage of females among the total qualifiers, improved substantially over the base year 1987-88 during the 1989-91 academic year, and more modestly over the base year in 1991-92 and 1992-93. Finally, the percentage enrolling reached over 40% in 1989-91, and is currently about 35%. Again, this has exceeded the Program's goal of 30% in each year. Moreover, the success of the intervention activities is reflected in the percent of the entering Algebra I class who were female near-qualifiers the previous year. As shown in Table 2, this percentage rose yearly from 1990 culminating in 25% of the 1993-94 Algebra I class consisting of females from pre-program intervention activities.

Table 2 Impact of Near-Qualifying Females on Entering Algebra I Class

Near Qualifier Activity Year	% of the Following Year's Entering Class who were NQs
1989-90	16
1990-91	15
1991-92	10
1992-93	25

The overall changes in the Program have also led to improvements in retention rates. As shown in Table 3, the retention rate from Algebra I to Algebra II for girls is now slightly better than for boys. Prior to any interventions (the 1987-88 school year), the corresponding retention rates were 85.7% for girls and 90.7% for boys. The retention rate from Algebra I to Geometry varies from year to year between boys and girls, and no clear pattern has emerged. On the other hand, retention rates from Geometry to Math Analysis again slightly favor females over males. Most importantly, qualification and participation in calculus for females has dramatically improved. The percentage of female students in Calculus I has risen from pre-intervention percentages of 17–20% to current percentages between 28% and 36%. In the difficult 1993 Calculus Qualifying Exam, three of the top five scores were achieved by females.

Table 3 Retention rates for students involved in the Bush interventions.

Retention Rates:	**Group I 1988**	**Group II 1989**	**Group III 1990**	**Group IV 1991**	**Group V 1992**
Algebra I to II					
Males	92.7 %	92.4 %	86.3 %	97.4 %	100 %
Females	93.4 %	98.1 %	90.2 %	95.1 %	100 %
Algebra II to Geometry					
Males	79.2 %	76 %	80.6 %	78.9 %	87.7 %
Females	70.6 %	80 %	71.4 %	74.4 %	80.6 %
Geometry to Math Analysis					
Males	98.3 %	79.9 %	98.0 %	100 %	
Females	100 %	83.3 %	100 %	100 %	
Math Analysis to Calc I					
Males	56.4 %	64.0 %	64.7 %		
Females	28.6 %	64.3 %	61.5 %		
Calc I Sem I to Sem II					
Males	79.5 %	97.6 %	100 %		
Females	69.2 %	94.7 %	100 %		
Calc II Sem I to Sem II					
Males	80.6 %	96.7 %			
Females	75.0 %	92.3 %			
Calc III Sem I to Sem II					
Males	77.8 %				
Females	50.0 %				

Finally, we briefly discuss female performance as measured by grade distribution. The Program's goal in this case was that the additional female students recruited under the Bush Intervention Program would maintain and enhance the academic structure of UMTYMP. Program data indicate that within the current academic environment, female students overall are indeed achieving at a level equal to that of the male students. This level of achievement has been realized at the same time that the Program has strengthened its overall curriculum. In UMTYMP, equity has supported and encouraged an enhanced curriculum.

Conclusions

To achieve UMTYMP's goal of equity and excellence with regard to females, the Program has formulated a clearly defined approach toward the education of all its students.UMTYMP provides, in addition to nationally recognized quality course work, a support system and network of students and faculty that enable students from any background to find companionship with other students in a success-oriented environment. It engages them in activities and interventions that help boost their self-esteem and self-confidence, challenge them intellectually in a highly demanding and highly supportive culture of mathematics, and fosters an atmosphere for students and faculty that cultivates self-worth.

The Program will continue to use this litmus test as a guide for future goals and directions of intervention projects for females, underrepresented students, and economically disadvantaged students. Moreover, UMTYMP has become more aware of the need to remain innovative, flexible, and responsive to creating new procedures and structures as we learn more about educating such extraordinary students. For UMTYMP, equity has strengthened excellence.

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Plenary speech by Karin Beyer *Photo: Lennart Jonson ©*

MATHEMATICS ANXIETY/CONFIDENCE AND OTHER DETERMINANTS OF COLLEGE MAJOR SELECTION

Susan F. Chipman
U.S. Office of Naval Research
David H. Krantz
Columbia University
Rae Silver
Barnard College

At least since Tobias published her well known book, *Overcoming Math Anxiety* (1978), many have believed that anxiety about mathematics and the associated desire to avoid the study of mathematics affect the career decisions of many students. This is widely believed to be especially true of female students. Yet past research on the determinants of enrollment in optional high school mathematics courses has not succeeded in demonstrating a significant effect of mathematics anxiety upon enrollment decisions (Chipman & Wilson, 1985). The difficulty is that strong correlations usually exist between the measures of affect about mathematics and the measures of cognitive ability that are the strongest predictors of enrollment in advanced mathematics. Furthermore, from the available research evidence, the hypothesis that early developing general interest patterns and/or more specific vocational interests determine the decision whether or not to enroll in advanced mathematics courses appears at least as tenable as the popular view that aversion to mathematics filters (especially female) students out of the career paths that require mathematical preparation. There are large sex differences in the general interest patterns (roughly characterized by Dunteman, Wisenbaker and Taylor (1979) as relative interest in things versus interest in people) that predict the selection of a scientific or technical major within both the male and female populations of students. For this reason, such interest patterns seem an excellent candidate for explaining observed sex differences in scientific career participation, a better candidate than the relatively small sex differences in cognitive abilities or attitudes toward mathematics (Chipman & Thomas, 1987; Chipman, in press).

On the other hand, it is also true that the studies reviewed by Chipman and

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Wilson (1985) consistently showed a sex difference in confidence (or anxiety) about oneself as a learner of mathematics. A broader meta-analysis by Hyde, Fennema, Ryan, Frost and Hopp (1990) confirmed that result. Therefore the possible impact of these psychological variables – mathematics anxiety/confidence and general interest patterns – still merited investigation under circumstances more favorable to demonstrating and measuring their possible effects.

A questionnaire that was administered to nearly all entering Barnard College freshmen in 1985, 1986 and 1987 provided the opportunity to investigate some of these questions in a population whose relatively high level of cognitive ability might increase the relative importance of affective factors. Furthermore, this population is quite similar to those from which the anecdotal reports about mathematics anxiety emerged. SAT scores were available to provide measures of cognitive ability comparable to those used in prior research. Building on the conclusions of past research concerning the variables of greatest interest, the questionnaire was designed to explore career predilections as well as beliefs about the amount of mathematics required for various careers, mathematics anxiety/confidence, mathematics course background, and the people-vs.-things dimension of personal interests. These questions were embedded among other items, including ones related to general anxiety, perceived locus of control, and diverse career-related values. This context may have served to mask our special interest in mathematics anxiety. There were also items probing computer-related anxieties.

In striking contrast to the results of previous research, cross-sectional analyses of these questionnaire data showed a large effect of Mathematics Anxiety/Confidence on openness to science careers but no appreciable or systematic effect of the cognitive variable, Quantitative SAT score (Chipman, Krantz & Silver, 1992).

These results held for both physical science and biological science careers. Regression analyses showed that people-vs.-things interest also affected the degree of interest in physical science careers.

These results lent strong support to the view that interest in science careers can be inhibited for students with a negative affective response to mathematics, independent of their objectively measured mathematical competence. They were also consistent with a subjective report obtained in the questionnaire: over 25% of the respondents agreed with an item, "The desire to avoid mathematics is affecting my career choice."

By now, the majority of the students in this study have completed college. Complete transcript data is available, recording courses taken, grades received, and final choice of major field of study. The present report considers how the variables measured at the beginning of college (as well as course experiences during college) are predictive of actual student outcomes, rather than of subjectively expressed career interests.

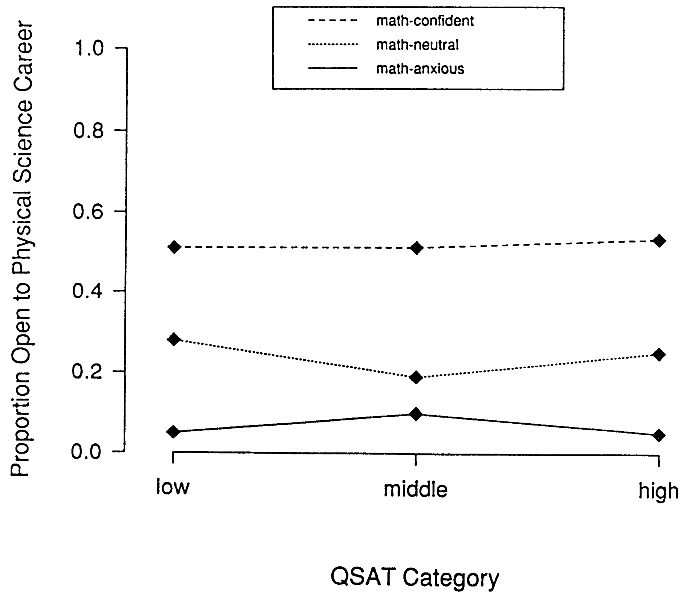

Figure 1 Openness to physical science or engineering careers as a function of both QSAT and Math Anxiety/Confidence levels.

Method and Results

Barnard College, affiliated with Columbia University, is one of the few remaining women's colleges in the U.S. The questionnaire was completed during an orientation meeting of the entire entering freshman class in 1985 (N = 448) and 1986 (N = 456) and 1987 (N = 462). Nearly complete questionnaires were returned by 1366 women (over 95% of the entering first-year students). Nearly 90% of entering students were U.S. citizens or permanent residents. Of these 90%, around 80% were white, 8-9% black or Hispanic, and 11-12% of Asian origin. As an indication of social class, 79% of mothers of freshmen, and 88% of their fathers graduated from college; about 55% of freshman received some financial aid. Quantitative SAT (QSAT) for this population was well-described by a normal distribution with a mean of 605 and a standard deviation of 70, as compared to a mean of about 450 for all U.S. women.

The Mathematics Anxiety/Confidence variable (MANX) used in this study was derived from five items, rated from strongly agree to strongly disagree and scaled with Mathematics Confidence as high scores. The items were:

1 I dread mathematics class.
2 Working on mathematics problems makes me tense.
3 Mathematics is easier for me than it is for most people.
4 I just cannot understand mathematics.
5 If I ever need to learn new mathematics for my job, it will be easy for me.

The estimated reliability of this scale was .89, and the stability over four years was estimated to be .65, based on follow-up questionnaires from a sample of the class of 1985. Despite a correlation of about .30 between QSAT and MANX in this population, 4.2% (57) of the respondents were simultaneously in the highest category for math confidence and the lowest for QSAT; 3.2% (43) showed the reverse combination, high math anxiety with high QSAT. People vs. Things Interest was measured with two transparent items of opposite polarity.

Results: Effects on Majoring in Science

By now, the majority of the students in this study have completed college and their major fields of study are known. Of the 1074 students for whom we have complete data, 45 majored in a physical science or engineering field, 69 majored in a biological science, 572 in a social science field, 357 in the humanities, and 31 in creative writing or similar fields. Figure 2 shows the proportion of physical science majors for students classified according to both their quantitative SAT score (QSAT) and their Mathematics Anxiety/Confidence score as measured at college entry.

In contrast to the interest data just discussed, these data for actual major choices show a very strong effect of QSAT ($p < .0001$), primarily concentrated in the higher rates of participation for those with QSAT over 650. The effect of Mathematics Anxiety/Confidence (MANX) is also significant ($p < .01$). The magnitude of these effects is of practical significance also. The odds of being a physical science major increase by a factor of about 16 (90% confidence interval 4-50) from low to high QSAT and by a factor of 5 (2-10) from math anxious to math confident.

The data for biological science majors present a much less orderly picture, shown in Figure 3. The effect of QSAT is not significant. Interestingly, however, there is a significant effect of Mathematics Anxiety/Confidence in these data ($p < .002$) also. Taking the large mid-SAT group as a basis for estimate, the odds of being a biological science major increase by a factor of 5 (90% confidence interval a bit larger than 2-10) from math anxious to math confident.

Additional path-analytic analyses showed that the predictive value of Mathematics Anxiety/Confidence is reflected in the measure of physical science interest at the time of college entrance. Similarly, the variable of People vs. Things Interest, predictive of physical science interest at the time of college entrance, does not have additional predictive value for major selection during college. Enrolling in a serious quantitative course as one's first quantitative course enrollment and receiving a high grade in that course does increase the likelihood of a physical science major. In summary, QSAT, physical science interest measured at the beginning of college and the first quantitative course experience all contribute significantly to predicting a physical science major. The comparable analysis for the

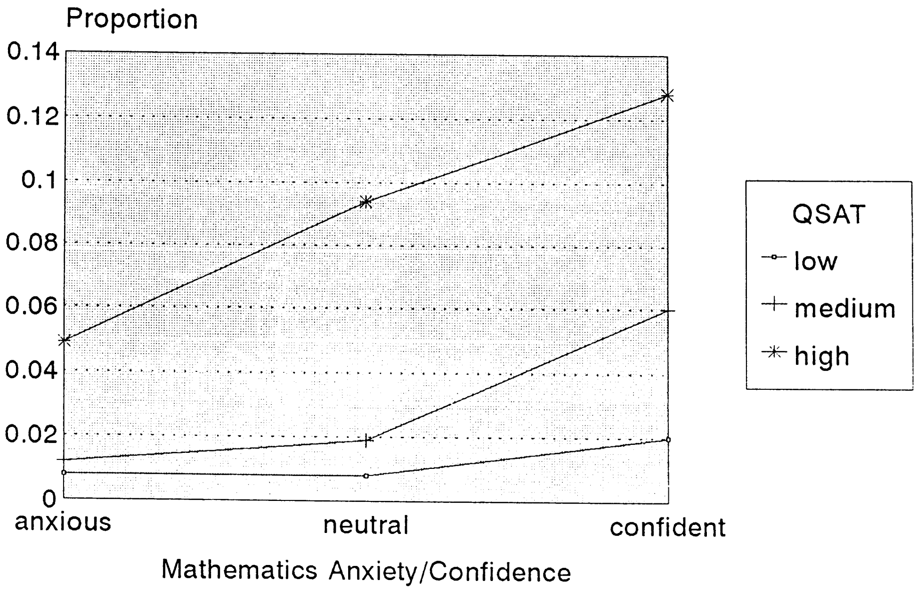

Figure 2 Physical Science Majors

biological science majors yields an even simpler picture. The measure of biological science interest at the beginning of college alone has highly significant predictive value. Neither QSAT nor MANX can improve the quality of prediction significantly.

Despite the impressive magnitude of the effects just described, we are still not doing very well in predicting precisely which individuals will become science majors. Perhaps the reason for this can be seen by inspecting the measures of science interest and their relation to the selection of a science major more closely. In this population, high interest in the science fields at the beginning of college was rare; low interest was very common. As one would expect, women who show high interest in science careers at the beginning of college were much more likely to major in science. However, since there were so few who showed high interest, the majority of actual science majors were found among those who did not show high interest at the beginning of college. For biological sciences, the relatively frequent career interest in being a doctor (physician) is a significant source of science majors. For physical sciences, there were actually more majors coming from those originally classfied as not open to consideration of science or engineering careers than coming from those with high initial interest. This fact implies that experiences during college are having a very substantial impact on the total representation of science majors in this college population.

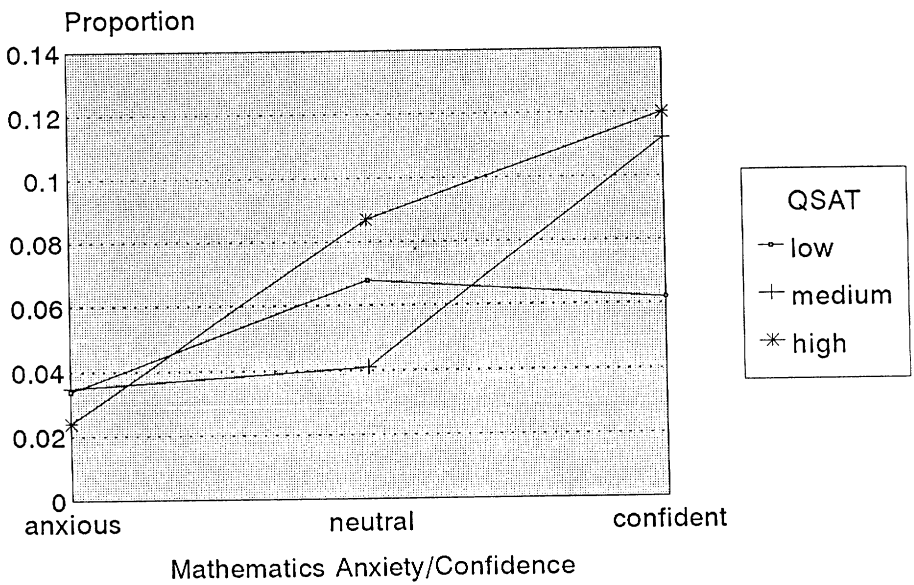

Figure 3 Biological Science Majors

Discussion

The emergence of a strong effect of QSAT on the actual pursuit of a physical science major, when it was not seen in the measures of physical science interest at the beginning of college, is an interesting outcome. This result is reminiscent of the results of the studies of participation in advanced high school mathematics courses reviewed by Chipman and Wilson (1985): measures of cognitive ability were the strongest predictor. There are several possible interpretations that may be given to this QSAT result. It may reflect the reality that mathematical competence really counts in the pursuit of a physical science field. Alternatively, it may reflect a strong belief in the college level community – both students and faculty – that mathematical competence really matters. Students with less than the highest QSAT scores may be counseled out of physical science majors or may counsel themselves out of those majors. In interpreting these results, however, one must consider that the Barnard College population is an intellectually select population. According to NSF data, the mean QSAT of this Barnard population is as high or higher than the mean QSAT of all U.S. males receiving BA degrees in physical sciences, engineering, or even mathematics (National Science Foundation, 1986). The majority of Barnard students, therefore, are probably capable of successfully completing a science major.

Perhaps it is more interesting that Mathematics Anxiety/Confidence seems to be affecting the pursuit of both physical science and biological science majors through its impact on interest as measured at the beginning of college, even though

QSAT does not seem to be having the same strong effect on the actual pursuit of a biological science major. For biological science, perhaps the generalized belief in the importance of mathematical competence for science affects pre-college formation of science interest more than is justified by the real demands. For the physical sciences, we see also that mathematics anxiety has an impact independent of actual mathematical competence; if anything, its effect is most evident within the subpopulation with the highest QSAT scores.

The most surprising element of our findings was the discovery that the majority of actual science majors was drawn from the population of individuals who did not show strong interest in science at the beginning of college; some of them envisioned no possibility of a science career. This runs counter to the general belief that most scientists are drawn from those who showed interest in science at an early age (Terman, 1954).

Although this study is of an entirely female population, many of the results may apply to males as well. There is a well-documented sex difference in mathematics anxiety or confidence, but it is a small one, particularly small among able college students (Hyde, Fennema, Ryan, Frost & Hopp, 1990). It should not be assumed that mathematics anxiety is a female problem. Other vocational interest factors remain more likely prospects for explaining the observed gender difference in science career participation. Nevertheless, the results of this study clearly demonstrate that mathematics anxiety is functioning as a barrier to entry into scientific and engineering careers, independent of its relation to objectively measured mathematical competence. These effects are already established at the time of college entrance. The attempt to overcome mathematics anxiety remains worthwhile.

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Note

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EMPOWERING YOUNG WOMEN IN MATHEMATICS: TWO IMPORTANT CONSIDERATIONS

Agnes Cordeau
Holy Angels High School, Toronto

This paper is a report of some of the findings of a case study conducted in 1987 at three senior high schools in Toronto. The research studied the interdependence between gender and mathematics and probed the reasons young women are underrepresented in advanced level mathematics courses at the senior high school level and, consequently, in the post-secondary careers that require a mathematical background. The study was based on the inadequacy of the inferior brain theory to explain differences in both ability and course selections between male and female students and accepts Gould's (1981, p. 325) premise that "different attitudes and styles of thought among human groups are usually the nongenetic products of cultural evolution." The research confirmed the observation of Fox (1980, p. 2) that "to seek the reasons why men and women may differ with respect to interest in the study of and pursuit of careers related to mathematics is to study a vast number of interesting and important questions about human behaviour."

This report gives a brief description of the research and considers two major factors that appeared to affect young women's attitudes towards mathematics: (a) the ethnic background of the young women; and (b) their attitude towards competition.

Methodology and Description of Sites

The case study was conducted in three high schools, two of which were coeducational and the third an all-girls high school. The schools are designated as X, Y, Z.

School X was a coeducational institution in a middle-to-upper class area with a population of 1240 students, of whom 655 were males and 585 females. Approximately 10% were of Chinese origin and many were Anglo-Saxons. There were no significant differences in the number of females and males studying mathematics at any grade level, or at any degree of difficulty, except at the Grade 13 level, where males outnumbered females in algebra, calculus, and statistics classes.

School Y was a secondary school in a blue-collar area with a student population of 933, of whom 525 were males and 408 were females. There was no signifi-

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cant difference in the percentage of females and males studying advanced mathematics, except at the Grade 13 level where the males outnumbered the females 5:1. School Y was a miniature United Nations, with a large number of "new immigrants" from Asia and Eastern and Southern Europe. Economics was a major consideration for parents and students. Most students had part-time jobs to help make ends meet. Many were born in Canada but were first-generation Canadians.

School Z was an all-girl, academic, secondary school in a high-income area with a student population of 850. It catered to academic students and did not offer basic or technical level programs. At the Grade 13 level, approximately 50% of the students took calculus. Both parents and students had high academic expectations and possessed the financial resources to bring these expectations to fruition. Although young women of Anglo-Saxon origin predominated, there were also many students of Oriental, Philippine, or Southern European descent.

Research Method: The Case Study

According to Yin (1984, p. 25) the case study has a distinctive place in research. It allows the researcher to investigate a contemporary phenomenon within its real-life context when the boundaries between the phenomenon and the context are not clearly evident, and also allows the researcher to use multiple sources of evidence. The research utilized a variety of sources of data.

The Interview

Interviews were conducted with 63 persons, 44 students (38 female and 6 male) and 19 educators, including principals, vice-principals, guidance counsellors, and teachers. Private interviews were held with all the staff members and 13 of the students. Interviews with 31 students were held in small groups of 5/6.

The students who participated in the interviews formed a non-random sample chosen by the guidance counsellors and mathematics teachers of each school, respectively. The educators chose the students to fulfil the criteria requested: for example, some very superior students, some average students, and some students who were failing mathematics, representing Grades 9 to 13.

Direct Observation of the Mathematics Classes

In each of the three schools, the equivalent of five 40-minute periods was spent observing mathematics instruction and student-teacher interaction in Grades 9 to 13, inclusive, in algebra, geometry, functions, calculus, and the mathematics of investment. The purpose of the classroom observation was to analyze the hidden curriculum by observing "the production of mathematics learning through social relations in the classroom" (Walden & Walkerdine, 1982, p. 9).

Statistics
Quantitative information was very useful in comparing the number of females versus males who participated in mathematics competitions and in researching the variety of ethnic students who were studying mathematics.

Field Notes
Extensive field notes were kept to assist in interpreting data and to minimize errors and biases by documenting all the procedures and descriptions of the social environment.

Method of Analysis
A qualitative analysis of the findings was made in an attempt to get a "holistic picture of the problem" (Klein, 1983, p. 93). An interpretive approach was used in an effort to understand the culture and social conventions that influence secondary school young women in their course selection decisions. The responses to the many questions in the interview schedule revealed that the gender factor in mathematics is a complex one, and that many forces are at play in influencing the curriculum and subsequent careers choices of young women.

The Ethnic Background of the Family
Funk and Wagnall's *English Dictionary* (1986, p. 455) defines "ethnic in Canada," as "an immigrant, especially one whose mother tongue is not English or French." For example, an ethnic student would mean a student "belonging to, or distinctive of a particular racial, cultural or language division of mankind." Of the 38 young women interviewed, 26 were ethnic students from the Orient (China, Vietnam, and Thailand), from rural areas of Southern Europe (Portugal and Italy), from Eastern Europe, or from the Philippines. The ethnic background of young women was a powerful factor in determining whether they liked mathematics, whether they expected to continue in their academic studies, and even whether the school thought them capable of mastering the subject.

The Oriental Students
Nine of the young women interviewed (approximately 24%) were of Oriental descent. Five of the nine students had been born in the Orient and four had attended school in their native countries. Those of Oriental descent were, almost without exception, excellent mathematics students. This was the expectation of both parents and educators. Educators made comments such as, "Oriental students are a very special group," "We expect and parents of Oriental students expect their children to excel in school."

The young women of Oriental descent were well aware of the high expectations for them and, given Western society, they received ambivalent messages because they were female and they could do mathematics. A guidance counsellor commented on how the masculine attribute of mathematics was influencing some of the young women of Oriental background. She had counselled two female students from Asia who were model students. However, when they reached Grade 12, one of them dropped out of school completely and the other complained to the counsellor: "You know, all Asian people are supposed to be good at math and science. It is so unfair." These students were getting "ambivalent messages" about their academic performance. Their background, culture, and traditions approved of their achievements, but some of their peers perceived their success as somewhat odd and out of the ordinary. For some of these students, their mathematical ability presented a dilemma. In separate interviews, two Oriental young discussed the problem of their lack of social life and their feelings of discomfort and unease because of their excellent academic records, especially in mathematics. A Grade 12 student stated that her success probably affected her social life. Another young woman, a Grade 13 student, also indicated she felt a "little different" because she was an excellent mathematics student, saying "I don't like to be that high in math." She was one of a group of four young women being interviewed, three of whom were Oriental. All signified agreement with her reply.

Other Oriental students made remarks such as, "The other girls think you are a little bit different," and "Teachers expect too much of us."

Young Women of Other Ethnic Backgrounds

Five students of Eastern European background, three female and two male, participated in the study. All were outstanding students in mathematics and displayed a healthy attitude towards the subject. Educators confirmed that students of Eastern European descent were among the best motivated in the schools. However, these students also complained about their peers' negative reaction to their success. The comments of a Grade 13 young woman, a recent immigrant from Eastern Europe, epitomized their feelings: "The boys discourage the girls so much."

Of the 38 young women interviewed, 14 (approximately 37%) were of Southern European descent. Interviews with both educators and students demonstrated that cultural values of their background affected the attitudes of these female students towards mathematics. Commenting on the few young women interested in technology in his school, a male principal noted that the problem was more pronounced in schools that served certain ethnic communities where "it is a whole different world for girls than it is for boys." "In these communities," he said, "a male is a very important person and a female still has the problem of being able to go to school and do math." To illustrate this difference, he referred to a very bright young woman who had been a much better student than her brother, but who discontinued her studies and went to work to help her brother attend university. He added that, as far as her family was concerned, she had done the right thing.

Educators are aware of the limits of what they can do to encourage young women in certain ethnic areas. They understand the twin problems of lack of tradition in mathematics and economic hardship. Many of the ethnic groups come from rural backgrounds, without university degrees, and do not have a university tradition for either males or females.

Certain schools that serve ethnic areas also have difficulty attracting teachers who specialize in mathematics. A mathematics teacher who considered applying for a department head position advertised by her school board commented that she lost interest in the appointment when she learned it was a school where there were many ethnic students who were not particularly interested in mathematics. She believed she would not be sufficiently challenged academically in such a situation.

Young Women and Mathematics Competitions

The research investigated the attitudes of both school personnel and students towards competitions as a means of encouraging young women to study mathematics. The results of this particular aspect of the study are not conclusive. Only one of the three schools, the all-girl high school, participated in the National Science Fair, and the school did so with national success. The staff of the two coeducational schools gave a variety of reasons for their non-participation: teacher workload, lack of student interest, the fairs involve projects done by parents.

Students from the three schools participated in various math contests. However, there were striking differences in attitude towards these competitions between females students in the all-girl school and in the coeducational schools. In the all-girl school, where more students participated with success, the teachers emphasized the activity itself rather than its competitive nature. Some young women simply wanted to be part of the team and, in the words of the coach of the school's Math League team, wanted "to have a feeling of belonging." Both the coach and the students remarked that the Math League "had a reputation and the students like to participate in it, with both junior and senior teams." The majority of the students interviewed (10 of 14) had participated in the Math League and several had written the mathematics contests. However, the educators in the school emphasized that "participation" rather than winning was the important element. Even weak students were encouraged to join, and cooperative learning was emphasized. The students in the all-girl school expressed enthusiasm for the competitions. They spoke of extra-curricular activities in mathematics as being more interesting than the regular curriculum and considered them a means of reinforcing what they had already learned. A Grade 13 student commented that the math competitions made her "competitive with the guys" and another stated, "I think girls should be encouraged to do extracurricular activities in math and science."

In the coeducational schools, the female students' attitude towards competition was almost completely negative. The experience in both schools was that young women, even top students, were generally not interested in mathematics competitions per se. Of the 12 female students interviewed in one school, only 2 had written the math contests, and none had joined the Math League. In the second school, 2 of the 11 female students who participated in the study had written the math contests and none had joined the Math League. By contrast, the three young men in one school had entered the competitions, and two of the three in the second school had also done so. In another school, of the 12 female students interviewed, only 2 had written a math contest and none had joined the Math League. A Grade 12 student, who was one of the best math students in the school, admitted that she had been asked directly to join the Math League but she had refused, saying, "I would never participate in a math contest." Another Grade 11 student in the same school, who loved both math and music, said, "No, I never thought I was bright enough to compete." In spite of her great ability, she was not interested in nor did she feel confident enough to participate in the competitions. Another young woman expressed dislike for the pressure of "timed competitions."

During a group interview with five female students, one conceded she might join the Math League the following year, but she would never advertise it because of her peers' negative attitude towards such participation: I might join (the Math League) next year. I wouldn't advertise it....The Math League is considered a detention. Even the yearbook photographer made fun of it and the other students made silly jokes about it.

The male students, on the other hand, seemed to equate the Math League with any other competitive extracurricular activity. Nevertheless, both male and female students discussed the negative attitude that seemed to prevail towards those students who participated in math competitions. They agreed with the comments of a female student who said students who do participate in such activities are known as "nerds" or "misfits," a common name for social outcasts. In both coeducational schools, the young women generally were not interested in competitions nor contests in mathematics. Only two young women in these schools spoke of their participation in contests as a favourable experience. Both were recent immigrants from foreign countries, one from the Orient and the other from Eastern Europe.

In both coeducational schools, all the teachers who coached the students for the competitions commented on the female students' aversion for the contests, and admitted it was a struggle to get them to participate. In the senior Math League, coaches considered it fortunate to have one or two young women on the team. One coach stated that, at the competitions, male students do much better, have a much greater competitive spirit, and are less hesitant than the female students to attempt answers.

The coach of the all-girl team noted that opposing coaches sometimes make negative comments about the "token females" on their teams. He related that during a competition, one of the male coaches called out, in a loud, clear voice so that the all-girl team from his school could hear: "Oh, our token female couldn't make the game today." Remarks like these contribute to young women's perception that there is something not quite right about their interest in mathematics, he said, and are not helping those who want to improve the percentage of females who participate in these competitions. No wonder, he observed, that for many young men, the Math League is replacing sports, but, for young women, it is just one more reinforcement of their inferiority.

Conclusion and Implications for Education

The Ethnic Background

The particular influence of ethnicity on the mathematics education of young women can be looked at from two points of view. On the one hand, the immigration of many Oriental and Eastern Europeans to Canada has helped eliminate the impression that young women naturally have special problems with mathematics. On the other hand, teachers, educators, and parents should be made aware of the pressures felt by many students of certain ethnic groups, and should understand the priority the various ethnic groups assign to mathematics education. Educators also need to be aware of the special tension some of these students experience trying to live up to the expectations of others. All social groups might learn from each other.

Cooperation Versus Competition

Female students react differently than male students in the social context of the classroom. They seem to favour collaboration over competition in learning. An important question for research might be: What changes can be made in the teaching process to establish a better learning climate for female students in the mathematics class?

According to Kohn (1986, p. 157), the difference between cooperation and competition is similar to "the difference between listening to each other's points of view and twisting each other's arm." Cooperation, then, should be an excellent method for young women to employ in learning, no matter what their field of inquiry. Cooperative learning experiences would offer young women a clear alternative to the competitions provided by the Math League and the math contests.

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GENDER AND MATHEMATICS EDUCATION: SOME IMPLICATIONS FROM FEMINIST SCIENCE

Suzanne K. Damarin
The Ohio State University

During the past decade, feminist scientists and philosophers have developed a substantial critique of some bodies of scientific knowledge and of the assumptions and procedures involved in "doing science." Several feminist approaches to science have been identified; in this paper I discuss briefly the application of two of these, feminist empiricism and feminist standpoint theory, to the science of mathematics education. Feminist empiricism begins with the position that science and its global methods are basically sound, but some practices, procedures, assumptions, and, therefore, findings of scientists are biased against women; because these practices, or abuses, are detrimental both to women and to science, they must be identified and curtailed. In contrast, feminist standpoint theory argues that a less biased account of the world can and should be constructed by beginning investigations with the experiences of women; this point is elaborated further below.

Feminist Empiricism

Within the fields of educational and psychological research, feminist empiricism has led to detailed analyses of ways in which sexism influences research (e.g., Eichler, 1988; Squire, 1989). These analyses make evident the potential for gender bias to affect studies at all levels: in the framing of research problems, in the methods of gathering information, in the coding and analyses of data, and in the interpretation of results. Feminist empiricists strive to eliminate all such biases from science. Further, they argue that the elimination of these biases allows the emergence within science of new constructs that can provide alternate descriptions and explanations of "the world."

Feminist Empiricism, Gender, and Mathematics

Much of the research that has contributed to the study of gender and mathematics

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can be seen as belonging to the tradition of feminist empiricism. Early writings on gender and mathematics by Fennema and others begin by addressing problems in the (then) current scientific literature and outline research issues and agendas for future, less sexist, study. It is important to note not only the coherence of most of these studies of mathematics and gender with the ideas of feminist empiricism, but also the substantial and continuing contribution of this work on mathematics to the larger body of feminist empirical work related to the global study of gender. Notwithstanding these successes, examination of feminist literature on bias in science raises several challenges for feminist-empiricist studies in mathematics education. Jagger (1987), in her discussion of studies of sex differences across many areas, points out that by accepting the tradition of designing studies to test null hypotheses of no sex difference, and accepting findings as significant only when the null is rejected, "a literature of differences" is created. By accepting this framing of research methodology, any question of gender is reduced to framing in measurable terms. Considering Jagger's insight in relation to the study of gender and mathematics education, it is not difficult to think of questions whose complexity does not admit this framing; the issues are far more complex than a "literature of differences" will allow.

The framing of questions as null hypotheses is not the only way in which bias occurs in the statement of problems for empirical investigation. Throughout the current literature on gender and mathematics, there is a persistent finding that girls excel at lower level computational skills whereas boys excel at higher level problem solving. Framing what girls do well as "lower level computational skills" (rather than as, say, "concern with, attention to, and appreciation of numerical detail" or "competence in handling numerical systems and their operators") invites the belief that girls' mathematical behavior is fully understood through lower level studies using lower level tests. Moreover, this framing is consistent with the general tendency to describe all female behavior as less competent than male behavior; this trend pervades the sex differences literature and has multiple effects. In particular, attention is drawn to the question of what the boys are doing, and the study of "gender" is shaped (once again) toward the study of the dominant group in order to identify mathematical behaviors of the achieving males that might be taught to females. What is left unstudied, and even unrecognized, is the nature and meaning of the behavior of women in situations laden with mathematical content. The responses of women to such situations are among the interests of feminist standpoint theorists.

Feminist Standpoint Epistemology

Feminist standpoint epistemology is a complex approach to the definition and description of a self-consciously feminist way of constructing and conducting science. The idea of a "feminist standpoint" was first introduced by Nancy Hartsock

(1983) and had its conceptual roots in ideas borrowed from Marxist epistemology. The following are important features and components of feminist standpoints as theorized by various feminists:

1. As theoretical positions for knowledge building, "'subjugated' standpoints are preferred because they seem to promise more adequate, sustained, objective, transforming accounts of the world" (Haraway, 1988, p.191)Women can know the world in valid ways that are not available to their oppressors; because they have less to lose in changing the status quo, they are lessbound to it and better able to examine it (Harding, 1991, and others).
2. A feminist standpoint is an achievement, not a birthright (Haraway, 1991). That is to say, women do not automatically or "naturally" look at or come to know the world in a particular (feminine) way. Moreover, a standpoint is different from a perspective that can be obtained simply by opening one's eyes (Harding, 1991) A feminist standpoint must be actively pursued and constructed as a way of knowing that begins with the lives of (particular) women in the world.
3. Multiplicity is implicit in feminist standpoint theory; there is not one "correct" feminist standpoint, but multiple feminist standpoints.For example, African American feminists (e.g., Collins, 1990) and Third World women construct feminist standpoints that are different, but not necessarily disjoint, from standpoints constructed by middle-class white women.
4. The "god-trick of seeing everything from nowhere" (Haraway, 1991, p. 189) that has been the goal of controlled experimentation in science is simply impossible. Scientists do not stand outside the systems they study; scientists and all knowers understand the world from some position and that position is material to the "truth" constructed.
5. Knowledge is always situated by the standpoint of the knower; from a feminist standpoint, knowledge begins with women's lives. The feminist standpoint is an imperative for women to construct knowledge beginning with their own lives and experiences.
6. Knowledge is always political, and never value free.
7. The idea that all aspects of the world or reality are socially construct *rejected.* It has been strategic for women to work from a social constructivist point of view in order to counter prior constructions such as "essential feminine virtue" and "biology is destiny." However, the view that every thing is socially constructed implies that our bodies are simply blank slates on which society writes its messages. This view is rejected by standpoint theorists because it leaves women (and men) without agency. Unless women have agency, they cannot construct and act upon a standpoint.
8. The feminist standpoint entails a "radical objectivity" (Haraway) or "strong objectivity" (Harding) in which there are real objects for study and under-

standing that exist outside the knower. Knowledge is achieved through the reciprocal relationship between the knower and the object of knowledge; the knower and the objects of study lie in the same "critical plane" (Harding, 1986). However, the objects of knowledge are not static andpassive, but are actors; knowledge is constructed dialectically through in teractions between object and knower (Haraway, 1991).

9. The nature/nurture dichotomy is rejected, as are other binary dualisms such as mind/body. Moreover, the traditional view of nature as fixed and unchanging is rejected. "The world" and "nature" are conceived, not as givens, but as actors that operate over time and space.

Summarizing these points, feminist standpoint theory is not a kind of relativism but acknowledges that "the world" exists and is knowable through the study of our relations with it. Interrogating our own position in relation to the objects of study is a critical part of the study of those objects. Unless we begin with that interrogation, we cannot understand the value-laden aspects of our own perspectives. Because women (and other "marked" groups) bring to their study less investment in continuing current theories, conceptions, and practices, their relations to the objects of study are less bound to the acceptance of present understandings as "true" or "natural." Indeed, the idea of the "natural" is rejected. Thus, the feminist standpoint idea allows for a multiplicity of truths, none of them complete, and finds most valuable those investigations that begin with the lives of women. For research on mathematics education, this implies a radical shift in underlying assumptions and standards; in particular, it requires a willingness to abandon beliefs about the nature of mathematics and how it must be taught and learned in order to be open to the "nature" of mathematics as it is experienced.

With respect to mathematics, there are clearly many standpoints from which particular women view the issues facing them as they do, or don't do, mathematics. Some of the disparity of these views is discussed by Longino and Hammonds (1990) as they contrapose the views of women within the professional domains of mathematics, science, and engineering to the views of academic feminists who address these questions from outside mathematically oriented professions.

Discourses of Distance and Feminist Standpoints

To arrive at a feminist standpoint with respect to mathematics and mathematics education, the primary question to be addressed is, "How do women experience mathematics?" A major part of this experience for many women is indirect; that is, the experience of mathematics does not take place within a community of creators or users of advanced mathematics, but rather in the general society. This experience is primarily discursive, not active, which is to say that the relation of most women to mathematics is constructed by the receipt of messages about mathematics. Thus, it is the content of these messages that creates the experience.

To begin, women (including young girls) experience mathematics as an area of competing discourses. Simultaneously told that it is important to learn mathematics, and that it is not important (for girls and women) to learn mathematics, women are subject to a multitude of other mixed messages about the importance of mathematics to their lives. In the following paragraphs I will examine some of these messages through the lenses of various feminist works and writings.

The Maleness of Mathematics

The effects of the idea that "math is a male domain" on the mathematical achievement of girls and women have been investigated empirically from 1977 (Fennema & Sherman) to the present (Ethington, 1992). By and large, empirical study has posited that the equation of mathematics with maleness is an individual attitude measured in individual females; as such, it is subject to individual remediation through instruction in the affective domain. However, the "maleness of mathematics" is a message that permeates our society; regardless of how mathematically competent a woman becomes, she can never escape discursive practices that reify the idea that mathematics is, indeed, a male domain. Thus, for a woman to continue to learn and to do mathematics, she must continually reject the messages that connote her "natural" position.

The message that "math is a male domain" links the present to a past in which a double argument insured the maleness of mathematics: the primary argument asserted that women *could not* do mathematics, while the second back-up argument asserted that they *should not*. Both arguments linked mathematical and reproductive capacities in a relation of logical exclusion in the historical construction of mathematics as outside the domain of women. Sociological explanations for women's separation from the study of mathematics speak to a need for adolescent women to choose among the demands to prepare for roles of wife, mother, and career (Maines, in Fennema, 1985). Moreover, in many ways the curricular and extracurricular teachings of schools speak to this bifurcation in sex education programs, for example.

The "othering" of women from mathematics extends beyond school and home. As Nelkin (1987) points out, the popular press describes scientists generally as male and remote from the realities as everyday life; women scientists, by contrast, are portrayed as homebodies who bake brownies and excel at all: career, wife, mother. Through these messages, the press creates for women a confusing picture. Adding to the confusion, what counts as mathematics and mathematical ability seems to change over time as historian Patricia Cohen (1982) documents.

Thus, within these contexts of history, society, the press, and schools, the message is alive and prevalent that "mathematics is a male domain." Even the continued study of women and mathematics (and the publicity that surrounds it) reminds women that their mathematical ability is a question worthy of scientific study. In this context, feminist researchers and theorists have responded with a variety of questions, ranging from "Well, is mathematics essentially male?" to "How is the maleness of mathematics socially constructed and reproduced in schools?" Next I consider some of these questions, beginning with the first.

The Female Mind-Body and Mathematics

The importance of combined physical and mental activity to the construction of mathematical knowledge is a theme that runs throughout the pedagogical and philosophical literatures of mathematics. Generally speaking, however, the question of what constitutes meaningful experience has not been addressed, at least not from a sex/gender point of view. The many uses of building blocks as manipulative materials might have differential meanings against the sex/gender-linked differences in prior experience with similar building materials (Damarin, 1991). Similarly, the relations of meaning making in block building itself to even prior experiences of sexuality and gender construction are basically unexamined.

In the context of the importance that mathematics educators attach to physical experience in relation to mathematics learning, feminist researchers might query sex-specific experience of the body in relation to the construction of mathematics. In their work, feminist philosophers Evelyn Fox Keller (1985) and Susan Bordo (1987) uncover metaphorical relations between the biology and practices of male sexuality and descriptions of the purposes and procedures of science, including the mathematical sciences. Feminist philosopher-psychoanalyst, Luce Irigaray (1987) queries the nature of mathematics itself in relation to the sexed bodies of women:

> The mathematical sciences, in the theory of wholes, concern themselves with closed and open spaces, with the infinitely big and the infinitely small. They concern themselves very little with questions of the partially open, with wholes that are not clearly delineated, with any analysis of the problem of borders, of the passage between, of fluctuations occurring between the thresholds of specific wholes. (pp. 76-77)

In this and other examples, Irigaray argues that logic, mathematics, and science encode principles that reflect and honor male, but not female, biological and psychological development.

Working from Irigaray's perspective, we can return to the findings of empirical study of women and mathematics with new questions. We might ask again, for example, why adolescence marks the increased separation of women from mathematics. In the context provided by Irigaray we can see an opposition between the linear time of mathematics problems of related rates, distance formulae, and asymptotic convergence versus the dominant experiential cyclical time of the menstrual body. Is it obvious to the female mind-body that intervals have endpoints, that parabolas neatly divide the plane, and, indeed, that the mathematics of schooling describes the world of experience in intuitively obvious ways?

A Multiplicity of Questions

If Irigaray's work leads us to question the assumptions concerning what is "obvious" that underlie the mathematics curriculum, other social scientists suggest different ways in which we might examine curriculum and instruction from a feminist standpoint. Based upon their study of "Women's Ways of Knowing," Mary Belenky and colleagues (1986) speak to women's relations to abstract knowledge:

> Most of these women were not opposed to abstraction as such. They found concepts useful in making sense of their experiences, but they balked when the abstractions preceded the experiences or pushed them out entirely. Even the women who were extraordinarily adept at abstract reasoning preferred to start from personal experience. (pp. 201-202)

These findings suggest that more attention be given in the teaching of mathematics both to the provision of opportunities to accumulate observations and to the acknowledgement of experiences with diverse ideas before these ideas are treated as obvious and formalized in definitions. How this might be achieved within the current structures of schooling and curriculum is unclear.

It is well-known that women often lack confidence in their mathematical knowledge and abilities. But how confident can women be if they lack an intuitive grounding for mathematical ideas, and if the value of their knowledge is continually undermined? Several researchers (Walkerdine, 1989; Willis, 1992) have uncovered evidence that teachers attribute female success in mathematics to hard work whereas they attribute comparable male success to ability. At the same time, there is an apparent arbitrariness to the sequence of mathematics instruction; the knowledge obtained as "prerequisite" for a course or topic is often not applicable in the subsequent context, and often leaves students confused about their own mathematical power (Willis, 1992). We might ask, then, what can be the source or grounding of women's confidence in their own mathematical ability? In the absence of societal messages affirming that mathematics is a female domain, of personal intuitions and the opportunity to build on them, of teacher recognition for ability as evidenced in accomplishment, and of the opportunity to apply knowledge in subsequent courses, on what base might a woman build a sense of confidence in herself as a mathematician? Again, we have an enigma and a challenge for mathematics education.

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A fuller discussion of these issues is presented in Damarin (forthcoming), from which this paper was adapted.

WHY ARE BOYS AS AFRAID OF MATHEMATICS AS GIRLS?

Andrejs Dunkels
Luleå University

The title of this lecture is deceptive. How should one perceive its message? Does the question concern all boys? Is it about all girls or some girls? Questions beginning with *why* are particularly deceptive. It is easy to accept, unconsciously, the claim made in the question, in this case that girls *are* indeed afraid of mathematics. It is easy to accept, without noticing, hidden claims in prejudiced statements of this kind.

Here I will touch upon one aspect only of the huge issue of mathematics anxiety, or fear, if you wish, and discuss certain parts and certain sources that affect everybody alike.

First, a comment on prejudiced statements, such as the British empty phrase "Boys will be boys." Who can contradict something like that? Who can refute something so self-evident? Something is hidden here that I cannot accept. Of course I understand that there is a difference between boys and girls – *that* particular difference is obvious. Sometimes this biological difference is used as an argument for differences in, for example, problem-solving ability in girls as a group compared to boys as a group. There must be differences in all kinds of other areas as there is such a big biological difference, the reasoning goes. Facts or prejudices?

Sometimes we hear that girls have more difficulties with geometry than do boys. This view is old. There was a time in Sweden when geometry was not taught in girls' schools. In mixed classes the girls had time off from mathematics when geometry was the topic of the lesson. There was no point in keeping the girls there. They were not capable of learning anything of that subject anyway. We may smile at such an idea – or weep. Yet it lies in wait and pops up even now among teachers of mathematics. Do such views influence one's actions in the classroom? What message does one then convey to the girls? How does a person who is considered a failure feel, never even having had a chance to try? Could it be that some of the girls get worried and gradually develop a fear of mathematics?

In this lecture I will not dwell on the problem of possible differences between boys and girls. My basic view is clear. I agree with the view of the Swedish basic school, as I have perceived it. Boys and girls are to be treated equally, all given

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Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

equal opportunities for developing their own ability. All pupils are to be given the same kind of encouragement, nobody made to feel that he or she is not good enough because of his or her biological gender. Each one is to be given the chance to build new knowledge based on his or her pre-knowledge and previous experiences.

I am sorry to say that my experience as a parent and a teacher-educator is that girls and boys are not treated equally in the Swedish school. This has also been reported by several researchers. It seems hard to break traditions. In my view, the variation within the group of girls is very large, as it is within the group of boys. There are some girls who are more like some of the boys than like other girls by, for example, being interested in mathematics, and there are some boys who are more like some of the girls than like other boys by, for example, being interested in dogs and horses.

It is impossible for me to accept the idea that there would be one way of thinking about mathematics for girls and another for boys. It is hard for me to accept that girls, for biological reasons, think and feel otherwise than boys within the area of elementary mathematics. Recently I read an article from research about "mothering" (i.e., taking care of babies). There is the view that women do this by instinct and that men can never learn "mothering." The conclusions of the researchers are that the instinct women had in ancient times is lost in today's humans, that women must learn "mothering," as must men; there is no difference in the ability to do this. Personally I am not at all surprised.

Some Sources of Mathematics Anxiety

Replay Pedagogues

When discussing mathematics anxiety, we do not have "Readin', 'ritin', 'rithmetic, taught to the tune of the hickory stick" in mind. The fear, as we see it today, does not have to do with physical punishment. However, the pupil may get *a replay pedagogue* as a mathematics teacher. Such a teacher knows just one explanation for each concept or problem. If a pupil asks a question, this kind of teacher will give the same explanation he or she gave the previous time. Should the pupil ask a third time, the same explanation is replayed. This is a source of fear. After a handful of such events most pupils choose to be silent.

Future teachers of mathematics need to reflect upon their own and others' ways of thinking. By reflecting upon one's own way of thinking, one becomes aware of the fact that thinking can be modified. This leads to metacognition. Somewhat oversimplified, this means that one puts oneself outside of oneself and observes one's ways of knowing. One thinks and reasons about and around the knowledge, not only about its contents. If one practices thinking and reflection in this way, there is may avoid becoming a replay pedagogue.

A variant of the replay pedagogue is the *crescendo pedagogue*,who not only repeats the same explanation once more, but does so in a louder and louder voice. Such pedagogues effectively bar all further questions from the pupils, the mathematics lessons pass in silence, exactly the way they *should*, and some pupils in the class develop an uneasy feeling about mathematics – they like the subject less and less, and eventually this may lead to fear and hatred. This influences boys and girls equally, although possibly there is a difference to be noted. Both boys and girls are brought up with the idea that it is acceptable for a girl to show fear; a boy is not afraid of anything, or at least he does not reveal his fear.

To Cultivate Thinking and to Follow Rules

For me teaching means taking interest in other people's thinking and sharing with them my own thinking. My main task as a teacher – at any level of teaching – is to cultivate my pupils' thinking. In this respect I consider all learners – pupils, students, teachers at in-service meetings, and so forth – equal. Boys as well as girls of all ages must be encouraged to practice thinking. In order to do so, boys as well as girls need stimulation from their teacher, so that they can add new knowledge to their old and sometimes modify the old knowledge with new insights.

Each era has its megaphones calling out for more basics, which in mathematics often means more traditional written tests. This encourages rule-based mathematics, a rich source of misunderstandings and hence of mathematics anxiety – affecting boys and girls with equal power. A famous rule is, "like signs give plus," which is very dangerous when taken out of context. I have encountered the following reasoning, "– 4 – 2 = +6, for, to begin with it is –6, but as there are like signs then that gives plus."

The work and activity of the pupil must be based on thoughts and not on mechanically moving symbols and signs about. Only then is there a basis for developing confidence with mathematics.

A well-known way of visualizing the solving of equations, well suited for the overhead projector, is shown in Figure 1.

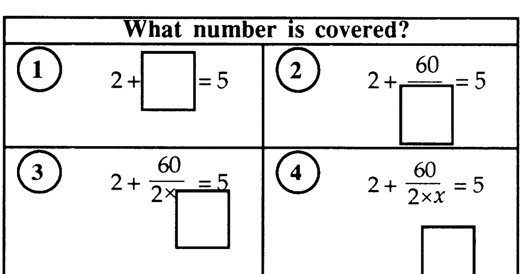

Figure 1 Part of an equality is covered with a piece of paper. This piece is successively moved so that more and more is seen of the expression that was completely covered at the outset. At each step one asks the question, "What number does the piece of paper cover?" Each pupil who is familiar with division and fractions kan easily follow and give the answers.

If the problem had been posed starting with frame number 4 in Figure 1, that is, as "Solve the equation ...," then many pupils would have been worried and felt that the task was too hard for them. In particular, this is true for all those who have learnt solving equations through a huge number of rules. For each of the many types of equation there are special rules of how things may be moved about. The rules are not phrased in terms of ways of calculating or ways of thinking about numbers but as rules for moving, deleting, changing signs, and similar ingredients. The equation that appeared in frame number 4 in Figure 1 belongs to one of the hardest types with x in the denominator. For this kind of equation, rules far down the list would be applicable.

All these rules form a source of worry. The beginner does not see any coherence in the system of rules, there is no way of reconstructing a rule, since it is not based on thoughts and intuitively acceptable reasoning. And it is hard to memorize when a certain rule is to be applied. It is hard to identify the relevant type of equation. Everybody suffers – boys and girls alike. If, however, the treatment of equations is based on ways that develop true thoughts about numbers then the learner gets the opportunity to build useful knowledge.

Another important example that I want to mention here has to do with "false metaphors." Humans often think in pictures, or images.

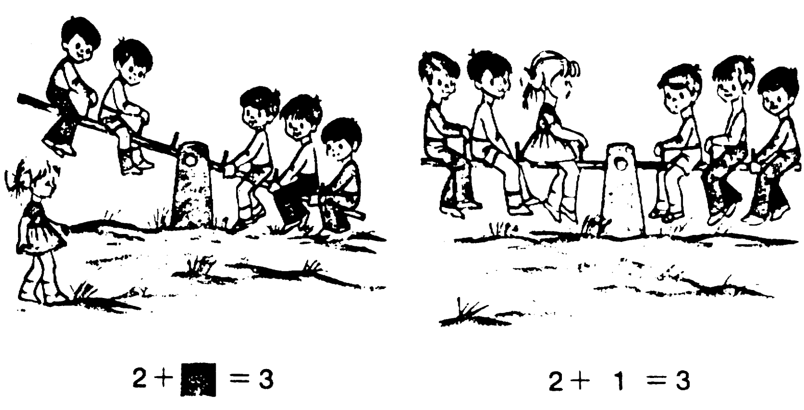

Figure 2 A sequence of pictures from a book for elementary school. What a surprise it must have been to the children when the seesaw ended up in a horizontal position with three children on each side. This very seldom happens in actual fact. Mathematics seems not to be connected to the real world.

The sequence of pictures in Figure 2 is from a textbook for elementary school. It wishes, obviously, to illustrate the relation $2 + 1 = 3$ and makes use of the equilibrium position of the seesaw as a metaphor for equality between numbers. The crux is that the written equality refers to *how many* children there are whereas the seesaw metaphor does not. There is something wrong here. All children have experiences of seesaws. They know that the same number of children on both sides does not imply equilibrium. Rather, their experience tells the opposite. Here mathematics teaching is not in accord with reality. The unpronounced message is that mathematics is one thing, reality another. This may seem strange and therefore scary. It affects girls and boys equally.

Conventions and Mathematical Truths

Piaget says that we acquire conventional knowledge in a different way from mathematical knowledge. Conventions can be learnt by being informed about how something is agreed upon. For example, you can learn that Christmas Eve is on December 24 from somebody who tells you so. You can learn that the number zero is written 0 in Sweden but • in Egypt. You can learn that the symbol P stands for the first sound of the word PAL – at least as long as we are dealing with English or Swedish. In the sequence CCCP that the Soviet ice hockey players used to have on their shirts, however, the symbol P signifies the r-sound. All these are conventions, that is, agreements. Conventions are, in general, more or less local.

A source of worry and mathematics anxiety is uncertainty concerning the status of statements appearing in the learning of mathematics. Some statements are mathematical truths, others are conventions, agreements. Conventions cannot always be understood. Sometimes they are logical, or some inevitable consequence of development, sometimes they depend on arbitrary choices or on chance. Mathematical truths are logical consequences of the properties of the basic concepts. There are no arbitrary choices, and so mathematics can be understood.

If the distinction between conventions and mathematics is not clear to the pupil then uncertainty arises. Then there is a risk that the pupil does not understand. Fear of mathematics is then close. It strikes girls and boys alike.

An example of a convention is the way we write fractions. In printed texts fraction bars are set so that they are aligned with plus signs, minus signs, and equality signs. Many teachers, including myself, believe that it is best to adhere to this convention also when writing by hand. We introduce a local convention saying that the alignment should be made so that it is possible to draw a horizontal line through plus signs, minus signs, and equality signs, as in Figure 3.

Figure 3 Writing fractions is not easy. Everybody knows who has tried to do that in a small word processing program or on a traditional type writer. In printed texts fractions are set with fraction bars aligned with plus signs, minus signs, and equality signs.

If I as a teacher find it important to write fractions according to this convention, then I must discuss this with my pupils. I myself always make it clear to my pupils that the way of writing does not belong to the mathematical truths. I make clear to them that it is not *wrong* to write according to the crossed-out variant of Figure 3, but that I would get irritated and bad-tempered about that way of writing. The only reason is not that it looks ugly. It is primarily because that way or writing is risky, there is a risk of losing denominators, in particular if one's expression contains both terms that are fractions and terms that are not. I have seen enough of such mistakes to consider it necessary to issue a serious warning.

The result of the addition in Figure 3 is a mathematical truth; we cannot change it to something different. We cannot even discuss any change of it. That truth is the same everywhere. Nobody can learn that truth by being informed. That truth has to be built up, like any other piece of knowledge, by each pupil inside of him or her.

Another important convention is the direction of reading and writing, in Sweden from left to right. Of course one must be aware of this convention in order to learn reading and writing Swedish or English. When it comes to numerals, we also read from left to right; there is nothing curious about that. (See Figure 4.) It is, however, of the utmost importance to be aware of this, so that one pays the right amount, so that one dials the right telephone number, so that one finds the right house number that one has been given on the phone.

705 + 829 = 1534

Directions of reading and writing of a text as a whole.

١٥٣٤ = ٨٢٩ + ٧٠٥

Figure 4 In Swedish (top line) the direction of reading and writing is from left to right. The results of the addition is a mathematical truth and depends on the properties of the numbers involved, but the ways of writing the numbers are just conventions. In Arabic (bottom line) the direction of reading and writing a text is from right to left.—If one knows the Swedish digits, is it then possible to work out what the corresponding Arabic digits are, if it is known that the equalities are the same?

We use Arabic numerals, we usually say. The name comes from the fact that the Europeans learnt these digits from the Arabs who in turn had brought them from India. Those who speak and write Arabic today use digits that look a bit different from ours. They say that they use Indian digits. Arabic is written and read from right to left. (See Figure 4.) This goes for text as a whole. The strange thing is that each individual numeral is read exactly the same way we do, that is, from left to right, see Figure 5. It is possible that the Arabs originally wrote their numerals beginning with the smallest unit, from right to left, and that the Europeans read the written numerals in their fashion. Certain traces of this are found in Danish and German, where, for example, *twenty-three* is *tre og tyve* and *drei und zwanzig*, respectively, that is, *three and twenty*.

These conventions are crucial knowledge if one has to make a phone call in Cairo or make out how much the hotel room costs. It is also important for teachers with immigrant pupils with Arabic as their vernacular. These pupils might believe that everything in Swedish reading and writing is the other way round, and so they may start reading individual numerals from right to left. I have experienced this. If the teacher is not aware of the different conventions then uncertainty and worry may arise. With that, there is a risk of mathematics fear. For us in Sweden who are used to reading in one direction all the time it is odd to have to change for numerals. For those who are used to the Arabic convention it is confusing to read our way.

Direction for entire sentence.

١٥٣٤ = ٨٢٩ + ٧٠٥

Direction for each numeral.

Figure 5 Each individual numeral is read from the left. Here we first have, that is, to the extreme right, 705. Then comes the plus sign followed by 829. After that we have the equality sign and finally the sum 1534.

Selective Viewing

In my experience from schools and from teacher education, the mathematical language of symbols contributes to worry and fear. It is alarming when pupils – and student teachers – believe that symbols are introduced to deliberately make mathematics difficult. The formulas provide something that causes trouble rather than being of help. If the worst comes to the worst, pupils might perceive trouble-causing as the very meaning of mathematics.

It is an advantage if formulas and notations are introduced when the learner experiences a need and feels that the new topics do indeed assist in one way or the other. When notations are introduced prematurely they may become a hindrance and be experienced as a drawback rather than something useful that simplifies and clarifies.

Once the symbols have been introduced, the learner must practice selective viewing. This means that one alternately allows one thing or another to stand out while others become hardly noticeable. It is almost like looking at a puzzle picture, where one at first sees a witch and then suddenly two vases instead. The formulas in Figure 6 illustrate selective viewing. Before one starts looking at details, one should get an overall idea by looking at the expressions without seeing the ingredients as such.

(A)

$$\frac{x}{x+1}+\frac{x-2}{x-1}=\frac{2x^2}{x^2-1}$$

$$\frac{x}{x+1}+\frac{x-2}{x-1}-\frac{2x^2}{x^2-1}$$

(B)

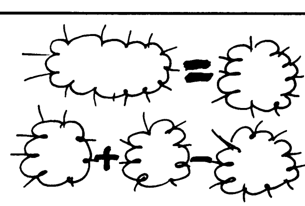

Figure 6 The two rows in (A) differ by just one single dash. Yet they convey completely different messages to the experienced formula reader. In the top row there is an equation, that is, an equality, true for certain values of the variable x. The bottom row has an expression—not an equation—containing a variable. Here there are three numbers to be added. In (B) it is shown how the experienced formula reader sees the two rows at first sight with his or her selective view turned on.

The experienced person, the teacher, has to discuss this with his or her pupils and tell them how he or she reacts when a formula appears. (See Figure 6(B).) Pupils, girls or boys, who have not heard about, or discovered by themselves, the phenomenon "selective viewing" may develop worries about formulas, and so there is a risk for mathematics anxiety to appear.

Future teachers must be made aware of all this. They must themselves reflect about their own way of reading formulas and to themselves make the selective viewing more pronounced and active.

Recipes in Mathematics Education

In a middle school I was once discussing with a pupil, standing slightly bent over her desk, the way teachers of mathematics all over the world do. I wrote this and that down to support our discussion. I wrote in the pupil's notebook. After a time, I noticed that she did not care about the mathematics. Instead I seemed to see and hear, clearly, her thoughts flowing out of her head, "... and his writing is so ugly ... with a ball-point pen and everything ... phew, I hope he will leave soon, so that I can tear the page out – for there is probably no way of erasing ..."

Since that day I have not written in a pupil's notebook. Never. I never will. Instead I recycle some of the used papers – backs blank – that my office is full of. I make small pads, approximately 1/3 of A4-size to be used for writing discussion comments – as unattractively as I like, with any kind of pen. I can draw a sketch, write one or two symbols, tear off the page from my pad, leave it with the pupil or group of pupils, and walk off to the next pupil or group, allowing for reflection and peer discussion. After a while I return to enquire if there has been any progress and continue the discussion.

The standard pattern is that one stands and breathes down the student's neck awaiting a quick answer so that one can hurry to all the many others who are eagerly waving their hands. There is then a risk that the learner feels worried having to produce quick answers, and that the teacher, meaning well, gives away all the answers to questions that the learner has the right to cope with himself or herself.

Do our traditional ways of working contain built-in dangers of spreading fear of mathematics? Is it fear of mathematics that we thereby spread or is it possibly fear of the teacher?

In Sweden from Grade 7 on we know that most teachers of mathematics are males. Could it be that boys more than girls are used to – from sports, for example – being pushed by men? Could this be a reason for girls seeming to be more afraid of mathematics than are boys?

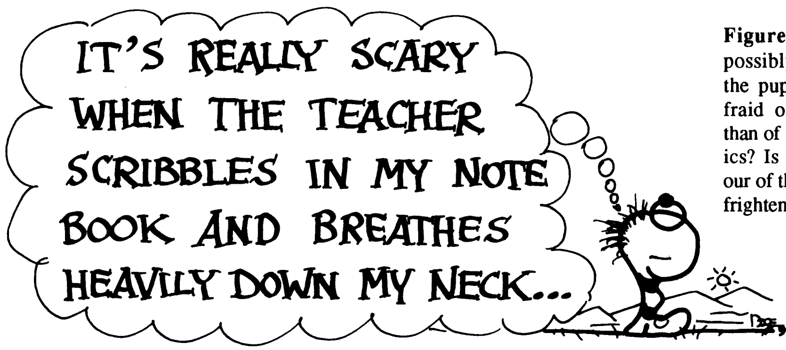

Figure 7 Could it possibly be so that the pupil is more afraid of the teacher than of the mathematics? Is it the behaviour of the teacher that frightens the pupil?

I feel somewhat like a doctor tearing out a prescription from the prescription pad when I hand over my comments on a sheet from my pad. The doctor's prescription is the basis for the pharmacy's recipe of the medicine prescribed. I do not want my sheets to be turned into recipes. They are meant to inspire thinking. Recipes that tell you to do this and to do that without thought should be banned from mathematics education – like the famous "velocity triangle" (see Figure 8).

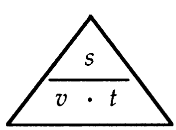

Figure 8 "The velocity triangle" that helps you produce any number of correct answers without giving you any idea of how problems are solved.

What message do those convey who advocate the use of such triangles? What do the triangles support except production of answers? What right does one have as a teacher to delete the context and to reduce certain problems to mechanical pushing of symbols written in a triangle? Where are the thoughts when such methods are practiced? How can this method be used with other problems? How scared does the person get who cannot recall the order in which the three letters in Figure 8 are to be written?

A milestone, also with recipes involved, is division by a fraction. By a "milestone" I mean a place where many learners simply give up, because they feel that mathematics cannot be understood anyway. Boys and girls are hit to the same degree by the "turn-upside-down-and-multiply" recipe, in particular with a replay pedagogue in the classroom – even if one manages to get the correct answer to each and every exercise. Not everybody can work and work and produce correct answers without understanding.

In fact the "turn-upside-down-and-multiply" recipe should not be introduced until it is time for algebra. Until then, one can manage using the least common denominator in each separate case and reason the same way as one does when adding and subtracting fractions. Then one wants to get a unified way of expressing the fractions involved. It is advantageous to do so at division as well, where the ratio aspect is the only possible.

Each division of two positive integers, a divided by b, has a ratio division interpretation:

$\frac{a}{b}$ answers the question "How many times does b go into a ?"

The same idea may be used with fractions: "How many times does what stands under the division sign go into what stands above?" We can, for example, think like this:

$$\frac{2\text{ thirds}}{1\text{ half}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{\frac{4}{6}}{\frac{3}{6}} = \frac{4\text{ sixths}}{3\text{ sixths}} = \frac{4}{3} = 1\frac{1}{3}.$$

By using the least common denominator and rewriting the fractions so they get equal denominators, the problem is reduced to the division of two integers: How many times do 3 sixths go into 4 sixths? This must be as many as the number of times that 3 goes into 4. This reasoning might help some pupils to stop and reflect at this milestone and hopefully come to understand what division of fraction is all about – and not drop mathematics. The traditional – and mystical – "turn-upside-down-and-multiply" recipe can wait until much later. It can be derived with the aid of the least common denominator and will in that case build on something that by then will have become the true property of the learner.

The section on division of fractions – treated traditionally – may enhance the impression that mathematics has come to us through stone tablets comprising all mathematical truths (see Figure 9), truths that cannot be questioned, truths that cannot be understood – and there is no use trying. It is just a matter of learning what to do to get a correct answer. However, as I see it, meaningful learning has to build on procedural knowledge as well as conceptual knowledge. The two types of knowledge go hand in hand, supporting one another. They can never be separated – just one of them will never suffice.

Conclusion

In general discourse one hears more about mathematics fear with girls than with boys. The girls' fear is more noticeable, because girls dare to admit and to discuss their mathematics fear and anxiety. In this lecture I have wanted to show that my experience indicates that the teaching of mathematics tends to affect everybody alike. My view is that differences that might exist originate in differences in up-bringing, differences in availability of models, and influence by society, the world around us, rather than differences in biological gender.

There are several reasons for it being particularly important to cater to girls' mathematics anxiety. Of the countries of Europe, Sweden has the most segregated work market. It is still predominantly females who become preschool or elementary school teachers, where many of our children's ideas about various phenomena and occurrences are founded. As I see it, it is then important that those teachers have a positive attitude towards mathematics without any aversions or fears.

My experience is that it is possible to cure mathematics fear by consistently letting the pupil develop her or his thinking. It is not an easy task. The pupil really must exert herself or himself to the utmost to think independently, the teacher must be silent on the right occasions. For pupils of elementary, middle, and high school it is, of course, an advantage if there is cooperation in their homes. Often so very little is needed for the pupil to adopt a way of working with mathematics based on thinking rather than moving symbols. Discovering the power of thought promotes the creation of motivation, which in turn contributes to reducing worry and fear. This holds, in my experience, for all levels of mathematics education.

It is important not to present mathematics in its polished form, so that it appears to be inscribed on stone tablets. Mathematics has not been created the way it

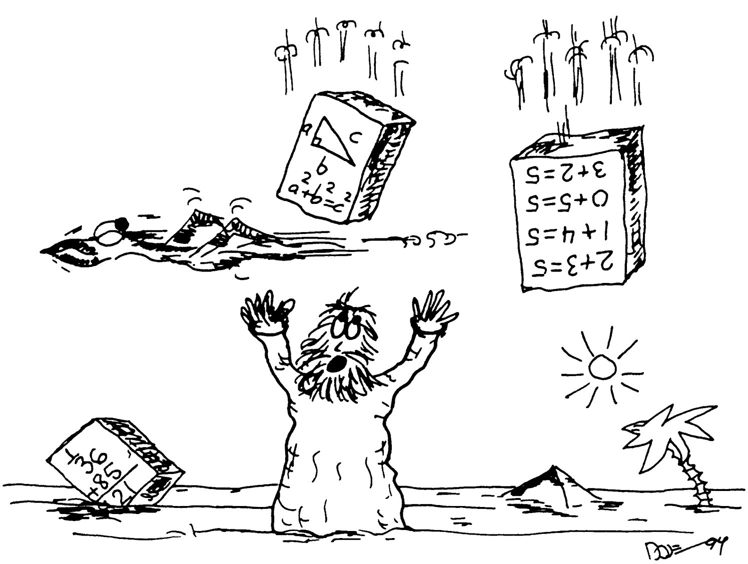

Figure 9 Is this the way mathematics has come to us? Or is mathematics an activity in which people are engaged and that emerges out of insightful thinking? What image do we want to convey to our pupils?

most frequently is presented in the textbooks. Here there are high demands on the side of the teacher's knowledge about concept formation, didactically as well as historically.

Many student teachers who are going to be elementary teachers have not chosen that career because they love mathematics. I have met student teachers with severe mathematics anxiety. They have believed that one has to memorize everything in mathematics and that the only way of learning the subject is through memorization. They had been trained throughout their own schooling to avoid thinking. Mathematics has been answer production according to particular recipes and rituals.

The retraining demands lots of energy. It is painful but possible, in my experience. When the result is positive, then there is no limit to the joy mathematics brings – free from frightening elements. Mathematics has become what it is for those already initiated, namely insightful thought – created by people.

GENDER AND MATHEMATICAL THINKING: MYTHS, DISCOURSE AND CONTEXT AS POSITIONING IN PRACTICES

Jeff Evans
Middlesex University

Introduction

This paper draws on a study of cognition and affect in mathematical activity among adult students in a social science degree program at a London Polytechnic in the mid-1980s (Evans, 1993). Most were in a compulsory mathematics (pre-calculus) and statistics course, and all were familiar with a range of numerate practices outside school. This study allows us to consider ideas concerning gender and mathematics.

We can define "myths" as ideas that are powerful, though not necessarily "true." One example is the idea of gender differences in maths performance that favour males (cf. Walkerdine & Girls and Mathematics Unit, 1989). This characterisation of myths suggests that research may be used (a) to critically scrutinise or to attempt to refute such ideas; (b) to study their *genealogy* or origins; and (c) to explore their *effects*.

Quantitative Research Procedures and The Idea of Context

Subjects (n > 900) completed a questionnaire at a lecture in the first two weeks of the course. The questionnaire included questions on gender, age, qualifications in school maths, and (in later versions) social class measured by occupation, performance, and mathematics anxiety. I aimed to distinguish performance in school maths from that in "practical maths" contexts, using questions for the latter from the survey of adults done for the Cockcroft Committee (1982); see ACACE (1982).

The maths anxiety items were selected from items of the Mathematics Anxiety Rating Scale (MARS) analysed by Rounds and Hendel (1980) into Maths Course / Test Anxiety (MTA) and Numerical Anxiety (NA). MTA and NA were considered to relate to school and everyday contexts, respectively, parallel to the

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distinction between school maths and practical maths performance. Thus, in the quantitative phase of this resesarch, *context* was specified by the *wording* of performance or anxiety items in the questionnaire.

Results From the Quantitative Research

We can use this data to scrutinise myths about gender differences in maths performance. First, the men's and women's average score on School Maths Performance (10 items) were compared, yielding an uncontrolled difference of 3/4 of a question (statistically significant; $p<.001$) in favour of the males; see Table 1.

In order to control for qualification in mathematics (i.e., course-taking), age (as more of the women were mature students), maths test anxiety and confidence (self-rating) in maths, a multiple-regression model was constructed. Now the difference for younger students (aged 18-20), about 1/6 of a question, was no longer statistically significant, and that for mature students (21+), just over half a question, was borderline; see Table 1.

Table 1 Gender Differences in School Mathematics Performance: Uncontrolled Group Means and Estimates Controlled for Qualification in Mathematics, etc.

	Number of questions correct		
	Men's Average	Women's Average	Difference
Uncontrolled	8.78	8.07	0.71 (Std. Error 0.11)
Controlled			
Young (18-20)			0.17 (S.E. 0.17)
Mature (21+)			0.57 (S.E. 0.24)

Thus the gender differences in school maths performance, which are fairly substantial (3/4 of a question, and statistically significant) become much less substantial after controlling for qualification in maths, age, and so on – and the quantitative analysis allows us to do this. (Quantitative research with similar aims is presented in Chipman, Brush, & Wilson, 1985).

Rethinking the Idea of Context – Discourse and Positioning:

Instead of myths, I find it useful to think, more broadly, in terms of *discourses*. We can think of discourses as ways of thinking, sets of ideas, goals, values, and techniques, "competing ways of giving meaning to the world and of organising social institutions and processes" (Weedon, 1987, p.35). Thus we may have discourses on work, on raising children, on autonomy, on gender, and so on. As all our activities are based in discourses, and are regulated by the social relations of power associated with them, we sometimes speak of *discursive practices*.

Basing our analysis on discourses allows us to develop a much fuller idea of the context of mathematical thinking or affect, to include not only the wording of problems (as in the quantitative approach above) – though *language* is crucial – nor merely the physical setting, but also the *goals* of the activity (Newman, Griffin, & Cole, 1989), the relations of *power* exercised (as in the work of Foucault (1977; Weedon, 1987), and the material and institutional *resources* (e.g. training, equipment, professional associations) made available for the activity.

In the approach developed here, the context is understood as *positioning in practice(s)*. Each practice produces positions subjects take up; in some contexts / practices, the availability of a particular position may depend on the subject's gender or age (cf. Hollway, 1989). For example, in the activity of feeding children, the child is positioned as demanding ("more") and the parent as having to regulate the child's consumption; the form of this regulation is likely to depend one's income and social class. For a discussion of how positioning in such practices might provide the context for children's thinking in primary school mathematics, see Walkerdine and Girls and Mathematics Unit (1989, pp. 52-53).

In my project, I wanted to avoid the tendencies towards excessive determinism that I found in the idea of subjects' *being positioned by* practices. Thus, I argue that, in a given setting, subjects in general are *positioned by* the practices at play in the setting. However, a particular subject will *call up* a particular practice (or mix of practices) that may differ from that called up by other subjects and that, through the associated positions, will provide the context for that subject's thinking and affect in that setting. Thus the subject's positioning depends both on the practice(s) in which s/he is positioned, and the practice(s) called up (see below, and Evans, 1993, Chs. 8 and 10).

Research Procedures – The Interview:

Semi-structured interviews were conducted in my office with a randomly designed sample (n=25) of students at the end of their first year. The interviews had "life history" and "problem solving" phases; in the latter, subjects were presented with a number of "practical" problems: for example, deciding how much (if at all) they would tip after a restaurant meal (Evans, 1993 –Appendix, Qu.4); deciding which bottle of tomato sauce was the "best buy", etc. (cf. Sewell, 1981). But this interview differed, in its use of *contexting questions:* when the student was first shown the "props" for the problem – for example, a facsimile of a restaurant menu in Qu.4 – before being asked anything "mathematical," s/he was asked, "Does this remind you of anything you currently do?" After discussing the question, s/he was asked, "Does this remind you of any earlier experiences?" Subjects' responses to these questions helped me judge the context of their thinking about the problem, of their emotional responses, etc.

My analysis of positioning in the interview setting was that subjects would tend to be *positioned* in two main practices: academic maths (AM), with positions: teacher / student; and research interviewing (RI), with positions researcher / respondent. To the extent that RI, rather than AM, predominated, I thought the subject would *call up* ways of thinking, emotions, etc. from further non-academic practices, with associated numerate aspects I called "practical maths" (PM). For Qu.4, I expected these to be practices of "eating out," with several configurations of related positions possible: host / guest; friends each paying their own share; or perhaps customer / waiter. Each positioning in practice will support different ways of thinking and feeling, including different kinds of numerate or mathematical thinking.

In order to judge which practice the subject called up, I drew on various indicators:

1. The "script": for example, how the interview, problems, etc. were introduced – as "research," "views," "numbers," rather than "test," "maths,"etc.
2. Unscripted aspects of the researcher's performance: for example, any tendency to respond differently to "right" and "wrong" answers.
3. The subject's talk: for example, "graphs" and "gradients" would indicate academic maths and "charts" and "trends" would suggest business maths; and especially the responses to the contexting questions.
4. Reflexive accounts: for example, whether I had been in the position of maths teacher to each student before the interview. (Walkerdine, 1988, Ch.3; Evans, 1993)

Two "qualitative" approaches were used. First, a *cross-subject* approach, based on Miles and Huberman (1984), aimed to consider, in a comparative way, the results from all of the interviews (see next section). Second, each of the interviews was considered as a *case study*, so as to study features of the particular subject's thinking and affect (see section following the next).

Results from the Qualitative Cross-sectional Research

Let us consider the results for all interviewees for problem 4, which began with the presentation of a facsimile of a restaurant menu. For this analysis, I aimed to record the practice that *predominated* in each subject's positioning, if possible. Indicators for a predominant academic maths (AM) positioning were considered to be:

- the use of written calculations; or
- the giving of an answer which involved a fraction of 1p.

Indicators for a predominant positioning in "practical maths" (PM) were:

- the use of mental calculations; or
- the formulating of an answer in practical, i.e. money, units.

The results from the cross-subject analysis of the relationship between gender, predominant positioning and performance judged as correct for problem 4 (a 10% tip when eating out) are given in Table 2. We must first stress the small numbers involved – 23 students in all. (Two students did not reach Qu.4, because life history material intervened.) Also, the overall level of performance, among both women and men, was rather good. The gender differences were *very* small (especially considering the small numbers). However, there are two findings that– if they can be replicated – would be interesting. First, a slightly lower level of performance among those judged to have a predominant positioning in academic maths than among those positioned in practical maths / eating out (60% – 3 out of 5 – compared with 89%). Second, almost half of the women – but no men – called up academic maths. Taken together, these very tentative findings pose the question as to whethersome of the gender differences in performance could be explained by differences in *positioning*, and whether this might be true in other settings (e.g., in large-scale testing).

Table 2 Gender, Positioning and Performance: Cross-tabulation of number of questions judged correct by gender and predominant positioning for problem 4

	Number (and Percentage) Correct		
Predominant Positioning	Women	Men	Total
Practical Maths	6 / 7	10/11	16/18
(eating out)	(84 %)	(91 %)	(89 %)
Academic Maths	3 / 5	---	3 / 5
	(60 %)		(60 %)
Total	9 /12	10/11	19/23
	(75 %)	(91 %)	(83 %)

Results from the Qualitative Case Study Research

One case study will show the potential of this approach. "Ellen" (not her real name) was aged 19 at entry, and middle class (by parent's occupation), with an A-level in mathematics (a specialised 18+ exam and rarely taken or passed by this sample). A student of Town Planning, she was also working part-time as an electronics assembler, and had worked previously in a shop.

In the interview, Ellen expresses overwhelming confidence after almost every question – except for problem 4! When I ask what a 15% service charge would be for her "chosen" meal (seafood platter, costing £3.53), she says "Well, I'd have to use pencil and paper." Then -

S: [7 sec. gap / inaudible / coughs / 6 sec.] Well, 23 1/2 pou- no, that's wrong [12 sec.] what I've done wrong, oh (JE: Is it wrong?) Yeah, umm [laughs nervously] I don't know what I'm doing.... [She realises she has *divided* 15% into £3.53, and begins to redo the calculation by multiplying the two numbers.]

JE: You might not be used to doing these questions in such a setting as this, so (S: Yeah) take ... there's lots of time, don't worry.

S: [15 sec.] 52.95, 53 pence.
(interview transcript, p. 7 - Appendix in Evans, 1993)

But she recovers from her slip and explains her rejection of the answer produced by dividing: "I just saw that it was obviously not right – it was far too small."

Thus, my **First Reading** of the episode: The "slip" may be just that, an aberration, a response perhaps to the relative difficulty of calculating 15%, as compared with the first three questions. For the first time in the interview she momentarily expresses confusion – surprising for someone who has A-level in maths. A reasonable conjecture is that she is feeling some anxiety, and it appears to be "maths anxiety" (or perhaps "interview anxiety"). This is the sort of analysis that a "clinical interview" would produce. Let us consider this episode more closely.

What is the context of her performance and of her apparent anxiety? In response to the first contexting question, whether she ever went to a restaurant with a menu like that shown, she replied very quietly and hesitantly – and this was *before* any numerical problem was posed. After she "chose" the seafood platter [£3.53], I had asked how much she would tip for a restaurant meal. She had replied, again somewhat hesitantly, " Well, 15%, I suppose...."

After she has performed the calculation with the result described above, I ask

JE: Does this sort of situation here remind you of anything, any earlier experiences, a restaurant, a meal, thinking about service, and so on ...?

S: No, I don't usually pay – I mean, I usually look at prices and things, add them up in my head....

JE: Even if you're not paying?

S: I don't want to be an expense.
(interview transcript, p. 8)

We can offer a second interpretation based on the ideas to do with positioning in practices. At the beginning of this episode, she seems to have called up a practice (or cluster of practices), which might be called "eating out at restaurants". She chooses a dish and produces a rule for tipping. However, it is not clear how familiar she is with positions that require tipping, as she reverts to using pencil and paper – which is likely to indicate academic maths (AM) – to calculate a 15% tip. Thus, I would argue that her positioning is "inter- discursive," that is, in a mix of more than one discourse, because she calls up "eating out" as well as being positioned as student in AM, and as interviewee.

Using her responses to the contexting questions and other indicators, we can also understand her anxiety as specific to one, or more, of these practices. Further support for her positioning, and hence her anxiety, being related to eating out comes from her hesitation, etc. at the presentation of the restaurant menu; that is, the anxiety seems to be exhibited *before* the 15% calculation has been mentioned, and to interfere with her doing it. Her final comment in this episode is also indicative of anxiety related to eating out: "I don't want to be an expense."

Now, calling the anxiety "mathematical," as in the first reading, would be accurate only if we were to assume that she was positioned *solely* in an academic mathematics discourse. It would not be appropriate if, as I argue, her positioning is based, at least to some extent, in eating out as well.

We can note the specificity of her positioning in gender, and other terms. She "doesn't usually pay" in restaurants. The position of not paying when you eat out is one that is (in the British culture) relatively more available to women, and/or to people younger than their hosts. Also, her specified rule of tipping 15% would have been very unusual for most people, especially students, in the mid-1980s in London; therefore, the restaurants where she ate seem likely to have been for middle-class or financially well-off customers.

She mentions that she doesn't want to be "an expense." The term "expense" signifies in different ways – as an amount that could be arithmetically calculated within a mathematical or related discourse, *and* as being a burden within a relationship with other(s) more wealthy or powerful, and/or on whom the subject is dependent (a parent or partner). This "key signifier" thus functions at the intersection of these two discourses. Its negative connotations in the latter context also suggest anxiety – anxiety associated with the activity of eating out, with the related social relationship(s), and with the operations involved – for example, choosing a dish, calculating the total cost of her meal.

Thus, my **Second Reading**: It is reasonable to suppose that Ellen's positioning is interdiscursive – in eating out, in academic maths, and in being interviewed. Her "performance" – her thinking, methods used, possibilities for critical reflection – needs to be understood in this context. The same holds for the anxiety she seems to be exhibiting.

Could there be other readings? There still seems to be a conflict or contradiction between the picture Ellen gives of overwhelming confidence about maths and the use of numbers, and the indications of anxiety in this (and other) episodes. Is she perhaps "protesting too much"? This question, which (with others) suggests a need for insights from psychoanalysis for the interpretation of my interviews, cannot be pursued here, but see Evans (1991, 1993) and Evans and Tsatsaroni (1994).

Conclusions

In this paper, I have argued the need, when considering the context of mathematical thinking and affect, to move beyond the mere wording of problems, what the French call *habillage* or dressing-up, to consider the aspects of language as discourse, the goals of the activity, power relations, the wider social and institutional setting. Thus, a subject's performance in problem-solving situations, her thinking, her "critical reflection" need to be understood in the context of her positioning in discursive practices; in Ellen's case, her interdiscursive positioning included being a "guest" in eating-out practices. Similarly, I argue that anxiety, which might on first reading seem "mathematical," may be considered instead to be specific to another context, to a positioning in other practices; in Ellen's case, this may have been related to "being an expense" as a guest.

In this research, both quantitative and (two types of) qualitative methodology have been used. Rather than polarising the discussion by asking which method is "best," we can note the relative strengths of each, and attempt to combine the different approaches in a way that is effective for the problem at hand. The *quantitative* approach is useful when we wish to make comparisons across subjects, or groups of subjects, and we aim for some degree of generality. We have seen the importance, and the power, of the controls used in the quantitative research. For example, this approach is useful in studying gender differences in participation or performance, and outcomes of policy interest more generally. The *qualitative case study* approach is useful when we wish to display and explore the richness, coherence (i.e., not being separated into variables), and process of development of a limited number of cases. Here we can display episodes of problem solving, in order to understand the process – for research purposes or to improve teaching and learning. The sort of semi-structured life-history / problem-solving interview with contexting questions used here allows the tracing of multiple signification (as with "expense"), and the making of judgments about the positioning of subjects. The *qualitative cross-subject* approach provides an intermediate approach, for cases where it may be challenging to produce comparability across subjects (as in judgments about "predominant positioning" – see above), but where some amount of generality in our findings is sought.

What suggestions does the analysis here produce for the classroom? The case study described here suggests that the attempt to put maths "in context" through careful wording of problems may be difficult to control (Adda, 1986; Tsatsaroni & Evans, 1994) because of interdiscursive positioning and the flows of meaning along particular chains of signification (Walkerdine, 1988). Case studies such as the one reported here can help make teachers and researchers more sensitive to the unpredictability of some of these flows of meaning.

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Paper presentations. Agnes Cordeau and Helga Jungwirth.

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ARE ASSESSMENT PROCEDURES IN MATHEMATICS GENDER-BIASED?

Sharleen D. Forbes

Ministry of Mäori Development, Wellington

Background

Society and employment have changed greatly over the past decade. In particular, computers now dominate many work activities and many more jobs require some mathematical or technical skill. However, although quantitative techniques now permeate almost all disciplines, the skills required of mathematics students have changed. There is an increasing need for students to be able to communicate about mathematics, undertake independent investigations, and write reports. But are these skills being evaluated, or valued, in our current assessment procedures? At its best, assessment can be a valuable diagnostic tool for teachers and students, but at its worst, it is a superficial device for separating students into those "with knowledge" and those "without knowledge."

Many of our most important life skills (such as talking, walking, cooking, or driving) are learned outside the formal schooling system. Most of these skills are not "tested," but, when they are, it is usually through certification that a certain standard has been reached (for example, attaining a Driver's Licence).

The function of schooling within education has been to impart knowledge deemed by the majority society as essential. In New Zealand, mathematics is one of the seven essential areas of learning promulgated by our Ministry of Education.

Assessment within schooling is used for a number of purposes, including:

- diagnosis of students' skills and/or understanding
- sorting or streaming of students
- monitoring of national standards
- monitoring of teachers'/schools' performance
- awarding of qualifications.

Numeracy and mathematics testing, even in early primary school, has been generally both timed and written. The results of mathematics assessment are either formally or informally used (and misused) for all these above purposes. This paper will focus on the assessment of mathematics at the national examination level. The primary purpose of national examinations is the awarding of qualifications to individual students, but these results are also used as a barrier to further progress in mathematics, and occasionally misused to evaluate the performance of

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both individual teachers and schools.

Only a limited amount of research has been done comparing diverse assessment methods in mathematics to determine those which may best suit women, ethnic minorities, or indigenous peoples. It also needs to be acknowledged that women themselves are not just one homogenous group of mathematics learners. In general, it cannot be appropriate to have assessment procedures in mathematics consistently designed by one minority group of the population (generally academic, middle-class men) applied to the whole population.

The international literature (de Lange, 1987; Burton, 1993) suggests that girls may perform better in mathematics when assessed on procedures other than timed, written tests. The purpose of this paper is to look specifically at two types of assessment (internal assessment and written examination) in common use in mathematics in New Zealand to determine whether there is any inherent gender bias within the procedures themselves.

In New Zealand, internal assessment in mathematics varies enormously. Some is based on the outcome of a single, in-depth, practical project; some on the amalgamation of a number of small project and/or test results. National examinations at the end of the 5th form (year 10) and the 7th form (year 12) in mathematics are generally end-of-year three-hour written examinations. One of the two mathematics papers available in the final year of secondary schooling (year 12) contains both internal assessment and final examination components in the final marks attained by students. This paper, Bursary Mathematics with Statistics, is generally aimed at students *not* intending to pursue mathematics in their tertiary education.

Introduction

The two components of the New Zealand Bursary Mathematics with Statistics paper are :

> 80% external assessment by a traditional end-of-year-hour written national examination
>
> 20% internal assessment, the format of which is left to individual schools but generally consists of one or more practical projects.
>
> *(New Zealand Qualifications Authority, 1993)*

In order to investigate whether girls perform better in internal assessment than in written examinations on the same subject material, raw and scaled examination marks (percentages – out of 100) and project scores (out of 20) were obtained for all candidates in the 1988 Mathematics with Statistics paper.

Overall Examination Performance

There were 10,152 candidates in the 1988 bursary paper, and full information was determined for 9,900 of these. The majority of these candidates (54%) attended state co-educational schools. The overall mean score for boys was 51.1 and for girls was 47.0. That is, there was a significant gender difference in mean performance (at the 5% level of significance). However, as shown in Table 1, the size of the gender difference in means depended on the school authority (state, integrated [mainly Catholic], or private) and the school type (single-sex or co-educational). A detailed gender analysis of the 1988 and several other years of Bursary Mathematics with Statistics papers is included in the research report of the women's collective, EIME - Equity in Mathematics Education (Forbes, Blithe, Clarke, & Robinson, 1990).

Table 1

School Authority			State		Inte-grated		Private	
School Type		Single Sex	Co-Ed Sex	Single Sex	Co-Ed		Single	Co-Ed
MALES	N	1598	3003	413	116	500	86	5716
	Mean	52.54	51.52	45.44	47.01	50.55	51.89	(59 %)
FEMALES	N	1008	2361	337	105	300	73	4184
	Mean	46.63	46.66	46.15	42.51	52.54	51.04	(42 %)
TOTAL		2606	5364	750	221	800	159	9900
		(26 %)	(54 %)	(54 %)	(2 %)	(8 %)	(2 %)	

It should be noted that the overall marks analysed to produce Table 1 are a combination of both the internal assessment and examination scores.

As different types of schools have different intakes of students, and also to enable a comparison of girls and boys in teaching situations that were as alike as possible, the project and examination marks for a sample of students in state co-educational schools only were analysed. Over 54% of the students taking this examination were in state co-educational schools.

Project Versus Examination Performance

I Analysis

As the internal assessment marks submitted by each school are *moderated* (scaled to have the same mean and standard deviation) by that school's performance in the examination, the *actual* marks obtained by each student could not be used. The analysis was, therefore, confined to measuring the *change* in each student's ranking in her/his school, between the internally assessed and examination components. As the two assessment procedures are testing different skills in mathematics, the practice of moderating the results of one procedure by performance in the other is itself questionable.

In order to account for differences in school size the difference in rankings was divided by the number of candidates in the school, giving values between -1 and +1. A zero indicated that the student's ranking in the school was the same for both the internal assessment and examination. A positive value indicated a higher ranking in the examination than the internal assessment.

Thus the variable analysed was

$$\text{CHANGE} = \frac{\text{EXAM RANK - INTERNAL RANK}}{\text{NUMBER IN SCHOOL}}$$

Two samples from the 244 state co-educational schools (73% of all schools) with candidates in the examination were selected for analysis. The first (Sample 1) consisted of every fourth school chosen from an alphabetical listing (that is, 61 [25%] state co-educational schools and 1,571 [30%] state co-educational students entered in the examination). The second (Sample 2) consisted of the first 25 schools in the alphabetical listing (that is, 25 [10%] state co-educational schools and 760 [14%] of state co-educational students) and was chosen to check whether there was any cyclic effect resulting from the way the first sample was selected.

In addition, to investigate whether any apparent gender differences were just the result of a few schools, or indeed a general trend, the changes in rankings were also investigated for all individual schools in either sample that contained 10 or more students.

II Results

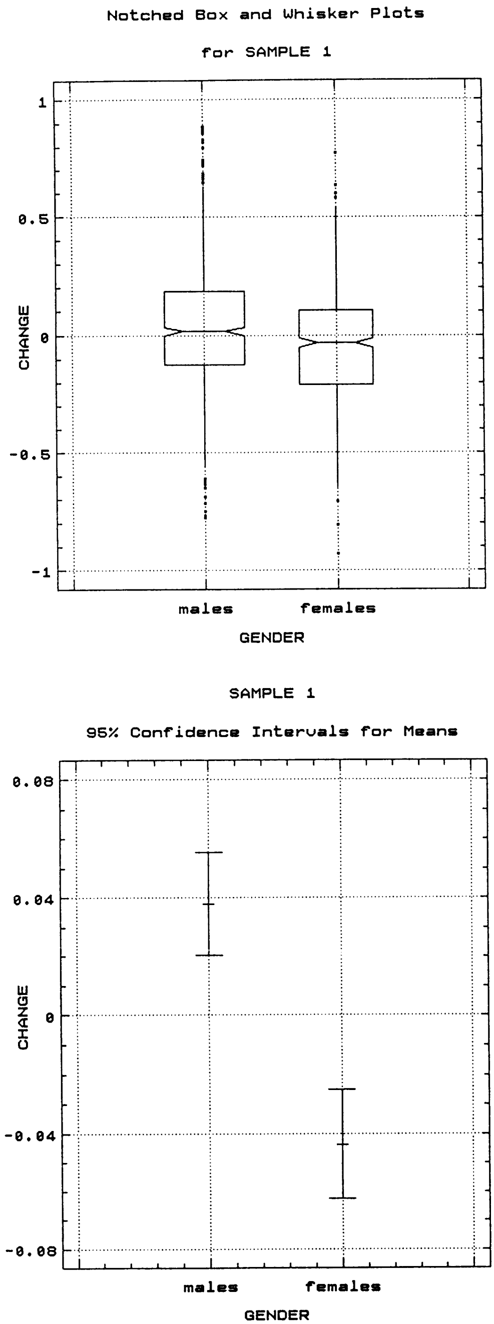
In both samples there were slightly more males than females, as Table 2 indicates:

Table 2

	MALES	FEMALES	**TOTAL**
SAMPLE 1:	842 (54%)	729 (46%)	1571
MEAN CHANGE	+0.04	-0.04	
SAMPLE 2:	396 (52%)	364 (48%)	760
MEAN CHANGE	+0.04	-0.04	

The following box plots and 95% confidence intervals (Figure 1) show there were significant gender differences (at the 5% level of significance) in the change in rankings between internal assessment and examinations cores. On average, girls have a higher ranking in their schools on the internal assessment component than on the examination, and boys have a higher ranking in their schools on the examination than the internal assessed component.

Gender differences within individual schools were also investigated for the 62 schools (25% of all state co-educational schools) in total from both samples that had 10 or more bursary candidates. There were eight (13% of all state co-educational schools) schools with significant differences (at the 5% level). In all of these the boys, on average, increased their ranking from the internal assessment to the examination, whereas the girls decreased theirs. Indeed, there were only 11 schools (18% of all state co-educational schools) where this pattern was reversed


Notched Box and Whisker Plots
for SAMPLE 1
1
0.5
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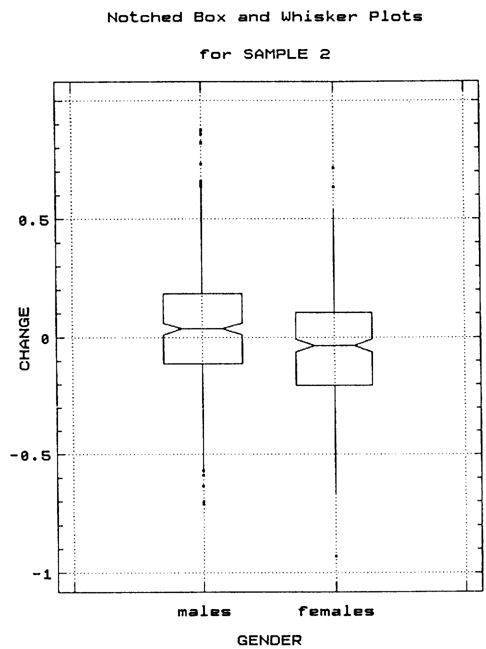
-0.5
-1
CHANGE
males
females
GENDER
SAMPLE 1
95% Confidence Intervals for Means
0.08
0.04
0
-0.04
-0.08
CHANGE
males
females
GENDER

Figure 1 a


Notched Box and Whisker Plots
for SAMPLE 2
0.5
0
-0.5
-1
CHANGE
males
females
GENDER

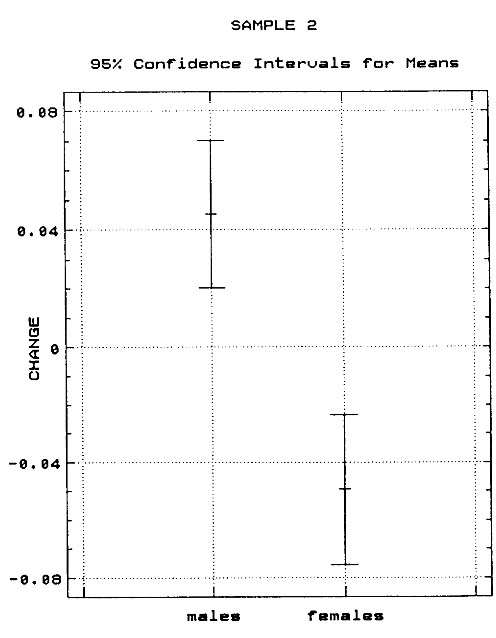

SAMPLE 2
95% Confidence Intervals for Means
0.08
0.04
0
-0.04
-0.08
CHANGE
males
females
GENDER

Figure 1 b

(that is, the girls increased on average and the boys decreased their rankings), and for none of these was the result statistically significant.

Conclusion

It appears that there is a small but consistent tendency for girls to perform better in the internally assessed component of the Mathematics with Statistics component course than in the examination and for the boys to perform better in the examination than in the internal assessment.

These results reinforce that different skills are indeed being tested by the different assessment procedures and indicate that the practice of moderation should be ceased.

Many practising statisticians would hold the view that it is exactly the research, problem solving, and report-writing skills tested in the internal assessment that are needed, not the ability to regurgitate or manipulate formulae in a set time.

At the very least, we need to question what we test, when we test, and how we test, and we need to adapt our testing procedures so that they fairly incorporate a number of procedures so that no one sector of students is unfairly disadvantaged. In general, it would be fairer and more informative to use a variety of methods to assess a student's attainment in mathematics, rather than rely on a single three-hour written performance to indicate the outcome of, in some cases, several years of study.

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Panel 1, Gender and Mathematics Education with moderator Mogens Niss and panelists Gila Hanna, Calle Jacobsson, Christine Keitel, Anna Kristjánsdóttir and Gilah Leder. (above)
Press conference with Gila Hanna, Mogens Niss and Miguel de Guzman. (below)

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GIRLS' ATTITUDES IN MIXED AND SINGLE-SEX MATHEMATICS CLASSROOMS

Helen J. Forgasz
Monash University, Clayton

Over recent decades, the trend in Australia, as elsewhere, has been towards co-education. Research indicates that co-education, per se, does not provide equity in mathematics education, and learning settings for mathematics are being re-examined. Linked to several models explaining gender differences in mathematics learning are a number of affective variables, including attitudes towards mathematics and about oneself as a learner of mathematics. The attitudes of students in Grade 7 who learnt mathematics in single-sex and mixed learning groupings were surveyed and the results are presented in this paper. The students attended comparably resourced private schools in metropolitan Melbourne, Australia. The results indicated that females' attitudes did not differ significantly across the learning settings. Significant gender differences were found on some affective variables for students from co-educational schools with mixed mathematics classes and from one which provided single-sex classes. Males' beliefs appeared to show greater variation across learning settings than did females'.

Introduction

Marked differences between countries in the ratio of males to females studying mathematics at the pre-university level were found in the first *International Study of Achievement in Mathematics* (Husén, 1967). School type, single-sex or co-educational, was considered to be one of a number of interacting and contributing factors. Countries also varied in the relative proportion of students attending single-sex and co-educational schools.

A complex of indigenous social, political, and economic factors have been found to influence the extent to which females and males are educated separately or together and the quality and resourcing of the education provided. The traditional rejection of co-education in Roman Catholic schools was said to account for the predominance of single-sex education in France, Belgium, and the Netherlands (Husén, 1967). Lee and Lockheed (1990) claimed that "in countries where education systems are still expanding, as in many developing countries, economic

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factors encourage the development of coeducation" (p. 211). Some developing countries provide single-sex education, the standard of which has been found to be lower for girls. In traditional societies that question the value of education for women, governments are often less willing to invest in education exclusively for females. For example, in Kenya where most government schools are single-sex and in Saudi Arabia where "coeducation is culturally unacceptable" (Lee & Lockheed, 1990, p. 210), females' schooling is reported as inferior. The socio-economic background of students can operate as another interacting variable in some societies. In Thailand, where Western-style education dates back to the late nineteenth century, education for females is widespread, co-education is acceptable where there are no alternatives, yet for some ethnic groups and the middle classes "single-sex education is apparently preferred for girls, particularly at the secondary level" (Jimenez & Lockheed, 1989, p. 119).

For meaningful comparisons of the educational outcomes of students from co-educational and single-sex schools to be made across international boundaries, local societal factors must be considered. Not accounting for social and economic factors within the one country may also render comparisons doubtful. In the Australian context this point was clearly put by Gill (1988, p. 3): "The distinction between single sex and co-educational schools is interwoven with the division between private and public schooling which has been the site of a long and at times particularly acrimonious debate in the history of Australian education."

The Australian Context

With colonisation, British traditions in education were transported to Australia. Church-based fee-paying schools in the British grammar school tradition were established relatively early for boys, and later for girls (Forgasz, 1994). Although the vast majority of single-sex schools are still private (Catholic or Independent), some private schools are co-educational. Historically, the government took much responsibility for the education of the working classes (Leder, 1992) and today, with only a few selective single-sex, post-primary schools, the vast majority of public schools are co-educational.

School Type and Gender Equity in Mathematics Education

Leder (1992) argued that:

> In recent decades, gender-segregated education has come to be viewed as an anachronism, reflecting outmoded beliefs that males and females have different educational needs. Co-education was assumed to be the avenue through which parity of treatment could be achieved. (p.610)

Particularly with respect to mathematics education, calls have been made to re-examine the equity assumptions inherent in the arguments favouring co-education (Leder, 1992). While inequities have also been found in single-sex schools in Australia (see Shorten, 1990), co-education has not resulted in equity for students in mathematics education. Findings on the relative benefits of single-sex or co-educational schools are inconclusive and the issue remains unresolved (Gill, 1988). With respect to mathematics education, the issue of optimal learning settings for girls is also unclear.

Research On Single-Sex And Mixed Settings In Australia

Participation and Performance

Enrolment statistics in Australia reveal that more females than males remain to complete secondary education, but twice as many males as females enrol in the most demanding mathematics courses (Leder & Forgasz, 1992). A paucity of data exists on subject enrolment and achievement levels across educational sectors (Government, Catholic, and Independent) in Australia. Acknowledging a complexity of intervening variables, Teese (1989) reported that in each educational sector at the Grade 12 level in Victoria, female participation in the physical sciences and mathematics was lower than for males. In New South Wales, Jones (1990) found that enrolment patterns in mathematics were affected by both gender and school type (Government vs Catholic) and that school setting (single-sex vs co-educational) was a factor influencing participation and performance in a range of school subjects (including mathematics).

The Gender Composition Of Mathematics Classrooms And Affect

Considering the setting for this study, only Australian research on affective dimensions and the gender composition of mathematics classrooms is discussed.

During the four-year transition when two single-sex government high schools in Sydney merged to form one co-educational school, differences in mathematics achievement favouring males were unaffected, but multidimensional self-concept measures increased for both males and females (Marsh, Smith, Marsh, & Owens, 1988). In Foon's (1988) study of Grade 10 students from 16 independent schools in Melbourne, males rated their mathematics achievement higher than did females, and females in single-sex schools rated their mathematics achievement higher than their co-educational counterparts. Females' self-esteem was lower than was males', but there was no difference in the self-esteem of females in co-educational and single-sex schools.

Student/teacher interaction patterns for females and males in mixed mathematics classes were investigated by Leder (1990). More time and a greater number of high cognitive questions in particular were directed to boys. Gender differences in attitudes towards mathematics and in beliefs about themselves as learners of mathematics were also found.

Government policy in Victoria has been in favour of co-education. The anticipated benefits for females from learning in single-sex settings have prompted some schools to set up segregated mathematics classes. In one school where students were randomly allocated to single-sex or mixed classes, Rowe (1988) found significantly higher gains in confidence for students in single-sex classes than in mixed classes with the most notable gains for girls. Rowe (1988) cautioned that the longer term effectiveness of single-sex groupings has yet to be established.

Gender differences in mathematics achievement are not usually apparent at the Grade 7 level but differences in beliefs about themselves as learners of mathematics have been found (Leder & Forgasz, 1991). This age cohort seems a particularly crucial one. In the study reported in this paper, the attitudes and beliefs of Grade 7 students from single-sex and mixed mathematics classroom settings, across school types and within the same school, were investigated.

The Study

The results reported here are from a larger study in which the relationship between affect and classroom factors is under investigation. A subset of the data from private co-educational and single-sex girls' schools was extracted. This enabled comparisons to be made between girls in the two types of whole school setting,while plausibly controlling for socio-economic background. Additionally one co-educational Catholic school was found that conducted single-sex mathematics classes taught by the same teacher.

Method

Grade 7 students completed a questionnaire that assessed their beliefs about themselves as learners of mathematics. The beliefs targeted were those common to several models explaining gender differences in mathematics participation and high cognitive level achievement (see Leder, 1992). The questionnaires were administered mid-way through the academic year.

Sample

Data were gathered from nine independent co-educational schools and from seven independent girls' schools. Students from only one class in each school completed the questionnaires. The total number of students in the co-educational schools was 215 (M = 110, F = 105). The number of females in the single-sex schools was 148. In the co-educational Catholic school (School X), there were 26 students in the girls' class and 24 in the boys' class.

The instrument

Slightly modified and reduced (6 items per subscale) versions of the Teacher, Confidence, Perceived Usefulness of Mathematics, and Mathematics as a Male Domain subscales of the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) were used. A 6-item subscale was developed (Cronbach α = .71) and used to assess persistence at mathematical tasks. The Mathematics Attribution Scales (Fennema, Wolleat, & Pedro, 1979), slightly modified for Australian students, were included to assess the students' attributions for success and failure in mathematics to ability, effort, task, and environmental factors. Sample questionnaire items are shown in the Appendix.

Results and Discussion

One-way multivariate analyses of variance (MANOVAs) were conducted to investigate whether the mean scores on the combination of affective measures differed across groups of students. The groups compared were: males and females in School X; males and females in the co-educational schools; and the females in the co-educational and single-sex schools. In MANOVA, $SPSS_x$ (Nie et al., 1975) uses listwise deletion of missing data. This resulted in reduced sample sizes but ensured that the results of the same female co-educational students were used in the two analyses in which their results were involved. The results of all associated one-way analyses of variance (ANOVAs) comparing mean scores on each of the affective variables for the students in the various learning settings were also examined.

Results of Multivariate Analyses

School X:

N = 42 (M = 23, F = 19)

Wilks $\lambda = .47$, $F = 2.47$, hypoth df = 13, $p<.05$, $\omega^2 = .53$

Co-educational schools: N = 193 (M = 98, F = 95)

Wilks $\lambda = .75$, $F = 4.49$, hypoth df = 13, $p<.001$, $\omega^2 = .25$

Single-sex and Co-educational females: N (S/sex) = 135, N (Co-ed) = 95

Wilks $\lambda = .92$, $F = 1.54$, hypoth df = 13, ns, $\omega^2 = .08$

The multivariate results indicated that for the optimal linear combination of affective variables:

- Student gender contributed significantly to the differences in mean scores forstudents in School X and in the co-educational schools. In the former, student gender accounted for 53% of the variance in the optimal linear combination of affective variables, and in the latter student gender accountefor 25% of the variance.
- No significant difference was found for girls in single-sex and co-educational schools.

These results implied that for these grade 7 students:

- There were gender differences in beliefs for students in co-educational schools irrespective of whether the classroom setting was mixed or single-sex.
- A co-educational or single-sex whole school setting was not a factor related to females' beliefs.

Table 1 shows the mean scores and significant differences by gender for all associated univariate ANOVAs.

Table 1

Mean scores on each subscale for related univariate ANOVAs conducted: by student gender for School X and the co-educational schools; by school type for females in single-sex and co-educational schools.

	Scool X		**Co-educational**		**S/sex**
Subscale	Males	Females	Males	Females	Females
Maths as a Male Domain	24.1 ##a	27.5	25.5+++b	27.7	27.6
Teacher	22.1	22.2	21.6	22.2	22.5
Persistence	24.7	22.7	22.8	23.6	23.2
Confidence	25.3#	22.2	24.2	23.2	23.1
Usefulness	25.9	25.3	25.9	25.5	25.4
Success/Ability	14.7	14.3	15.1++	13.9	13.8
Success/Effort	13.9	14.3	14.0++	15.2	14.2*c
Success/Task	15.3	13.7	14.5	14.3	14.1
Success/Environment	15.3	14.1	14.3	14.8	14.8
Failure/Ability	10.4	11.5	11.3	11.6	11.7
Failure/Effort	11.6	11.1	11.9	11.3	12.5**
Failure/Task	12.6	12.6	12.4++	13.6	13.4
Failure/Environment	11.5	12.0	11.3	11.2	11.2

a significance levels for differences in scores for males and females in School X:### p<.001, ## p<.01, # p<.05

b significance levels for differences in scores for males and females in co-educational schools: +++ p<.001, ++ p<.01, + p<.05

c significance levels for differences in scores for females in co-educational and single-sex schools: ***p<.001, ** p<.01, * p<.05

The data in Table 1 reveal that in School X males were more stereotyped about mathematics (p<.001) and more confident about themselves as learners of mathematics (p<.05) than the females. With the small sample in School X, the discussion of these findings is necessarily tentative. These students shared the same teacher who, presumably, taught the same curriculum. The teacher believed that students in the two classes were treated similarly. It is noteworthy that, although not statistically significant, the direction of the differences in the mean scores for males and females for usefulness, and for success and failure attributions to ability and effort correspond to those frequently found in the literature (see Leder, 1992).

One possible explanation for the results is that, despite having the same teacher, students are not sharing similar learning experiences. The timing of the data collection may also have been too early in the year for any anticipated benefits of single-sex mathematics settings to have been fully realised. It is of interest to note that the males in School X appeared more stereotyped about mathematics than the males in the co-educational settings.

In the co-educational schools males were more stereotyped about mathematics ($p<.001$) than females and attributed success to ability ($p<.01$) to a greater extent (Table 1). Females attributed success to effort ($p<.01$) and failure to task ($p<.01$) more than did males. With the exception of females' attribution of failure to task, the gender differences found are among those consistently reported in the literature (see Leder, 1992).

As the MANOVA results showed no significant difference for females in single-sex and co-educational settings, the significant differences between the two female groups on the associated univariate ANOVAs are noted as worthy of further investigation: co-educated females scored higher on attributing success to effort, and single-sex females attributed failure to lack of effort to a greater extent. Perhaps classroom experiences or expectations for success differ in the two settings. In single-sex schools the additional attention the girls might receive may induce a sense of security coupled with higher expectationsfor success, which might account for their higher mean score for attributing failure to lack of effort. In co-educational schools, the girls may perceive their expectations for success to be lower and thus feel the need to work harder in order to succeed. Note that, of all the groups, male or female, co-educated females scored highest on attributing success to effort and single-sex females highest on attributing failure to lack of effort. The two groups of males scored similarly on both variables.

Although not tested statistically, general trends were noted that replicate reported findings on gender differences on affective variables related to mathematics learning (see Leder, 1992). When the mean scores of both groups of males were compared with those of the three groups of females, both male groups were more stereotyped about mathematics, more confident about themselves as learners of mathematics, perceived mathematics as more useful, and attributed success to ability to a greater extent. The three female groups scored higher on attributing success to effort and failure to lack of ability than did the two groups of males.

The report, *Listening to Girls* (Milligan, Thomson, & Ashendon & Associates, 1992), lends support to some of the interpretations reported above. Some girls in co-educational schools who had experienced single-sex classes were less concerned about lost social opportunities but were "wary that single-sex classes give a false sanctuary, affording temporary protection from the dominance or competitiveness of males" (p. 22). Some teachers also believed that single-sex settings per se were no guarantee against the development of dysfunctional beliefs: "Girls and boys can learn just as easily in single-sex classes as in co-educational classes that boys are smarter, stronger, more valued by teachers, and have much greater opportunities in work" (p. 22).

Final Words

The mean scores for the males in the single-sex class in School X and for the co-educated males seemed to show greater variation than those of the three groups of females. The similarity in scoring patterns for the female groups was noteworthy. Except for attributions of success and failure to effort, neither whole school nor classroom setting seemed related to females' scores on the affective variables investigated. On average this may be true. But the conclusion should not be extended to the individual school, individual classroom, individual teacher, or individual student. "Teacher perception and teaching style are likely to be crucial in an all-girls class *just as they are in a mixed class* in order to achieve the desired ends" (Gill, 1988, p. 12). Effective intervention strategies have also been found to impact on females' beliefs (Leder & Forgasz, 1992).

Although single-sex and co-educational school environments differ, important differences might also exist among schools in each group (Gill, 1988). Future research, Gill suggested, "should attempt to delineate intra-school differences beyond whether or not the school is single sex or coeducational and attempt to factor in such differences into an explanation of outcomes" (p.130). More also needs to be known about co-educational schools that offer single-sex mathematics classes.

The results of this study were inconclusive. For females, neither single-sex nor mixed mathematics settings seemed more effective in promoting those affective beliefs identified as central to explaining gender differences in mathematics learning. For males, however, the setting in which mathematics learning occurs might have had a bearing on their beliefs. To gain a better understanding of the underlying reasons for gender differences in students' affective responses, quantitative data such as those reported here need to be supplemented with qualitative data. For example, the more subtle classroom processes that may be contributing elements need to be explored.

Societies concerned with achieving gender equity in mathematics learning need to identify local factors and constraints that serve to disadvantage females. Informed by research from elsewhere, an appropriate mix of options may need to be considered to redress inequities. Leder (1984) wrote:

> Much more careful monitoring of students' learning of mathematics in different classroom environments is required before final organizational decisions are made. At this stage flexibility of and variety in class compositions seem indicated.... Providing the most appropriate environment(s) so that all students can maximize their potential is a goal worthy of pursuit. (p. 172)

The results of this study suggest that, at least in the Australian context, little has changed in a decade and Leder's (1984) insightful comments remain equally pertinent today.

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Appendix

Questionnaire Subscales

The Teacher, Confidence, Mathematics as a Male Domain, Perceived Usefulness of Mathematics, and Persistence subscales each consisted of 6 items; three positively worded and three negatively worded. Responses were indicated on a 5-point Likert-type scale ranging from Strongly Agree (SA) to Strongly Disagree (SD). Negatively worded items were reverse scored. The subscales assessing attributional style consisted of eight stem statements, four for success and four for failure. Each statement was accompanied by four alternative statements, one related to each of Ability, Effort, Task, and Environmental Factors.Students were required to respond to each of the four statements on the same Likert-type scale as for the other subscales.

Sample Items From Questionnaires Administered to Students

My teacher encourages me in maths (Teacher, positively worded)

I'm no good at maths (Confidence, negatively worded)

Boys are naturally better at maths than girls (Mathematics as a male domain, negatively worded)

Maths is one of the most worthwhile and necessary subjects to study at school (Perceived Usefulness of mathematics, positively worded)

When I make a mistake in maths I try to work out where I went wrong before asking for help (Persistence, positively worded)

Event C: You had trouble with some of the problems in your maths homework (Failure event)

a. There was no time to get help from the teacher (Environment)
b. You don't think in the logical way that maths requires (Ability)
c. You didn't take enough time to do each problem properly (Effort)
d. The problems were very difficult (Task)

Elizabeth Fennema *Photo: Lennart Jonson ©*

WOMEN IN HIGHER EDUCATION IN AUSTRALIA: CHANGING PATTERNS OF PARTICIPATION

Janice M. Gaffney
The University of Adelaide

This paper presents data on the participation of women in higher education in Australia. From these statistics it is apparent that, although women are choosing mathematics, the physical sciences, and engineering in ever-increasing numbers, the improvements in the proportional participation rates in these disciplines have not matched the improvements in the proportional participation rates for law, medicine, or economics.

The Australian government has set targets for 1995 of an increase in enrolments of women in higher education to 15% in engineering and to 40% in non-traditional courses other than engineering. The role government can play in promoting change should not be underestimated.

Introduction

A recent discussion paper on the objectives, targets, and strategies for achieving equity in higher education in Australia by the Department of Employment, Education and Training and the Training and National Board of Employment, Education and Training (1990) gave the following targets for the participation of women in higher education in Australia (pp. 2-3).

- An increase in proportion of women in non-traditional courses other than engineering from their current level to at least 40% by 1995;
- An increase in the proportion of women in engineering courses from 7% to 15% by 1995
- An increase in the number of women in postgraduate study, particularly in research, relative to the proportion of female undergraduates in each field by 1995.

Do the statistics, however, suggest that these targets will be easily achievable?

Australia's Higher Education Sector

The higher education sector in Australia includes all institutions offering courses

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that lead to at least a first degree. This particular grouping of institutions is equivalent to the system of universities and four-year colleges in North America and to the system of universities and polytechnics in Britain. The sector was created in the late 1980s to replace a binary system of universities and colleges of advanced education established in 1965. Major structural change, including the amalgamation of institutions, occurred as a result of the formation of this sector and almost all institutions were affected.

At the present time there are 39 publicly funded institutions in this sector and 2 privately funded. As the Commonwealth Government is the major source of funding for the publicly funded institutions (over 70%), it has significant power to influence the operation of institutions within this sector.

Further details on the basic features of the sector can be found in *National Report on Australia's Higher Education Sector* by the Department of Employment, Education and Training (1993).

Participation Rates 1930 - 1960

Australia-wide data are not readily available in an appropriate form. However,

Table 1: Enrolments - Percentage of Females Queensland University 1930 - 1992 (By faculty)

	PERCENTAGE OF FEMALES																
Faculty	30	40	50	60	70	75	80	82	83	84	86	87	88	89	90	91	92
Agricultural Science	3.6	9.1	1.8	5.2	11.6	16.0	21.7	26.3	27.9	27.5	31.0	28.5	25.8	26.8	28.0	29.5	34.9
Applied Science															28.0	30.9	32.6
Architecture & Planning		12.5	3.4	2.5	6.5	18.7	26.1	32.3	32.5	33.2	26.2	30.0	32.5	33.3	35.4	38.6	40.0
Arts	35.4	29.4	30.2	42.0	57.9	57.8	60.8	61.9	62.8	63.2	63.7	64.9	66.3	67.5	67.0	68.5	67.4
Business															41.3	42.1	44.5
Commerce & Economics	4.0	10.9	12.2	7.5	10.6	17.1	22.2	27.5	29.7	30.4	31.2	33.9	35.3	36.7	37.2	36.7	37.5
Dentistry		2.8	3.8	8.8	5.0	19.4	20.1	24.7	28.2	32.0	35.1	31.9	32.6	32.2	30.7	33.2	35.1
Education		20.0	16.2	28.8	37.2	48.1	60.0	59.8	61.3	60.7	62.3	62.5	63.7	61.5	63.3	65.5	70.0
Engineering			0.6	0.2	1.1	2.7	4.1	5.5	6.0	6.7	8.1	8.6	10.7	11.4	13.2	15.3	15.4
Law		9.7	0.0	6.6	12.1	20.4	33.2	37.2	38.3	40.5	42.4	44.6	44.8	46.2	44.8	47.6	49.2
Medicine		24.0	18.7	22.6	39.4	52.6	53.7	52.2	52.2	51.3	53.8	54.0	54.8	55.7	56.4	58.1	58.9
Music					72.7	72.5	66.7	79.5	71.6	70.9	66.0	69.6	70.3	66.9	69.8	72.2	68.6
Science	32.6	23.1	18.9	23.6	26.5	32.2	34.0	38.4	38.4	38.1	37.1	38.9	40.9	40.7	41.5	42.2	42.6
Social Work						72.6	76.2	79.8	81.6	78.8	82.3	81.9	81.2	80.8	82.0	82.9	82.5
Veterinary Science		3.6	4.8	5.5	12.3	24.4	37.2	43.0	44.2	46.8	52.5	49.7	50.4	47.9	49.3	48.3	51.0
Ph.D.				22.8	32.9	40.5	45.5	46.7	47.2	47.4	48.4	49.0	49.7	49.6	48.3	49.1	49.5
Total	29.8	26.7	16.7	22.8	32.9	40.5	45.5	46.7	47.2	47.4	48.4	49.0	49.7	49.6	48.3	49.1	49.5

Source: Statistics 1992, The University of Queensland, p147.

data from the University of Queensland (Table 1) suggest that the patterns of female participation in the non-traditional areas of science and the professions were relatively static throughout this period. It would appear from these data that, although the structure of universities changed significantly in this period, particularly following World War II, these changes did not have much impact on the participation rates of females within the various disciplines.

Although a complete data set for all Australian institutions does not as yet exist, a perusal of data from other Australian institutions suggests that the qualitative features of the Queensland data are reasonably typical. For example, the report on graduation ceremonies in the calendars of the University of Adelaide shows that in 1930 there were no female graduates in engineering in 1930 and in 1955 the situation had not changed!

Participation Rates Since 1960

A picture of the pattern of participation of women in the various disciplines at the University of Adelaide is provided in Table 2 and Figure 1. It can be seen from Table 2 that total enrolments expanded greatly between 1960 and 1969, reflecting the significant expansion in the student population at Australian universities in that period, and also in the late 1980s, reflecting mainly the amalgamation of institutions.

The female share of enrolments for the disciplines under discussion in this study is shown in Figure 1. In medicine, the female share was 17.6% in 1960, but by 1992 it had grown to 43.4%. Law shows a similar change, with the female share of enrolment increasing to 52.2% from 11.5% in 1960. These data compare with the figures for science, in which the female share was 19.3% in 1960 and 48.4% in 1992. Some care, however, is needed in interpreting the data for science because mathematics and computer science have not been included in the science figures since 1973, when a separate faculty of mathematics and computer science was created.

The figures for the participation of females in engineering for this period also show considerable change. Although women remain significantly under-represented, the female share of enrolment rose from 0.2% in 1960 to 16.0% in 1992. A more rapid change, however, has occurred in economics and commerce. Women increased their share of enrolments to 37.2% in 1992 from just 4.4% in 1960.

In summary, the enrolment patterns for women show that change has occurred at a rate that, for the physical sciences, including mathematics and computer science, has been significantly slower that in the professions and in economics and business.

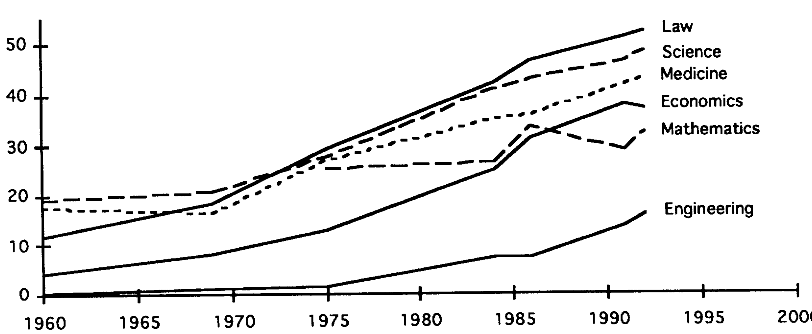
Some statistics for the participation of females in higher education in Australia are given in Tables 3 and 4.

Table 2: Enrolments in Bachelor Degrees - Percentage of Females Adelaide University 1960 - 1992 (By faculty)

Year	**60**	**69**	**75**	**84**	**86**	**91**	**92**
Ag.&Nat. Res. Sciences	4.5	9.5	31.0	35.4	33.7	31.4	32.6
Architecture & Planning	8.9	9.6	12.3	42.4	32.2	38.7	39.5
Arts(Ed.)	45.3	44.1	55.9	64.0	64.6	66.6	66.4
Dentistry	13.9	10.0	16.4	31.3	41.9	40.9	41.1
Economics & Commerce	4.4	8.1	12.7	24.8	31.4	38.0	37.2
Engineering	0.2	1.1	1.6	7.5	7.5	13.5	16.0
Law	11.5	18.2	29.3	42.0	46.4	51.2	52.2
Math.&Comp.Science	-	-	25.4	26.4	33.4	29.0	32.5
Medicine	17.6	16.5	26.9	35.2	35.9	41.7	43.4
Performing Arts(Music)	28.6	57.6	60.6	57.1	61.1	63.2	60.4
Science	19.3	20.5	27.7	41.1	43.0	46.3	48.4
Total %	19.2	25.5	32.6	40.4	43.7	46.0	46.5
Total students	4270	7113	7300	7162	7035	9816	10064

Source: Statistics publications of the University of Adelaide, 1969-1992.
Calendar of the University of Adelaide, 1961, p. 421.

Figure 1 Enrolments in Bachelor Degrees - Percentage of Females Adelaide University 1960 - 1992 (Various Faculties)



The enrolment patterns have the same qualitative features as those observed in the University of Adelaide data. It is interesting to note that, although numbers and proportional participation rates have increased in Arts and Education, as a proportion of female students in the different faculties, the fall has been quite dramatic.

Table 3: **Enrolments in Higher Education - Percentage of Females Australian Universities 1979, 1985 and 1992 (By faculty)**

Year	**1979**	**1985**	**1992(1)**
Agriculture/Forestry	22	30	34
Architecture/Building	15	23	34
Arts/Humanities/Social Sciences	61	65	68
Dentistry	21	32	38
Economics/Commerce/Business	22	32	43
Education	66	66	73
Engineering/Technology	3	5	12
Health Sciences	82	79	85
Law	31	42	47
Medicine	38	42	45
Science/Applied science	32	36	40
Veterinary Science	36	49	54
Visual and Performing Arts	59	63	
Total	**45**	**48**	**53**

(1) A new field of study classification scheme was introduced in 1987 and data may not be entirely comparable.

Source: National Report on Australia's Higher Education Sector, p. 20
DEET (Unpublished data).

The Dynamics of Change

Before World War II, the Australian university system largely served a social elite. The situation that exists today is vastly different, with about 30% of the population aged 17–22 years undertaking university study.

The statistics presented in this study, however, show that despite the massive increases in female enrolments and major structural changes in the organisation of the higher education system, females remain significantly under-represented in mathematics, the physical sciences, and engineering. Furthermore, any explanation as to why that should be so has to account for the contrasting dramatic changes in the share of female enrolments in medicine, economics, and business.

It is conceivable that, as the social barriers that exist towards the involvement of women in the professions are being dismantled, girls with ability in mathematics and science and some expectations towards higher education are now actively being encouraged to choose medicine, economics, business, and, indeed, engineering. At present, there are likely to be biology teachers who, in the context of society as it exists today, might instead have chosen to be doctors, science teachers

Table 4: Female Enrolments in Higher Education Australia 1979 and 1990 (By faculty)

Year	**1979 Nos**	**1990 Nos**	**1979 %**	**1990 %**
Agriculture	1028	2825	0.7	1.1
Architecture	1057	3586	0.7	1.4
Arts	49378	74480	34.8	29.1
Business	10631	42643	7.5	16.7
Education	52283	54143	36.9	21.2
Engineering	523	3632	0.4	1.4
Health	9025	39367	6.4	15.4
Law	2865	6385	2.0	2.5
Science	13784	26122	9.7	10.2
Vet Science	527	806	0.4	0.3
Non-award	646	1666	0.5	0.7
Total	**141747**	**255655**	**100**	**100**

Source: National Report on Australia's Higher Education Sector, p. 213.

who might have wanted careers as engineers, mathematics teachers who might have pursued a career in banking.

The dynamics of change is complex and in the short term we may see perverse effects.

The Role of Government

The Government in Australia has become directly involved in the issue of the under-participation of girls and women in science, engineering, and technology.

Senator Chris Schacht recently announced[1] the appointment of Professor Ann Henderson-Sellers as Chair of the Women in Science, Engineering and Technology (WISED) Advisory Group. This advisory group has been set up to report on strategies to increase the participation of women in these areas, with the following terms of reference:

> The Advisory Groups should address issues relating to:
> - improving the participation of women in senior science, engineering and technology positions in both the public and private sectors;
> - improving the level of participation and retention by women in science, engineering and technology education and training at both the vocational and academic level; and
> - improving awareness of the contribution women can make to science, engineering and technology in Australia.
>
> The Advisory Group should take account of current Government and other organisations' policies and programs, and draw heavily upon its own experience and expertise in reporting to the Minister assisting its views on short and long term strategies to address these

issues. Relevant reports produced in Australia and overseas should be taken into account in the formulation of the Advisory Group's advice.

This initiative of the Australian government provides evidence that the present level of female participation in science, engineering, and technology is now explicitly an item on the political agenda. As government not only reflects popular opinion but also has the capacity to lead it, it is significant that these currently existing levels of participation are said to be an "under-representation" and are therefore deemed to be at an undesirable level.

Future Directions

The overwhelming majority of students attending the University of Adelaide has been educated in the South Australian school system. This system is an interesting mix of single-sex and co-educational schools, both public and private, and across varying socio-economic and identifiable cultural groups. As one of the questions that the ICMI Study on Gender and Mathematics Education is considering concerns the benefits of a single-sex education, it would be interesting to survey girls and their families at schools representative of each type to find out if differences in attitude and expectations at the single-sex schools could be established.

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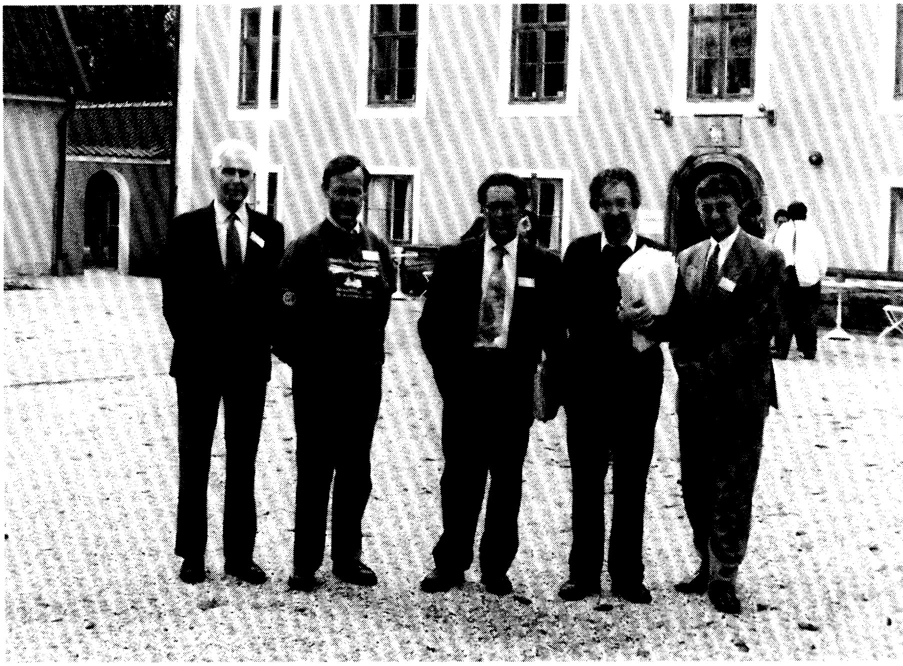

The executive committee of ICMI. Jean-Pierre Kahane, Jeremy Kilpatrick, Miguel de Guzman, Mogens Niss and Jack van Lint. *Photo: Lennart Jonson ©*

GENDER AND MATHEMATICS EDUCATION IN SWEDEN

Barbro Grevholm
Lund University

The Situation in Compulsory School

The education system in Sweden is going through great changes. A new curriculum for compulsory school will be implemented in 1995/96. Official documents state that "the main task of the nine-year compulsory school is to convey enduring knowledge and basic values, such as respect for human dignity, individual liberty and integrity, and concern for those in adversity" (Ministry of Education and Science, 1994a). It further states that

> another foundation stone is that of greater equality between the sexes, with girls and boys enabled to study in accordance with their individual needs and abilities.
>
> Developments in the society in which we live are making new demands on teaching in schools. The government is now proposing a thoroughgoing reform of the Swedish education system. (p. 3)

This is the official voice. Equal opportunity in education is not a new concept in Sweden. As far back as 1969, a tenet of the curriculum was that school shall work for equity between men and women. In compulsory school girls seem to do well. At the end of compulsory school girls have better marks in mathematics than do boys on average (and in most of the other subjects too) (Grevholm & Nilsson, 1994).

Until now there was a gender difference in participation in the advanced mathematics course. In schoolyear 7-9 students chose between the general and the advanced courses. Both courses have the same number of teaching hours, but the general course is less demanding. The new curriculum has only one course. Figure 1 shows how choice of course for pupils in year 9 has changed over the years (based on data from Statistics Sweden 1988, 1992).

What is the explanation of the disappearing difference? No official report has been published about this. Maybe changes in society have made girls think that mathematics is important to them. Maybe teachers nowadays more often advise girls to choose advanced mathematics. Maybe girls' in school of year 1-6 experience mathematics to be interesting and rewarding. Further investigation would be of interest.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

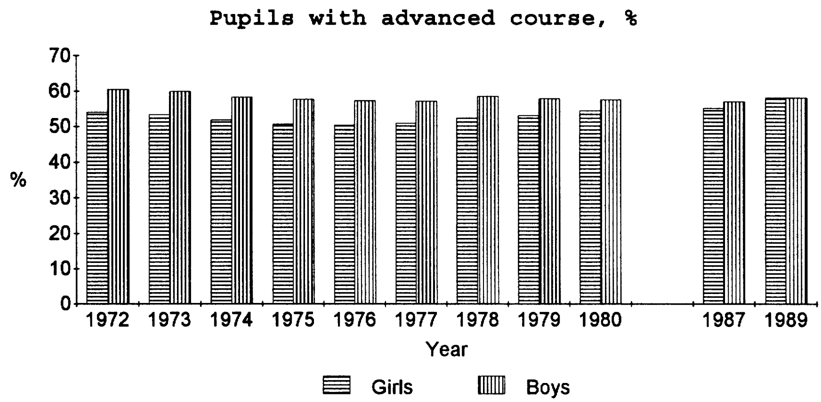

Figure 1

A standardised test in mathematics is given in year 9. In this test boys have slightly better test results than do girls (Grevholm & Nilsson, 1994). Despite that, teachers' marks are in favour of girls. Maybe this is because the curriculum demands that marks be based on all work, not only written tests (Lgr80). The situation is similar to what Kimball (1989) found in many other studies about gender differences in tests and marks in mathematics.

Swedish pupils took part in the 1964 and 1980 IEA (International Association for the Evaluation of Educational Achievement) studies. The average achievement of girls was slightly, although not significantly, below that of boys in 1964. In 1980 girls were at the same level as boys (Hanna, 1989). Other investigations (Feingold, 1988) have also shown vanishing differences in achievement during this period. The IEA study assessed attitudes towards mathematics. Both girls and boys considered mathematics an important subject in which they wanted to achieve good results. Both claimed to the same degree that knowledge of mathematics is important to get a good job. Indeed, with two exceptions, all attitudes towards mathematics were similar. Boys were more positive towards the use of calculators and computers. Boys were more uncertain about girls' need for and capacity for learning mathematics (*Matematik i skolan*, 1986).

Thus achievement and choice of courses in mathematics in compulsory school is not gender related. However, there are many other perspectives to worry about. Studies have shown that teacher-pupil interaction is gender dependent. Girls get less of the teachers' time and interest, less scolding and less appreciation, and they are spoken to differently than are boys (Kilborn, 1976; Molloy, 1990; Engström, 1994). Gender differences in this kind of interaction have not vanished over the years.

Mathematics textbooks show a world that consists of at least twice as many men as women. Areskoug & Grevholm published (1987) an investigation of textbooks for years 6 to 9 that aroused intense discussions. The ensuing debate brought about revision of most textbook series. However, renewed investigation in 1994 shows that this gender bias in pictures and names still exists in textbooks. Picture editors seem to have forgotten the debate and authors do not interfere. One book for natural science in upper secondary school shows 43 pictures of boys/men and 5 of girls/women. The contexts of problems are also gender biased, being more often contexts with which boys are familiar (Areskoug & Grevholm, 1987; Rönnbäck, 1992).

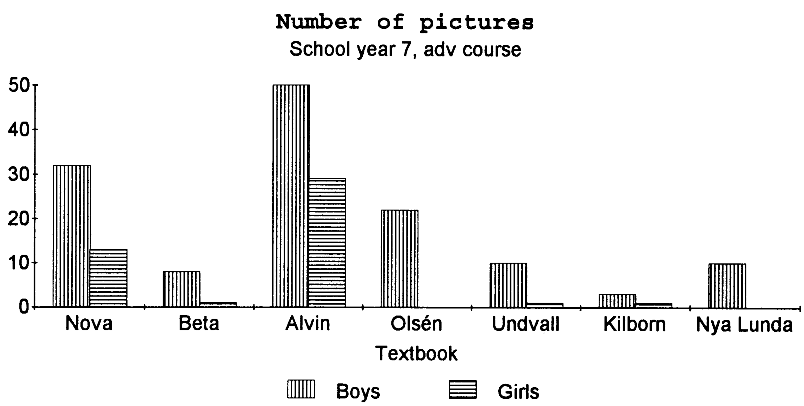

Figure 2

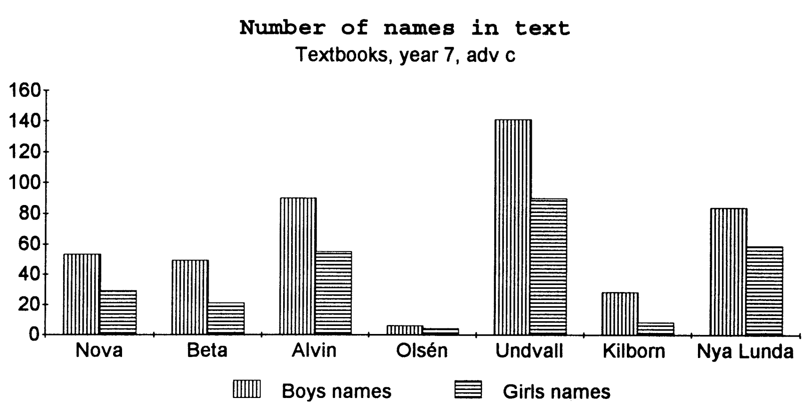

Figure 3

Are there any acceptable reasons to keep these gender differences in textbooks? Has there been research concerning the influence gender-biased books have on pupils, attitudes to mathematics and learning results? Is this situation worse in Sweden than in other countries? Publishing companies may claim that pictures and contexts have nothing to do with pupils' achievements in mathematics. However, if the books convey a hidden message that mathematics is not important to girls, the effect may be devastating.

An investigation of what Swedish teachers of mathematics believe about the gender facts mentioned here implies that most teachers are unaware of gender bias in teacher interaction, textbooks, and achievements in mathematics (Grevholm, in press). Much remains to be done before the conditions for learning mathematics are the same for girls and boys. First, more teachers must become aware of the situation; then maybe they will want to change things. Finally, we have to find ways to bring about change once the desire is there.

Upper Secondary School

A new curriculum and a new marking system for upper secondary school have been decided about. The new curriculum is geared to the new goal- and achievement-related governing system for the public schools.

The foundation values of the upper secondary school are respect for human dignity, the freedom and integrity of the individual, concern for those less fortunate, and personal responsibility. "In addition to that the school shall increasingly take an international perspective, work for equality and strive to reduce environmental problems" (p. 6) (Ministry of Education and Science, 1994b).

Almost all pupils go to upper secondary school. Until now it has been organised along 27 different education lines. Four of them are theoretical and preparation for higher education at university. The new model is a system of 16 programs, with 2 theoretical and the others more vocational. There are proposals to enable the municipalities to make their upper secondary schools more course-oriented.

Looking back it is easy to see that only two lines have been equally popular with boys and girls. One is the natural science line with the most advanced course in mathematics, leading to university studies in mathematics, science, technology, and medicine. Most lines can be looked upon as either boys' education or girls' education, because pupils of one sex predominate (Grevholm, 1992). One reason for change has been the wish to break this tendency of traditional career choice. It is doubtful if the new programs have been constructed in a way that will achieve this change. The new curriculum in mathematics does not mention this question.

The most advanced course in mathematics is taken in the natural science and technical lines. Approximately one third of the pupils in these two lines combined are girls. Twice as many boys as girls study the most advanced course in mathematics.

The achievement in mathematics of the girls who choose the most advanced course is good. Ljung (1991) has studied the results of the standardised test in mathematics for the years 1985-1989. The girls' average is either equal to or slightly better than the boys' for all years except 1986. The standard deviation is somewhat higher for the boys, about 1-11% more than for girls. Ljung comments that the group of girls is a positive choice, with higher marks in mathematics in compulsory school. Although on average the final marks for the girls are better than for the boys (Statistics Sweden, 1988, 1992), more girls than boys leave this line of education for another, with the second commonest reason being the teachers' negative attitudes towards girls in technical education (Skolöverstyrelsen, 1988).

The situation in mathematics in upper secondary school is not satisfactory. The number of girls taking the natural science program with the most advanced mathematics course should be 50%. There is obviously a proportion of girls able to study this program with good results. An interesting question is what role the experience of mathematics in compulsory school plays, when pupils choose their education for upper secondary school. For some years there has been a debate about how Sweden will manage to get enough students in science and technology (Grevholm, 1993). From this perspective, the question of girls' participation is interesting. How do we change the situation? Is it possible to influence young people's choice of career? The government has given a five-year task to the National Agency for Education and the National Agency for Higher Education to encourage more students to choose science and technology.

In Spring 1993 Statistics Sweden carried out a survey among pupils in the last grade of upper secondary school. The aim was to study the plans for higher education among young people. A little more than half of the pupils intend to apply for higher education within the next three years. A little more than one third of the males were interested in technology, whereas only 6% of the females interested in higher education wanted to study technology.

University Level

More females than males participate in higher education in Sweden. Women in Sweden are prepared to go through higher education, but they more often than men choose shorter programs. The choice of education is gender-dependent. Women dominate in teaching, nursing, and social welfare, whereas men dominate in science and technology. The situation in law and medicine has changed over the years so that now enrolment of women and men is almost equal. The Swedish labour market is claimed to be the most gender segregated in Europe (Statistics Sweden, 1994).

The figures 4 and 5 show the transition rates within three years for students from upper secondary school (Data from Statistics Sweden, 1992).

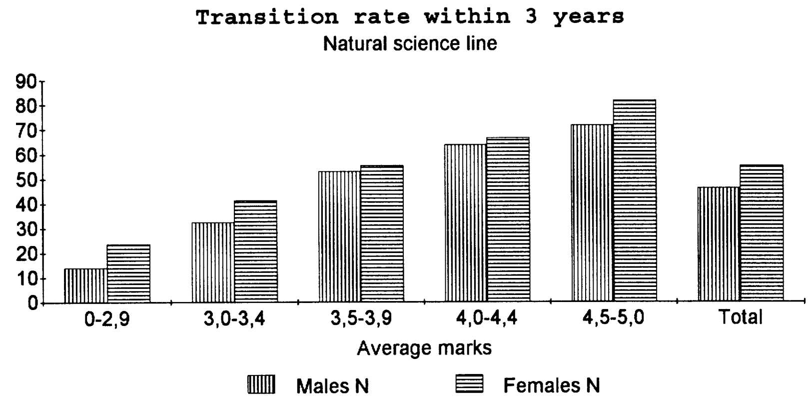

Figure 4

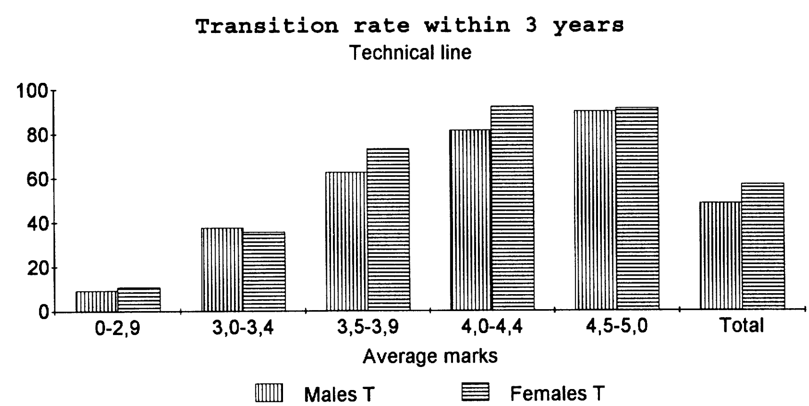

Figure 5

What happens when the students go to university? Few girls go to technical universities (except for architecture and chemistry); the rate has not changed much over the years, although many interventions have been carried through to influence more girls to take technology (Grevholm, 1993). About one third of the students of mathematics at the first year level are female, which is about the same proportion as in the appropriate lines in upper secondary school. In the expected study time 60% of the women and 55% of the men in Lund University finish their courses in mathematics. The women seem to get good results. At Lund University about 40% of the students in studies to become a teacher of mathematics in upper secondary school are women. 52% of the women and 58% of the men finish their courses in the expected study time.

In conclusion women in Sweden are prepared to go to university to at least the same degree as men. After their education the labour market welcomes them to employment in less favourable environments, with lower salaries and lower status than most employment for men with comparable education (Statistics Sweden, 1993).

At the graduate level in mathematics there are very few women. Of approximately 250 lecturers in mathematics at university with permanent positions there were in 1992 only about 8 women. In 1993-94 the number of women increased to 12. Compared to the situation in other subjects with about 20% female lecturers, this is very low (Wittenmark, 1993). Sweden has no female professor of mathematics. The amount of female professors in general is 8%.

In the Government Bills "Higher Education for Added Competence" and "Research for Knowledge and Progress" (1992/93), the low rate of women in research has been observed. The goal is that more women should be attracted to research. The government states:

> A ten-point programme to increase the number of women in research and postgraduate training is to be presented. The proposals include financial support for local measures to promote equality, the doubling of grants for research in women's studies and equal rights, a guest professorship in women's studies, prioritization of female postgraduate students in certain fields and the creation of a national equal rights prize (p. 15).

There is an official wish to change the unequal situation at the postgraduate level. However, this will be slow in bringing about change. Is it possible to influence the situation in other ways? Fennema and Leder claim (1990) that practices, values, expectations, and beliefs of individuals and society must be examined if the currently existing gender differences in mathematics participation and performance are to be understood and changed.

Women With Higher Education in Mathematics

Sonja Kovalevskaja was professor of mathematics in Stockholm University 1884-1891. Sweden has not had another female professor of mathematics. What is the explanation? The other Nordic countries all have female professors of mathematics (Piene, 1993). One reason may be that Sweden has comparatively few professors of mathematics. It is probably more difficult to obtain a professorship in Sweden .

Is it possible to learn from history in order to bring about change? Since 1990 I have carried out research to find out about all women in Sweden with higher academic degrees in mathematics. What can we learn from their experiences?

The first Swedish woman to obtain a doctorate in mathematics was Louise Petrén in 1911. Her thesis, written in French, dealt with an extension of Laplace's method to solve certain differential equations. It is said that she did not have a supervisor but that she worked independently. Neither did she get any support after her degree. At that time women were not allowed to work at the university. She maintained herself and her four children after her husband's death by working as a part-time teacher at an upper secondary school for girls in Lund. She lived until 1977 and was active writing and reading her whole life (Grevholm, 1990).

During the years 1947-1971 another nine women took degrees equivalent to the PhD. The first of them to work at the university level was Sonja Lyttkens, who got her PhD in 1956. All of these women have (or have had) tenured positions as senior lecturers at a university. Five of these women studied in Lund (including L. Petrén) and five in Uppsala, the two oldest universities in Sweden. After 1971, the system of higher examination in Sweden changed so that it was only possible to take one higher degree, not two as before. During the years 1974 to 1994 Sweden has produced another 12 women with degrees equivalent to the PhD. Most of these women are working at a university in tenured or temporary positions as senior lecturers (Grevholm, 1994).

What made these women go on with their studies in mathematics? Why were they successful? They have been invited to write a chapter about their lives in mathematics and outside mathematics. Can we learn from what they tell? Some of them can tell about moments in their career when some professor of mathematics told them not to go on with their studies. Why did they persist? What about those who did not fulfill their plans? Further investigation in this area may give us some insights.

At least 9 of these 22 women have been active as researchers in mathematics since their degrees. Nine of them have devoted themselves to teaching and at least six have published books or articles on mathematics education. The social competence of these women seems to be high. Two of them hold a prefecture in their institutions and one has become head of the university in Luleå and this is in concurrence with several hundreds of male mathematicians.

What kind of persons are these women in mathematics? In 1910 the section of mathematics and science in Lund University commented on Sonja Kovalewskaja in a debate about females' right to university positions. The official minutes recorded that "such single phenomenon proves less than nothing in the wanted direction. It only proves that sometimes feminine individuals are created, who in one or other respect have peculiar masculine qualities." (Grevholm, 1990) The female mathematicians we are discussing here appear to be ordinary people. Most of them have families and children and sometimes grandchildren. Their interests are varied and include music, literature, nature, sports and so forth. It seems to be possible to live a normal life and still be a female mathematician.

The Swedish Network of Women and Mathematics

Another way to intervene and raise consciousness about gender questions is the network Women and Mathematics, started in 1990 and which now has more than 500 members. The aims of the network are:

- to create contacts between those who are interested in women's/girls' conditions in studies or research in mathematics
- to spread information on projects and research about women/girls and mathematics
- to suggest speakers (preferably female) in subjects concerning women and mathematics
- to be a national suborganization of the international network IOWME

(International Organization of Women and Mathematics) (Grevholm, 1991)

The effect of the network is much stronger than anyone could have imagined when it started. The results multiply and are important not only for the women but for men and for the subject as such. The network has, among other things, organised two conferences on gender and mathematics education and published proceedings from these conferences (Brandell, 1994; Grevholm, 1990). As the traditional attitude towards pedagogical changes in mathematics departments at universities is reluctance (Jacobsson & Elvin-Nowak, 1994), the need for other forces in order to stimulate change is great.

For the Future

Research on gender and mathematics education has produced some obvious knowledge during the past decades. How much do teachers know about the results? In order to get a current picture of teachers' beliefs I have asked teachers of different age groups to give their opinions to a group of statements about gender and mathematics education. From the results so far it is evident that research results have not reached ordinary teachers in Sweden (Grevholm, in press). An important mission must be to disseminate knowledge about gender facts to teachers and decision makers. The next question is, what can we do once we know about the facts? Is it possible for teachers to change their behaviour, for publishers to be aware of gender, for parents to give the right support, for politicians to distribute resources in a better way, and so forth? Intervention programs of different kinds have shown that changes are possible. Many of them are carried out in a local school and the results are not communicated. The ideas and methods must be spread, and teacher education must deal with these questions. Hanna (1994) has asked the question "Should girls and boys be taught differently?", and we have to continue with this discussion. What ways do we want to choose for future development?

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The International Program Committee. Mogens Niss, Barbro Grevholm, Gila Hanna, Gilah Leder and Christine Keitel-Kreidt. Carlos Bosch and Geoffrey Howson not present. *Photo: Lennart Jonson ©*

AN OVERVIEW OF TIMSS FOCUSING ON GENDER ISSUES IN THE STUDY

Liv Sissel Grønmo
University of Oslo

Introduction

About 50 countries are expected to participate in the Third International Mathematics and Science Study (TIMSS). TIMSS is the latest, and largest to date, in a series of international studies by the International Association for the Evaluation of Educational Achievement (IEA). For very good reasons, there have been ongoing discussions about this type of study. Results from IEA studies usually get much attention in the media, as well as much criticism. Discussions about this kind of study are important from different viewpoints; nevertheless, I will not touch upon this in this paper.

I will briefly present an overview of TIMSS, the design of the study, the framework within which the study takes place, the target populations for the study, the time schedule, and so on. After this introduction, I will discuss some opportunities for focusing on gender issues. Since the study is still in a phase of planning and decision making, I will be presenting what is planned, as well as my hopes for what will take place in the study.

When focusing on gender issues in the study, I will first say something in general about opportunities to do so in this kind of study. I will then concentrate on two issues of special interest for me, upon which I very much hope to get some feedback from you. The issues are: (a) testing student abilities in mathematics in different test contexts, and (b) the concepts of "gender" and "sex."

About TIMSS

One reason for the criticism against IEA studies might be the way results from these studies have been presented in the media. The media has mainly focused on ranking of countries based on the achievement tests. Whereas the first IEA study carried out in the sixties had ranking as its main interest, the emphasis has now changed. Effort is now being placed on analysing the intended and implemented components of the curriculum (Figure 1). The study of students' achievement is only one part of the study. I will briefly present an overview of the conceptual framework and some of the research questions that are central in TIMSS.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

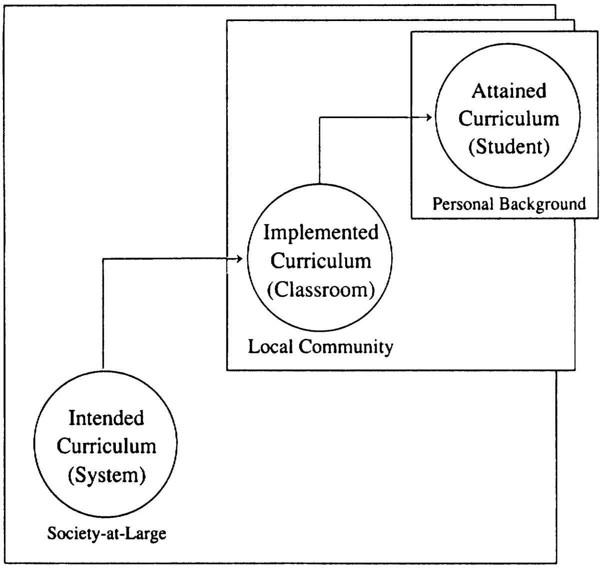

Figure 1 The conceptual Framework for TIMSS

The conceptual model is mainly derived from the models used in earlier IEA studies – especially the Second International Mathematics Study (SIMS; see Travers & Westbury, 1989), and The Second International Science Study (SISS; see Rosier & Keeves, 1991) – modified and updated to meet particular demands of TIMSS. Fundamental to the design of TIMSS is the centrality of curriculum as a variable. The relationship between science and mathematics as separate yet linked components of the curriculum is also fundamental to the design of the study. The model illustrates the sets of variables that impinge on educational achievement. We know, for example, that the social context in which the school is situated influences the goals and means of education. An appropriate description must draw on the important features of the system at all levels, as well as on the social contexts in which the system operates.

For the purposes of TIMSS, curriculum is conceived of as the concepts, processes, and attitudes in mathematics and science that are intended, implemented, or attained. In the most general terms, the research questions for TIMSS are the following:

1 What mathematics and science concepts, skills, and processes have students learned, and what factors are related to students' opportunity to learn those concepts, processes, and attitudes?
2 How do educational systems vary in the intended learning goals for mathematics and science, and what characteristics of educational systems, schools, and students are related to the development of those learning goals?

3 What opportunities are provided for students to learn mathematics and science, how do instructional practices in mathematics and science vary among educational systems, and what factors are related to this variation?
4 How are the intended curriculum, the implemented curriculum,and the attained curriculum related with respect to the context of education, the arrangements for teaching and learning, and the outcomes of the educational process?

For further information about the framework of TIMSS, the design and foci of the study see Robertaille et al. (1993) and TIMSS (1992). TIMSS deals with many new aspects of the curriculum. From a researcher's point of view, some of these aspects, and combinations of aspects may attract more attention than results from the students achievement tests.

At the beginning of 1994, before the student achievement tests have taken place, TIMSS will publish the first report based on data from the study. This report will present the analyses of the intended curriculum in different countries. The basis for this part of the study is an accurate description of the curriculum guides and the textbooks in all participating countries. One intention for doing this is to underline that the study is not mainly a study of student achievement.

The second precaution to be taken is in the way the presentation of the final report on student achievement will take place. The IEA studies presented a ranking list of all participating countries, based on each country's mean score for student achievement. One reason for changing this is that the difference between one country and the next on the list is often statistically insignificant. The second reason is the tendency to concentrate one's attention on the position of a country in such a list.

TIMSS will not report the results as one main score, but as subscores for several topics in mathematics and science. There will not be a list of consecutively numbered countries in each topic either, but instead all countries are divided into groups, according to their performance on that specific topic.

TIMSS will collect data from three different populations. Population 1 consists of the two grade levels with most 9-year-old students, population 2 consists of the two grade levels with most 13-year-old students, and population 3 consists of students in the final year of upper secondary school. A major strength of the design for TIMSS is that it permits both age and grade comparisons to be made for population 1 and 2, thereby overcoming a weakness of many previous international studies. This design makes it possible to do a quasi-longitudinal study of growth in achievement, and to relate this to the intended curriculum.

All countries in TIMSS will test students in population 2, while it is optional for each country whether or not to join the study for populations 1 and 3. TIMSS is planned in two phases, one from 1991 to 1995 and the second from 1996 to 1999. This design of the study gives a broader opportunity to do comparative studies than have any of the earlier IEA studies.

The study will result in an enormous collection of data. This collection makes it somehow difficult to talk about TIMSS as just one study. It is a study consisting of several reports, but it could in addition be looked upon as an international database where all data are collected in accordance with some basic principles. The database makes possibilities for research from a variety of perspectives.

I will underline the opportunities these data give researchers to do several analyses in addition to those done as part of the study as such. In Norway, for example, several students at the master's and doctoral levels are planning to use data from TIMSS as the bases for their theses. These studies cover a broad range of research questions and approaches. Later this database should give rich opportunities also to researchers outside the project.

About TIMSS Data Related to Gender

I will present what TIMSS can offer related to gender. Many of the previously mentioned research questions in TIMSS are, of course, of general interest from a gender point of view. It is not clear whether there will be an international report from this perspective in TIMSS. Alison Kelly (1978) published an IEA report called *Girls and Science* based on data from the First International Science Study (FISS), and Gila Hanna (1989, 1993) has analysed earlier IEA studies in mathematics from a gender point of view. Nevertheless, in most IEA studies there has not been a report especially focusing on gender. Gender issues have only been an implicit, and usually small, part of these reports. My hope is that such a report will be published in TIMSS.

One important question in relation to gender is, at what age are differences between boys' and girls' achievement and attitudes to mathematics statistically observable? TIMSS will provide data about students performance at five different school levels in 50 countries, thereby offering an excellent opportunity to study the development of gender related differences.

Differences between boys' and girls' achievement in mathematics have often been related to biological differences, for instance, changes that take place in puberty. If that is the case, we should expect the same differences to take place all over the world at about the same age. Are there any differences between countries in relation to this? Do gender-related differences appear in the same way in geometry as in algebra? Are there topics in mathematics where girls outperform boys? Can differences in the intended curriculum explain some of the differences between countries? TIMSS data offer opportunities to go deeper into such questions.

In addition to data about student achievement in mathematics, TIMSS will require that all students (and teachers) answer a questionnaire about attitudes and values. It is therefore possible to examine the relationship between values, attitudes, and achievement in mathematics. Can attitudes be examined as a significant factor for influencing achievement? How do attitudes toward mathematics vary from one country to another? Are some attitudes more effective for learning and understanding mathematics? What are the relationships between biological sex, attitudes, values, and achievement?

Testing in Different Test Contexts

One issue in research related to gender differences in mathematics has been how students' performances differ when the context is changed. Traditionally this has been tested with two items that are categorised as having the same mathematical content, but the context of these two items in which the mathematical task is presented is different.

Some reports have underlined that familiarity of context is of crucial importance to students' performance. Strassberg-Rosenber and Donlen (1985) report that girls do less well compared with boys in contexts traditionally seen as male domains. It also seems that context is a more significant variable for girls than for boys. Eddowes, Sturgeon, and Coates (1980) found that context influenced girls' achievement but not boys'.

I have suggested that TIMSS should examine another aspect of "what happens when we change the context." My suggestion is to use identical items, but to change *the context in which the testing takes place.* IEA has recently finished the Reading Literacy Study (Elley 1992). One part of this study tested the students' ability to read tables and diagrams. These items could as well have been items in a mathematical test. Including these items in the TIMSS mathematical test will create opportunities to compare students' performance on items when they are presented in a language test or in a mathematics test. Does this change influence the difference between girls and boys when it comes to achievement? What about high and low performers among girls and boys? Is the pattern the same from one country to another?

Earlier studies have underlined that confidence is a crucial factor for explaining achievement in mathematics (Fennema, 1980; Jones & Jones, 1989). Perceptions of sex roles have also been an important issue. A study of Skolnick, Langbort, and Day (1982) pointed out that 6- and 8-year-old students tended to do better if a task were thought to be feminine for girls and masculine for boys. If girls are more self-confident in language than in mathematics, and if they regard mathematics as a more masculine subject, it seems reasonable to believe that girls would do better on tasks presented as part of a test in language than as part of a test in mathematics. Presenting identical items in two different test contexts makes it possible to do a closer examination of questions of this kind.

It should be mentioned that in the Reading Literacy Study girls outperformed boys throughout the world in most parts of the test. Reading diagrams and tables were part of the category "documents," where boys tended to do just as well as girls. Nevertheless, in reading tables and diagrams girls also outperformed boys for some of the items (Munck, 1992). From my point of view, it is interesting to see whether or not this will be the case in TIMSS – my hypothesis is that it will not.

The Concepts of Gender and Sex

Gender was originally a concept used in grammar to distinguish between masculine and feminine nouns. For the past decades it has been commonly used when discussing the issues of girls and boys in science and mathematics. The G in GASAT (Gender and Science and Technology) changed from Girls to Gender around 1985. This conference is called the ICMI Study on Gender and Mathematics Education.

Sandra Harding (1986), Evelyn Fox Keller (1984), and others have made significant contributions to the concept of gender. Nevertheless, the way the concept is used seems problematic to me. Today gender is used more or less as a substitute for "sex," or "sex roles". The reasons for the common use of this concept are many. Two reasons might be that the term gender underlines the social nature of the issue and that it avoids unwanted sexual connotations associated with "sex" or "sex roles."

I find the use of the term gender especially confusing and problematic in reports from empirical surveys such as TIMSS. Usually what is done is that students or other respondents are classified in two groups according to their answer on a simple question about biological sex: "are you a girl or a boy?" But when conclusions are made, gender is the commonly used concept when the differences between girls and boys are described.

By using the term gender this way in reports, biological sex and gender seem to be somehow identical. I have used sex and gender as synonyms in this presentation so far. I have varied between using gender and biological sex in a way indicating that they are identical concepts. Even if commonly accepted, I find this to be problematic. To me gender is a social construct, biological sex is not. "Biological" boys might have traits associated with "sociological" girls and vice versa. In TIMSS the students will complete a questionnaire about attitudes and values. Their answers on this part of the study are related to what I associated with gender.

This is not at all an easy task. I did not take stock of myself to sort this out here, but I wanted to underline that a lot of work has to be done to clarify the relationship between "gender," "sex," and "sex roles." This is a great concern to me especially in relation to how to report on differences between girls and boys in a study like TIMSS. If gender is a synonym for biological sex, it should be made explicit. If gender is not a synonym for sex, differences between girls and boys in TIMSS should be reported as sex differences.

Another point is that by distinguishing gender as a social construct from biological sex, there should be rich opportunities to study the relation between these two concepts, as well as their interaction with achievement in mathematics in TIMSS.

Final Comments

In studies like TIMSS, which differences do we need to explain? From my point of view, it is important not only to focus on differences in mean scores between girls and boys, but also on how to account for the variation within each group. How to measure differences between girls and boys is not at all an easy task (See the discussion in *Review of Educational Research* between Feingold (1992, 1993) and Hedges and Friedman (1993a, 1993b). What I have said about a close examination of differences between girls and boys must, of course, be based on a very careful investigation of what is to be measured as a difference.

We do not need research that exaggerates small differences in mean values between girls and boys. In a quantitative study like TIMSS, there will always be a danger of overdoing small statistical differences. Even though I did not focus on these issues, I want to mention it as something of crucial importance.

I have given a brief overview of TIMSS, as well as focusing on some issues of special interest from a gender point of view. My main purpose has been to focus on issues in which I hope to get some feedback. In that way I can make TIMSS, as well as my own research, more interesting in relation to gender, and from the perspective of getting more girls interested in mathematics.

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GENDER ISSUES IN THE PRIMARY CLASSROOM

Maggie Haynes
Auckland College of Education

One of the courses offered to primary teacher education students at Auckland College of Education is entitled "Girls and Their Mathematics Education." Students who participated in this course in 1993 investigated gender specific issues through the reading of research, participating in observations, and interviewing and teaching children in an Auckland primary school. A heightened awareness was reflected in the range of issues the students selected for further investigation and the depth to which the students pursued these issues.

Girls and Their Mathematics Education

This unit is designed to raise the awareness of the issues related to gender that affect the mathematics education of girls. It is expected that students taking this course will develop ways to improve their teaching skills so that they are able to provide equal learning opportunities for all.

300 level courses in the Mathematics Education Department of the School of Primary Education at ACE raise an awareness of equity issues:

- Under Achievement in the Mathematics Classroom
- A Cultural Perspective in New Zealand Mathematics Education
- Girls and Their Mathematics Education

Other courses include:

- The Mathematics Children Bring to School – assessing children's ability on arrival at school in order to provide appropriate learning experiences
- Mathematics Education in the Classroom – satisfying the learning needs of a wide range of children
- Mathematics Education from Form 1 to Form 4 – develop own confidence in this area in order to provide for individual needs of form 1/2 children

In the first session we discussed how one issue of equity cannot be isolated from others, such as ethnicity and socio-economic, but that people on this course had chosen to focus specifically on the gender aspect. Students highlighted gender-specific issues that had interested them in their Education papers, particular issues they had read about in mathematics, such as performance, teacher behaviour, expectations, and so forth.

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Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

Results of a brainstorm session indicated the amount of prior knowledge students brought to the course – what did they think affected the mathematical experiences of girls?

family	expectations	teacher's gender	culture
textbooks	individual	exams	assessment
peers	timetable	careers	computers
resources	finances	media	experiences
teacher	school policy	employers	politicians
business people (round table)		overseas influences	

. . . what did they think we should look at /consider during the course?

visit classrooms	talk with teachers	talk with children
resources	learning styles	teaching styles
readings/research		

One student summed up this session with the question, "I hope we are going to look at some strategies for dealing/coping with these issues?"
Learning outcomes of the course were to

1 Develop an awareness of the issues related to gender that affect learning and achievement in the mathematics classroom
2 Develop ways of implementing classroom practices that enable girls to realise success in their mathematics education

Assessment for each learning outcome included:

Learning Outcome 1
The performance criteria demanded an observation of a class of boys and girls at work in a mathematics lesson and individual interviews with a group of children to ascertain their attitudes to mathematics.

We recognised that there could be a problem finding a school where teachers were willing to have us in the classroom; observation visits could be seen as a threat to their behaviour and their classroom environment. We were fortunate to hear of a school, through one of the mathematics advisers, that discussed the possibility as a staff and offered us the use of the whole school at times to suit our ACE classes. We were then able to visit children, from New Entrant to Standard 4 levels. It was decided that students would remain with the same children for the whole course.

Students Visits to School

Students were expected to write a report that included details from their observations, noting factors that might affect the success of girls, and a summary of their interview responses, reflecting on the implications that arose from the visit. Their reports were to be supported by statements relating their conclusions to relevant research.

Attitudes and Beliefs

Students spent a session sharing and discussing questions, aware of the need to frame them appropriately according to the level they were working with.

- Do you like maths?
- What kind of maths do you like?
- What is maths?
- What do you like doing out of school?
- What were your favourite toys before you went to school? ... when you were little?
- Did you go to Playcentre/Kindy etc?
- Who do you like doing your maths with? ... by yourself?
- What is your favourite subject /lesson at the moment? Why?
- Who tries to answer the questions in your group/class?
- Who helps you with your homework?
- Who do you see doing maths out of school? Do you do any maths out of school?
- Is there a best group?
- Who is best at maths? Why? Tell me about it.
- How do you find maths tests?
- Who does their maths quickly /quietly?
- Are you good at maths?

Learning Outcome 2

Students were asked to prepare and teach a lesson designed to be enabling and motivating for girls and to present a seminar focusing on one aspect of mathematics education from a gender specific perspective. The seminars covered a variety of topics, all well supported by research.

Are boys more capable than girls?	Assessment
Expectations and attitudes	Teaching styles
Role models	Stereotypical language
Texts and resources	Learning styles
Single sex groups	Computers
Outside the classroom	

Evaluations highlighted the importance the students placed on the course:

"... found the course enlightening and useful."

"... the assignments were challenging, relevant, and useful"

"... the best thing about this course was the seminar.There were very interesting topics and a lot of research was done, fantastic."

"... high interest level, extremely relevant, politically essential ..."

"... didn't harp on about deficit. Looked positively at what can be done with out putting down males."

"An important course – brought a strong focus to girls' needs."

" I came out with good ideas for enabling girls in maths."

"... and the content related to the class needs."

" The course was very useful because it opened our eyes to see the real issues that are present but not taken notice of."

I conclude with reference to one of the student seminars:

Since the Education papers in our first year we have been constantly collecting gender jargon and accompanying researchers names ... and what filters through is male domination and success in the schooling system, with particular reference to the area of mathematics ... so what can we actually do about the girls ... what about doing something FOR the girls ...

GENDER ISSUES IN JAPANESE MATHEMATICS EDUCATION

Setsuko Hazama
Osaka Kyoiku University
Hanako Senuma
National Institute for Education Research, Tokyo

This paper considers gender imbalance in mathematics education, which stem in part from social factors. It also suggests the view that matematics education should be changed so as to include the social context in which it is studied and taught. Gender issues should be considered in relation to individuality. Many data have been gathered concerning gender imbalance, and special attention has been paid to an analysis of mathematics textbooks. We hope that gender consciousness can be raised through many points of view.

Background of gender issues in mathematics education

Implicit gender bias in education

Gender issues in education in Japan are implicit. Most teachers, parents, and the nation's educational society or educational boards do not perceive why "gender" should be an issue. The social, cultural, and economic developments in Japan have not influenced a "consciousnessraising" of gender problems directly. The present Japanese culture still has a bias towards males, though not as strong a bias as before World War II (Harada, Jitosho, Kawata, & Susuki, 1992). In other words, prior to World War II the bias was explicit, now it is implicit.

In 1993, economic conditions in Japan are somewhat depressed. It is difficult for female graduates to get employment. Disadvantaged people and women suffer the waste in this respect. Figure 1 shows the rate of males and females whose first destinations after graduation from university are entering employment, advancing to high-level courses, or unemployment. Unemployed females who are not admitted to graduate study outnumber males (Asahi, 1993)

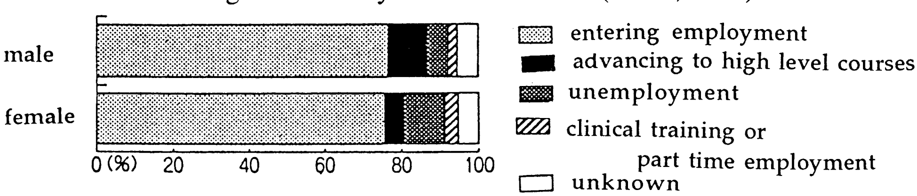

Figure 1 First destination of new graduates

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Gender and Mathematics Education, an ICMI Study

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It is difficult to find commentaries that note or discuss gender aspects in education. In government reports such as the Educational White Paper Japanese Culture and Educational Policy (Ministry of Education, 1990), which contains over 500 pages, we can find no relevant comments in the text; reference to gender occurs only in the appendix. Table 1 from the appendix of the report considers only females, not both genders.

Table 1 Proportion of female in educational institutions

Type of institution	Students	Teachers
Elementary schools	48.8%	57.5%
Lower secondary schools	48.8%	35.7%
Upper secondary schools	49.6%	20.1%
Junior colleges	91.1%	38.3%
Universities	26.4%	9.0%
Special training schools	52.7%	50.3%

(Ministry of Education,1990)

Gender bias in entrance examinations

Entrance examinations have strongly influenced mathematics education in Japanese schools. The main aim is for pupils to learn mathematics sufficienly well to get into a good upper secondary school or university, or to get a good job.

Howson (1993) notes two types of motivation to study mathematics – instrumental and significant motivation – mentioned with respect to mathematics education in Japanese schools. "Both types of motivation are important, but it would seem dangerous to rely too much on instrumental motivation. I suspect this is the case in Japan" (Howson, 1993, p. 67).

Also, the content of mathematics examinations is purely abstract and symbolic. Therefore not only gender problems but also real-world problems are common. It seems clear that we cannot place gender issues in true perspective until mathematics education is freed from the entrance examination and mathematics is set into a social and cultural context.

Participants' bias in the area of mathematics education

We are not in the situation in which gender issues can be addressed because the proportion of female participants in the educational society of mathematics and educational boards is generally very low. The number of female mathematics teachers in lower and upper secondary schools, colleges, and particularly in universities is small. In addition, there are few leading female mathematics educators in influential positions such as the Committee on the Revision of the Mathematics Curriculum for each school level, nor are there very many female authors of mathematics textbooks. Thus there is little opportunity to raise gender issues and too few female mathematics educators who can serve as models.

Setting gender issues in relation to individuality

The Japanese Ministry of Education revised the Course of Study in 1989. The new Course of Study was put into effect in 1992 for elementary schools, in 1993 for lower secondary schools, and will become effective progressively from 1994 onwards for upper secondary schools. One of four main objectives of the revision of the Courses of Study is "to enhance such educational programs as will enable each child to give full play to his or her individuality. A consistency in the curriculum for each subject area should be secured among different school levels from elementary to upper secondary school" (Ministry of Education, 1989).

We need to take into account one's gender as a factor of his or her individuality, though nothing is mentioned about this in the main objectives of goverments reports. Thus, it is necessary to study the development of each child's individuality in teaching-learning mathematics from gender perspectives.

Gender imbalance in mathematics education

Concern for gender

For many years, gender issues in mathematics education have not achieved respectability as an area of research. No handbook/dictionary on mathematics education published in Japan includes items/entries/headings concerning gender issues. Also, there are no items on gender issues included in the indices of these publications. Only rarely are there keyword concerning gender in research papers. This situation is quite different from that which pertains in the U.S., U.K., and many other countries.

But the situation is changing gradually. For example, a new handbook for research in mathematics education, being edited by the Japan Society of Mathematical Education, is supposed to have an entry on "gender and mathematics education" (Senuma, forthcoming). This will be the first instance of a major publication containing an entry on gender in mathematics education.

Gender of students

From kindergarten to upper secondary school, the ratio of male to female students is 1:1. It is 7:3 in universities and 8:2 graduate schools.

Table 2 Gender of students in educational institutions

Type of institution	Male	Female	Total
Kindergartens	50.8%	49.2%	100%
Elementary schools	51.2%	48.8%	100%
Lower secondary schools	51.2%	48.8%	100%
Upper secondary schools	50.3%	49.7%	100%
Universities	70.1%	29.9%	100%
Graduate schools	82.0%	18.0%	100%
Special training schools	49.4%	50.6%	100%
Junior colleges	8.3%	91.7%	100%

(Ministry of Education, 1993)

Gender of students who major mathematics

The radio of male to female students majoring in mathematics is 8:2 in universities and 9:1 in graduate schools, as indicated in Table 3.

Table 3 Gender of students who major in mathematics

Type of institution	Male	Female	Total
Universities (Undergraduate)	80%	20%	100%
Graduate schools (Master course)	90%	10%	100%
Graduate schools (Doctoral course)	91%	9%	100%

(Ministry of Education, 1993)

Gender of teachers

The ratios of female to male teachers are high in both kindergartens and elementary schools. But in secondary schools and in universities, the ratios of female to male teachers drop considerably, as indicated in Table 4.

Table 4 Gender of teachers in educational institutions

Type of institution	Male	Female	Total
Kindergartens	6.8%	93.2%	100%
Elementary schools	43.7%	56.3%	100%
Lower secondary schools	65.4%	34.6%	100%
Universities	90.6%	9.4%	100%
Special training schools	50.0%	50.0%	100%
Junior colleges	61.8%	38.2%	100%

(Ministry of Education,1992)

Gender of mathematics teachers

The ratio of male to female mathematics teachers is 4:1 in lower secondary schools and 9:1 in upper secondary schools, as indicated in Table 5.

Table 5 Gender of mathematics teachers in lower and upper secondary schools

Type of institution	Male	Female	Total
Lower secondary schools	80.1%	9.9%	100%
Upper secondary schools	91.3%	8.7%	100%

(Made by Senuma based on the data of Ministry of Education, 1992)

Gender of mathematics education specialists in universities

More than 50 national/public universities have courses in mathematics education and many professors teach them, but of these only one is female.

Gender of mathematics textbook authors

There are six mathematics textbooks published at both the elementary and lower secondary schools levels, all approved by the Ministry of Education. The number of female authors of mathematics textbooks at both the elementary and lower secondary levels is less than 10%, as indicated in Table 6. More than half of all elementary teachers are female, yet the number of female authors is quite small.

Table 6 Percent of female textbooks authors

	A	B	C	D	E	F
Elementary school textbooks (1992 use)	4%	3%	0%	3%	0%	0%
Lower secondary school textbooks (1993 use)	4%	3%	0%	3%	0%	5%

Sex-segregated schooling and mathematics curriculum before World War II

Before World War II secondary schools were segregated by sex, and the level of difficulty of mathematics curriculum for females was lower than that for males. Since 1945 we have had a coeducational system, equal rights, and the same mathematics curriculum for male and female students.

Gender differences in student achievement and attitudes towards mathematics

Some surveys reveal that there are no sex differences in elementary schools mathematics achievement and attitudes towards mathematics. But the achievement of male students tends to be higher than that of female students in secondary schools. For example, in the IEA/SIMS study, conducted in 1980, there were no sex differences in population 1 (13-year-olds), but there were great differences in population 2 (17-year-olds), both in achievement and attitudes towards mathematics.

Gender of mathematicians

The situation seems similar among mathematicians. The percentage of female mathematicians in Japan was 4% (Negishi, 1991)

Gender context in Japanese mathematics textbooks

Hypothesis and method

Gender context is a part of the social, cultural, and historical context, but not the mathematical context per se. Here we focus our attention on illustrations and words used for referring to persons in mathematics textbooks. Illustrations play the important role of gender references, helping pupils to become familiar with the content in his/her social and cultural context, and leading them to think mathematically or have an interest in it. In a study of science textbooks, it was revealed that rate of female illustrations is 24% in upper secondary school physics textbooks, and 31% in lower secondary science textbooks (Uenishi et al, 1991). There is no study of mathematics textbooks from the same point of view. So, we set forth some hypotheses as follows. The gender imbalance in many areas of mathematics education, already indicated in he data, would have some influence on mathematics textbooks. Types of figure illustrations (male only, female only, mixed) would be an indicator of gender bias. On the other hand, words used in reference to persons in the textbooks would translate from personal and the feminine and masculine to the common and neutral; in addition, in the lower secondary level, they would become "gender context-free".

We will analyze two series of mathematics textbooks, published by different publishers, A and B, for both the elementy and lower secondary schools. All of them are authorized by the Ministry of Education and have been used in schools since 1992 and 1993.

Figure illustrations in each grade textbooks

Figure illustrations in the textbooks may be categorized into three types: male-only type, female-only type, and mixed type. Examples of the three types are shown in Figures 2 through 4-2.

Tables 7 and 8 show the percentage distributions of male-only, female only, and mixed-type illustrations by grade. Table 7 shows the percentage distribution for the elementary textbooks, and Table 8 shows it for the lower secondary textbooks. Some comparative results are as follows.

For the elementary school textbooks:

1. The percentage of three types of illustrations, in total of 1 st-6th grades, show approximately that (M):(F):(Mix)=1:1:2 for the textbooks A and (M):(F):(Mix)=3:2:5 for the textbooks B, (here M, F and Mix indicate the percentage of the male-only, the female-only, and the mixed type, respectively).

2. The mixed type is used more than 50% for 1st-3rd grade of the textbooks A and B. On the other hand, the percentage of the male-only type is much higher than for the female-only in 5th and 6th grades of both the textbooks and it is nearly equal to the mixed in the text-books B.

Fig. 2 An example of male only type
(Elementary textbook A for 6th grade)

Fig. 3 An example of female only type
(Elementary textbook B for 6th grade)

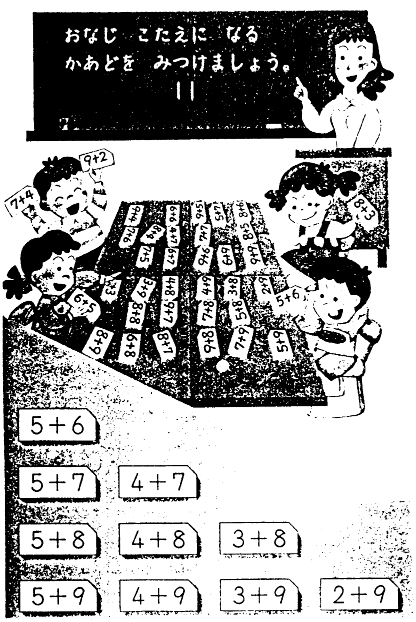

Fig. 4-1 An example of mixed type
(Elementary textbook A for 1st grade)

Fig. 4-2 An example of mixed type
(Lower secondary textbook A for 8th grade)

Table 7 Percentage distribution of three types of illustrations in the elementary textbooks A and B

	Textbooks A			Textbooks B		
Grade	Male	Female	Mixed	Male	Female	Mixed
1	14%	25%	60%	16%	13%	71%
2	15%	17%	68%	23%	26%	51%
3	21%	23%	56%	26%	21%	53%
4	25%	31%	42%	14%	28%	59%
5	40%	31%	29%	39%	20%	41%
6	37%	21%	42%	40%	22%	39%
Total	22%	24%	54%	24%	21%	52%

Table 8 Percentage distribution of three types of figure illustrations in the lower secondary textbooks A and B

	Textbooks A			Textbooks B		
Grade	Male	Female	Mixed	Male	Female	Mixed
7	27%	35%	38%	73%	19%	8%
8	36%	9%	55%	59%	17%	24%
9	4%	30%	67%	59%	11%	30%
Total	23%	23%	54%	63%	16%	21%

For the lower secondary school textbooks:

3. In the 7th - 9th grades, the percentage of each type shows approximately that (M):(F):(Mix) = 2:2:5 for the textbooks A and (M):(F):(Mix) = 3:1:1 for the textbook B.

4. In 8th and 9th textbooks A, the mixed type is used more than 50%, but an imbalance of the male-only with the female-only is remarkable. On the other hand, for 7th-9th grade textbooks B, the percentage of the male-only is higher than for the mixed and for the female-only.

The proportions of three types are different among two kinds of textbooks, especially for lower secondary textbooks. There is a bias in that the male-only type increases in the 5th and upper grades and that the percentage of male-only type for lower secondary grades is much higher than for both the other types in one set of textbooks.

From the quantitative analysis sketched out above, it is suggested that more analyses be carried out, different from the quantitative aspect, because illustrations in general reflect social and cultural stereotypes or gender role expected, and pupils are guided in the learning situation reflected in the textbooks.

Proportion of figure illustrations to all illustrations

Tables 9 and 10 show the proportions of illustrations, in elementary and lower secondary textbooks, that include geometric figures, number lines, graphs, diagrams, and so on.

Some comparative results are as follows.

1. In the elementary textbooks, for the textbooks B, the proportion of figure illustration in the 1st and 2nd grades is little higher than for the other grades (n total of 1st-6th grades is 18%).

Table 9 Proportion of figure illustrations to all illustrations in elementary textbooks A and B

Grade	Textbooks A	Textbooks B
1	–	27%
2	–	21%
3	20%	17%
4	8%	11%
5	9%	19%
6	8%	16%
Total	–	18%

Table 10 Proportion of figure illustrations to all illustrations in lower secondary textbooks A and B

Grade	Textbooks A	Textbooks B
7	7%	6%
8	8%	6%
9	6%	5%
Total	7%	6%

2. The proportions of figure illustrations are in subtotal of 3rd-6th grades, and 11% and 16% for textbooks A and B, respectively. The proportion for each 4th-6th grade of the textbooks A is lower than 10%.

3. In the lower secondary textbooks A and B, the proportion by each grade is lower than 10%.

The proportion of illustrations decreases to less than 10% in lower secondary level for both the textbooks. It may be that the relatively low percentage hides the gender bias observed in the previous section. Also, one may say that learning situations in mathematics textbooks is little related to society or culture, or they are all "context-free", especially in lower secondary textbooks.

We may note that illustrations appear in the introduction, the text, and the exercises of each chapter in both elementary textbooks A and B and lower secondary textbooks B; on the other hand, about 80% of the illustrations appeared in the introduction to each chapter in lower secondary textbooks A.

Neutralization of words used in reference to persons

We will list the kinds of words that are used in reference to persons in the textbooks, and how the words are translated from the personal and the gender to the common and the neutral by grade.

Words used in reference to persons in both textbooks can be classified into seven main groups, from the personal and gender to the common and the neutral, as follows:

1. Personal names (Miss Hanako, Mr. Masao, etc.)
2. One's brothers, sisters, father, mother, etc.
3. Boys, girls, men, women, etc.
4. Family names (Miss Hazama, Mr. Yamada, etc.)
5. Children
6. Words such as graders, pupils, and persons who do..., etc.
7. A, B, C, etc., using variable

Each group of words may be divided into male, female, and neutral.

The distributions of the words used in both textbooks are shown, respectively, in Tables 1 and 2 of appendix.

Some comparative results are as follows:

1. For the 1st grade in both elementary textbooks, personal names, "brother", "sister", etc., "boys", "girls", and " children" are used a few times but not family name and variable.

2. Personal names appear many times in the elementary textbooks A and B, except for 5th and 6th grades of the textbooks A. On the other hand, personal names do not appear in both the lower secondary textbooks A and B.

3. "Brother", "sister", etc. and "boys", "girls", etc. appear in each grade of two levels in textooks A and B; particularly in the upper grades of the elementary textbooks they are used many times.

4. Family names appear in 5th and upper grades and they also are used to call persons without distinction of sex in the textbooks A. On the other hand, in the textbooks B family names except for mathematicians do not appear in any grades.

5. "Children," "graders," etc. are used in most grades of both levels.

6. A, B, C, etc.appear in 6th and upper grades for referring to the common and the neutral, but they are also used for referring to the male (Mr. A) or the female (Ms. B).

In conclusion

In this paper, we described many factors contributing to a gender unconsciousness, such as the implicit bias in education as a whole, the entrance examination bias, the partipicipants' bias (i.e., some problems come from outside mathematics education). Job, students' courses, marriage, etc. all have an influence on it . We must consider gender issues under those situations and then suggest looking at gender issues in relation to individuality.

We demonstrated that in mathematics textbooks, gender imbalance increases gradually and also that the textbooks tend to be gender free and social context free, especially in the lower secondary grade levels. Qualitative analyses are needed to clarify the stereotyped role playing – such as males playing baseball, females doing cooking, etc. This is a task to be analyzed at the next opportunity.

We hope this paper is not only an initial stage for deliberation on gender issues, but also a milestone in mathematics education in Japan.

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Appendix

Table 1 Distribution of words used for referring to persons in the textbook A for both school levels

	Elementary textbooks				Lower sec. textbooks				
	1	2	3	4	5	6	7	8	9
	M F	M F	M F	M F	M F	M F	M F	M F	M F
1) Personal names	3 3	25 37	50 32	8 26	0 0	0 1	0 0	0 0	0 0
2) Brothers, sisters, father and mother	2 3	8 9	6 4	17 26	16 5	25 2	32 34	2 0	5 4
3) Boys, girls, men and women	5 5	8 10	8 8	2 2	2 2	43 22	6 7	8 13	5 9
4) Family names	0 0	0 0	0 0	0 0	50 73 25	36*22 18	7 0	1* 5	2 2
5) Children	3	10	2	0	4	0	2	0	0
6) Words such as graders, pupils	0	12	2	0	15	0	2	20	0
7) A, B, C,	0	0	0	0	0	3 3 1	0	0	4 4 9

* means number of mathematicians are contained

Table 2 Distribution of words used for referring to persons in the textbook B for both school levels

	Elementary textbooks				Lower sec. textbooks				
	1	2	3	4	5	6	7	8	9
	M F	M F	M F	M F	M F	M F	M F	M F	M F
1) Personal names	17 15	15 20	38 37	36 57	77 50	49 37	0 0	0 0	0 0
2) Brothers, sisters, father and mother	1 0	4 3	3 4	1 4	5 7	12 8	9 2	2 0	1 1
3) Boys, girls, men and women	3 3	3 3	3 4	4 4	0 0	28 5	0 0	12 6	5 5
4) Family names	0 0	0 0	0 0	0 0	5*0	1 0	8*0	0 0	0 0
5) Children	1	7	3	4	5	6	2	0	1
6) Words such as graders, pupils	1	9	6	17	25	26	10	9	5
7) A,B,C,	0	0	0	0	0	0 0 1	5 0 1	3 2 1	4 2 4

* means number of mathematicians only

THE CONFIDENCE FACTOR – INTERVENTION STRATEGIES DESIGNED TO ENCOURAGE POSITIVE ATTITUDES TO MATHEMATICS

Lesley Jones,
University of London
Teresa Smart,
University of North London

Introduction

In the many studies undertaken on pupils' attitudes to mathematics, confidence has been identified as a contributing factor to gender differences in participation in mathematics. We know of a number of issues that affect confidence:

- Children's attitudes towards their own success in mathematics
- The phenomenon of "learned helplessness"
- Working with contextualised problems
- Working with (new) technology
- Teachers' style and attitudes

In this paper we discuss the "confidence factor" and the part it plays in pupils' learning of mathematics. We then outline intervention strategies that may encourage girls to develop confidence with mathematics, drawing on our own experience of using these strategies in Britain :

1. Residential conferences for girls in the first year of Sixth Form (17-year olds)
2. Female only classes introducing new technology
3. Involving girls in school-based research demonstrating their own confidence levels

Children's Attitudes Towards Their Own Success in Mathematics

Mathematical ability has long been seen as a yardstick for "braininess" and as such it is not seen as a socially acceptable ability to demonstrate in the school culture. Leder's (1980) research suggests that "bright" girls demonstrate "fear of success" in mathematical situations in far greater numbers than do boys. In Jones and Jones' study (1989) girls admitted that clever was seen as "square" or "bor-

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Gender and Mathematics Education, an ICMI Study

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ing." They felt they had to be very well established with a group of friends before they could admit to being good at maths. They noted that there were changes in attitude between primary school and secondary school; in primary school it was acceptable to be considered "good at maths," but within the secondary school culture it became necessary to hide such talents.

It may be that this phenomenon is not so apparent in single-sex schools, and there is certainly some evidence to suggest that girls achieve better in a single-sex environment (Myers, 1992). Myers suggests that mixed schools should attempt to find out what girls think about their school experience and attempt to use this information to create a more "girl friendly" environment. A report in 1987 found that when girls were taught mathematics in single-sex groups for the first three years of secondary schooling their achievements at O-level increased significantly, yet they still maintained a negative attitude to mathematics (Sexton, 1987). In contrast to this, the report of a research project in Australia (Rowe, 1988) indicates that single-sex grouping did not affect achievement, but positively affected girls' confidence in their mathematical ability. In higher education there is some evidence to suggest that single-sex groups are effective. Isaacson and Smart (1988) describe the experience of a group of women who gave up their study of mathematics and then, as mature students, were on a women-only conversion course into science and engineering. At the start of their course the women were asked to describe their feelings about mathematics and they used negative terms such as, "It was a nightmare – quite horrendous," "... not very positive ... because I wasn't very good at it." However, after a year of working in a collaborative way with other women, one reported that her feelings now were "positive. I enjoy mathematics and look forward to continuing with the subject on a chemistry degree. I have also found myself 'selling' it to friends and relatives."

The Phenomenon Of 'Learned Helplessness': Do Teachers Teach Girls To Lack Confidence?

Dweck and Bush (1976) use the term "learned helplessness" to refer to the phenomenon where failure is attributed to factors outside one's own control. They found that girls were more readily influenced by failure feedback from adults than were boys. The girls tended to attribute their failure to lack of ability, whereas the boys blamed their failure on lack of effort. However, when they were given "failure feedback" by their peer group, the girls showed rapid improvement and the boys showed no improvement at all. It is difficult to draw useful conclusions from this aspect of the research, and indeed Dweck and Bush do not draw any inference from this, but it does indicate that adult evaluators can have a profound effect on girls' opinions of their own ability and there is evidence of this elsewhere (Jones and Jones, 1989).

Perhaps, instead of learned helplessness we should be thinking in terms of "taught helplessness." Confidence is one of those personality traits that is double edged. Whereas it is usually considered to be a positive quality, it is also possible for one to be viewed as overconfident. Demonstrations of confidence are seen as acts of assertiveness, which may or may not be seen in a positive light. We also have a set of expectations about the way we expect people of different sexes to behave. A man who freely expresses his emotions or freely discusses emotional issues is considered remarkable or out of the ordinary. A woman who behaves in a very confident way is considered extraordinary. As women and girls we are taught to seek reassurance, particularly in specific areas of our life, such as academia. Isaacson (1989) provides a very interesting discussion of the way in which women have to tread a delicate path through life. She explores the notion of "double conformity," which refers to the dilemma one faces when having to conform to two conflicting sets of expectations simultaneously.

Apart from their ability to influence pupils' self-image, teachers are in a powerful position in relation to pupils. Walkerdine (1989) has shown that they may make judgments about the pupils' mathematical potential influenced by their judgment of the pupils' own level of confidence. In the British education system this can have a long lasting effect. Until 1988 pupils were entered at age 16 for GCE or CSE examinations. Universities and employers adopted an unfavourable attitude to the latter, yet teachers tended to enter more girls for this examination as they thought it was a better route for those girls who they perceived as lacking confidence in themselves (Walkerdine,1989). In the new GCSE examination, designed to put an end to the two-tier system, teachers are still in the situation where they decide at which level to enter the children. Entrance at a lower level eliminates children from reaching the higher grades. In 1991 83% of the 16-year-old cohort entered for mathematics GCSE: 88% of the girls and 79% of the boys in the year group. However, there were fewer girls entered for the higher tier paper (13% of all entries, compared with 14.1%), and a greater number entered for the intermediate paper (28.5% compared with 22.9%) (ULEAC/ NFER 1992).

Working With Contextualised Problems

There is considerable research evidence on the effect of context on the performance of boys and girls, the context significantly altering the relative achievements of the two groups. For example, Strassberg-Rosenberg and Donlan (1985) found that girls do less well than do boys in contexts that are seen as traditionally male, such as space and electricity. Eddows, Sturgeon, and Coates (1980) found that girls perform better on questions on the area of a dress and quantities for a recipe than when the same questions were set in the context of the area of metal for a template and a blast furnace. On the other hand, boys performed equally well in both contexts. This seems to be once more a question of confidence. How have we arrived at a situation where boys are (presumably)able to disregard the context, yet girls are phased by it?

Working With (New) Technology

Since computers were introduced into schools there seems to have been a tendency for boys to monopolise their use. Gribbin (1986) describes how a large proportion of primary teachers attending computer courses in one local authority are men (40%), despite the much greater proportion of women in the primary teaching force. Men are widely seen as the people with technological know-how, both in school and outside. A visit to any computer conference immediately confirms this impression!

Elkjaer (1992) puts forward the interesting notion that information technology is identified with symbolic masculinity. She suggests that girls are "guests" in this curriculum area, and that this can afford them some advantages. Boys are seen as "hosts," in that they feel a need to maintain their dominating position in a sphere defined by masculinity. She considers that boys need to take up a male stance in information technology, that it is threatening to their gender identity if they are not allowed to. Elkjaer uses pupils' comments to support her argument that girls recognise this need and accept it but do not interpret this as technical expertise and dominance. As the subject is not tied in with the girls' gender identity, they are able to develop without any anxiety. If this argument is accepted, then girls can be considered to have a more confident approach to the subject than boys have. I find the argument beguiling, though it does not explain why more girls opt not to continue with their studies in information technology.

Elkjaer goes on to say, "It is important that girls in computer science do not perceive boys primarily as dominating in terms of subject content, but simply as persons who need to show off or pose in the public sphere of learning." If we are aware of this when we use information technology in the mathematics classroom then boys will not be able to set the agenda and pace of work. Conditions will be set up that permit girls to work in ways that they enjoy and feel confident with and, as Elkjaer says, "to walk around and look at the progress of each other's work and help one another."

Teachers' Style and Attitudes

Jones and Jones (1989) showed how much significance was attached to the teacher's style and attitude to the children. The girls in the study specified this as a factor affecting their confidence in their work, though this appeared to be in terms of the teacher's personality and individual manner in working with children, rather than specific teaching strategies. Other studies have confirmed the significance of the teacher in girls' perceptions of their own mathematical ability (e.g., Rodgers, 1990). In particular, girls seem to be affected by the teachers' opinion of their mathematical ability. " She didn't seem to think I was very good at maths so I didn't work very much at it. With the first teacher she seemed to think I was better at it and I had less difficulty with her." "I worked very well with him because he would say my maths was good. He encouraged me" (Rodgers, 1990).

Burton (1987) suggests that we need to make a fundamental change in pedagogy if mathematics is to be made accessible to a larger group of people. She suggests moves towards collaborative work and language-intensive processes and an extension of open-ended, problem-solving work. Rogers (1989) warns that moves in this direction would need to take account of girls' previous experience (and areas of success) or they may leave girls in a worse situation than before. Having produced "fun" materials designed to interest the girls in investigations and problem solving, one teacher received the response that they were "too scary all together," not having a recognisable framework within which to work.

Different Levels of Confidence for Different Ability Groups?

The APU survey of 1980 found that gender differences in performance were minimal in most topic areas *except in the top ability bands.* Nearly all the differences they found were accounted for by the top 10 to 20% of attainers in most topics. Amongst 15-year-old pupils, 61% of boys and 39% of girls obtained the highest 10% of scores on the concept and skills tests. This imbalance is not matched by any corresponding difference at the lower levels. In fact, there are fewer girls than boys in the bottom 10% in most topics. Fennema (1980) has suggested that brighter girls are more inclined to show a lack of confidence towards mathematics than are more able boys. Jones and Jones (1989) confirmed these findings in their study, particularly with questions in an unfamiliar context. They found that girls had confidence in their female peer group, but had less confidence in their own ability. This is also shown with women who are relatively successful. Becker's (1990) research with women pursuing graduate study in mathematical sciences shows that even at this level women have less confidence in their ability, and she quotes from one who says, "Every now and again I think maybe I am a little smart. But I sure pull the wool over their eyes ... having to perform these acrobatics so that they won't find out that I am really stupid. I do consider myself to be just a hack ... If I were brilliant then it would be easy for me." By contrast, the men were confident and never appeared to doubt their abilities. "I just did well in everything. I mean in the math sense I wasn't pushed, I was alone, I always managed to get an A, I always managed to do the best in the class. It was just never a problem."

Mura (1987) studied university students following undergraduate mathematics courses. She found that the majority of students overestimated the final grade they would receive, but that the men's expectations were even more unrealistically high than the women's. In the course they were presently following, there was no significant difference in students' confidence in their ability to succeed, but a significant difference was found in their view of their ability to obtain a PhD. Mura concluded that women needed a higher level of confidence than men in order to embark on a PhD programme, but also that there are factors other than confidence that affect women's decisions about studying for a PhD.

Intervention Strategies

We have been involved (jointly and separately) in a number of different intervention strategies designed to bring about change in girls' levels of self-confidence. A brief description of three types of intervention follows.

1. Maths conferences for 17-year olds

Over a number of years we have run a three-day conference targeted at 17-year-old students who are still studying mathematics, with the aim of encouraging them to continue their mathematical studies. At higher levels of mathematics there is a smaller proportion of girls who continue to study the subject. (See, for example Her Majesty's Inspectors, 1989.) The conference provides an opportunity for the girls to consider a range of career options with a mathematical bias or for which mathematics is a useful qualification. We invite working women along to describe their jobs and give the girls the opportunity to ask any questions they want. During the conference there are maths workshops in which they have a chance to explore some mathematics "off the beaten track." These are presented, in an informal way, with opportunity for collaborative work and discussion. The students' evaluation of the conference has indicated that it has been successful in providing a forum for the girls to consider the possibilities open to them and that it has, in some cases, been influential in helping girls to continue with their studies in mathematics. They have enjoyed the informal style of the mathematics workshops. It has been shown that, "undue haste seems to have a disproportionate effect on girls in terms of reducing their confidence in, and enjoyment of the subject" (HMI, 1989). Girls at this conference benefit from three days to work on mathematics in small groups at their own pace.

2. Classes introducing new technology

Imagine 50 girls exploring transformations and graphs with graphic calculators. Participating in a girls-only graphic calculator workshop, they sat on the floor in a large conference room in groups of six, each with her own calculator but sharing their suggestions with the rest of the group. This exciting session was one episode of the work done with teachers to try to introduce graphic calculators into the mathematics classroom. Over a period of three years one of the authors used graphic calculators with 11- to16-year-olds in mathematics classrooms. Their use reveals some of the same gender differences as with other calculators and computers, but when used carefully they can encourage collaborative work and mathematical thinking and help girls to develop a confident approach to technology. Although the graphic calculator is small and private, much involved mathematical discussion develops and collaborative work can be encouraged even if pupils work individually. For example, an introductory task is to investigate the role of m and c in the equation y = mx + c. After exploring straight line graphs, the pupils work through a puzzle sheet. Soon it becomes sensible for two or three pupils to collaborate. As they find out that a single wrong graph cannot be rubbed out, one calculator becomes the "try out" calculator and the other is used for the final version.

Most 11- to 16-year-old pupils do not come across a graphic calculator before they meet one in the classroom. Girls are not yet disadvantaged. It is important that we learn from past experience, as it is well documented that boys dominate the use of computers at home and at school (Culley, 1988). The graphic calculator is private to the pupil, and one head of department commented that "girls in my school prefer working with the graphic calculator because of the privacy it affords. They can make mistakes without the fear of ridicule from the boy computer experts."

Boys seem to feel a need to conquer the instrument, asking,"What else will it do? I have done this; can we go on to something new?" The emphasis is on finding out about all the facilities (the product), rather than on using any one facility to explore an idea further (the process). However, the value of the calculator is not only in enabling the student to plot graphs of whatever complexity easily and quickly, but in using the graph-plotting facility to generalise about families of curves and being involved in mathematical activity.

3. Involving girls in school-based research demonstrating their own confidence levels

This study was undertaken with 13- and 14-year-olds in an Essex comprehensive school. Data were collected from all the children in the year group, then the findings were presented to groups of able girls. A discussion of the findings ensued. The research had three stages. Initially we accessed NFER scores for the whole year group, from when they had arrived in the school at the age of 11. In the first phase of the study the children were presented with just four question and asked to say whether they thought they could successfully answer them.

In the next stage we classified the results according to gender and compared the predicted success rates. The results are shown in Table 1.

Table 1 The percentage of pupils who thought they could answer the questions correctly (draw scores in brackets)

Question	Boys (73) %	Girls (87) %
1	50.6 (37)	32.3 (28)
2	71.2 (52)	54.0 (47)
3	68.4 (50)	74.7 (65)
4	54.7 (40)	54.0 (47)

Following this, the pupils were grouped according to their NFER scores to form three groups broadly indicating ability levels. The results were again collated, this time according to gender and ability. At this stage they began to look very interesting, with the phenomenon noted by Fennema (1986) being apparent; the more able girls showed lower levels of confidence than their average peers on three out of the four questions. On the other three questions there was a noticeable difference between the girls' and boys' predicted success rates. The boys appeared to be far more confident than the girls. Once the data was gathered and analysed we moved on to the next phase of the research. Girls from the higher ability groups were grouped into eights, and one of the researchers met with them. They were reminded about the questions they had been asked (some weeks prior) and asked to predict, for each question, how many boys and girls they thought would have predicted success. They did this first for the whole age group, then for their (able) ability group. Their estimates had to be reached by consensus, the whole group estimating a figure for their peers. They were then given the actual results and asked to compare and discuss them.

The results were fascinating. As a group the girls had far more confidence in their female peers than they had in themselves as individuals. The prevalent feeling appeared to be: "It's only me who is not very able. I am not as good as others believe me to be." By facing the girls with their own evidence, we hoped to convince them that their experience was a shared one and that confidence has an effect on performance.

Discussion

Confidence can be considered to be a belief in one's own ability. Experience tells us that it is not a constant factor, but changes according to circumstances and situations. It is not always easy to assess one's own level of confidence in a situation and much more difficult to decide on someone else's confidence. Personal experience suggests that those people who present the most overtly confident front are often those who reveal much personal uncertainty as we get to know them better. It may be that the signs we read as lack of confidence do not tell the real story. This becomes significant if teachers base wide-reaching decisions on their assumptions about pupils' levels of confidence.

The concern about girls' participation in mathematics has in recent years moved up the age group. Large proportions of girls opt not to study mathematics at higher levels. One possibility is that this may be based, not on lack of confidence in their mathematical ability, but on their confidence and enjoyment of other areas of study. In modern languages and other language-based subjects girls outstrip boys in their studies at all levels. Perhaps they are exercising positive choices to study subjects they value, rather than negative choices about mathematics.

One of the students involved in the women-only conversion course commented on the difference between teachers of the arts and of mathematics, "I had two English teachers who were absolutely brilliant ... It is as if liberal arts teachers have got enthusiasm for their subject and that's what made it seem to me that maths didn't have the potential for sparking the imagination and being fun" (Isaacson & Smart, 1988). This woman has now qualified with a diploma in electrical engineering. We hope that this illustrates how intervention strategies such as we discuss in this paper can ensure that women start to see mathematics as an enjoyable activity and one in which they can participate fully on an equal footing.

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AN INTERACTIONIST AND ETHNOMETHODOLOGICAL VIEW ON GENDER-RELATED DIFFERENCES IN TEACHER-STUDENT INTERACTION

Helga Jungwirth
Johannes Kepler Universität, Linz

The aim of this paper is to present an interactionist and ethnomethodological approach to gender and mathematics. This approach is not yet widespread within the research on gender and mathematics, but it is very promising as it offers a new understanding of the interaction processes in the classroom and thus provides a further explanation of girls' relations to mathematics. In order to clarify this approach, it is contrasted with the classical perspective from which gender differences in mathematics are viewed. The paper begins with a typical example of the teacher-girl interaction in the mathematics classroom. This example is analysed from the classical point of view (see for example Jungwirth, 1993a) and then from the interactionist and ethnomethodological position.

Context of the example: Tenth graders (aged about 16) in an Austrian grammar school solve application problems to the geometric sequence. Just before the dialogue below begins the students solve the problem: "How much does the value of the money go down in one year if there is a rate of price increase by 6%? How much of its original value has a certain sum of money got after 5, 10, or 20 years if prices rise steadily?" The whole solution is still on the blackboard. Now the following problem is going to be solved: "How much is the rate of price increase if in five years the purchasing power decreases by half ?" The girl Wilma volunteered to solve this problem at the board. (The original dialogue was spoken in German; for the transcription code, see Appendix.)

11 Teacher:	how shall we [3 sec p, Wilma comes to the board]
12	let's see. what must we do'
13 Wilma:	I don't know <u>this</u>. [points to the denominator of the
14	formula for the purchasing power $(1/1{,}06)^n$, which
15	is still on the board]

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16 Teacher:	yes or I can say I call the whole fraction q.
17	[3 sec p, Wilma doesn't write] yes' I can do this
18	can't I'
19 NBoy:	*(can you?)*
20 Wilma:	yes- [shrugs her shoulders]
21 Teacher:	well you must have q to the' [2 sec p] well
22	how many years have we had'
23 Wilma:	the n I know that is five years'
24 Teacher:	five', and what must
25	that be approximately' , the half means'
26 Wilma:	0 0 point five.
27 Teacher:	0 point five. , and therefore q is' [3 sec p, Wilma
28	writes $q^5 = 0{,}5$]
29 Wilma:	well the fifth root of the (.) [writes q =]
30 Teacher:	the fifth root of 0 point five
31	and this I will calculate for you. [2 sec p]
32	so point five' [4 sec p] and that is 0 point
33	eightyseven 0 six' [2 sec p, Wilma completes
34	$q = 0{,}8706$] so which rate of price
35	increase have we got approximately- [2 sec p]
36 Wilma:	ahm [coughs, 3 sec p]
37 Teacher:	how much is the rate of price increase now.
38 NBoy:	eightyseven percent-
39 Teacher:	no.
40 Wilma:	no. [shakes her head] well this is p divided by
41	a hundred- [looks to the teacher]
42 Teacher:	you can read it off can't you.
43 Wilma:	and this is then-
44 Teacher:	yes' [4 sec p] so the 0 point [3 sec p, he takes
45	the chalk from Wilma's hand]

The Classical Interpretation

In lines 13-15 Wilma is about to introduce her procedure. Apparently she is going to give a comprehensive answer. But she can make the introduction only. The teacher does not allow her to explicate her idea of the solution of the problem. He immediately contrasts it with his approach (16-17). This reaction of the teacher to the girl's attempt to solve the problem is typical. As we know, it is difficult for teachers to give attention to answers or contributions of girls and to think about them seriously. Wilma's waiting now and saying nothing (17) shows her fear and uncertainty. She is anxious because of her wrong approach to the problem and does not dare to go on. This is not surprising as girls' confidence in their mathematical abilities is not very high in general. From line 21 on the teacher directs the dialogue. We can see his guiding activities, for example, in lines 21-22, 24-25, 27, 30-31. The teacher undertakes the necessary calculation (31-34) because Wilma, as she stated before this transcript starts, doesn't have a pocket calculator. To direct Wilma in this way corresponds with his teacher role as well as with the male role in a talk or in a discourse with a women. Wilma answers his questions (23, 26, 29) as a good girl does. In general, girls adjust to the steps of task-solution set by teachers much more than boys do. The questions of the teacher for the rate of price increase (35-36, 39) can't be answered in this stage of the solution process. It is impossible to read off the value of the rate of price increase. But nevertheless Wilma tries to do so (40-41). The teacher, however, does not pay attention to her considerations (44) – apparently he is not interested in them. We know that teachers are not very interested in girls' considerations to mathematical problems, at all events they have less interest in them than in those of boys, because they do not suppose girls to be bright in mathematics, and because they are not convinced that their ideas would be worth being taken up. Wilma tries again (43), but then does not say any more, probably because she is anxious not to say something wrong. From now on (line 44) the teacher does not give her a further chance to solve the problem. He does not expect her – a girl – to have the crucial idea. He takes the chalk out of her hand and begins to write down the solution himself.

The basic idea of this analysis: The teacher does not assume her to be smart in mathematics because she is a girl and only in exceptional cases are girls really good in mathematics in the view of their teachers. They are assumed to work hard, but not to be bright (see, for example, Walkerdine et al., 1989). Thus, the teacher treats her in this way. Wilma herself behaves like a good girl: she is anxious, but tries to give correct answers and allows herself to be directed by her teacher.

This analysis refers to the conception that there is a gender-specific socialisation that takes place in the mathematics classroom. Every case – for instance, the episode presented here – gives evidence of this general law of gender-role socialisation. This conception means that the gender-role stereotypes are considered as essential schemata of orientation within the socialisation processes that affect a typical female or male relationship to mathematics. It is assumed that the female stereotype causes women's mathematics avoidance. It provides girls with the "wrong" models, it leads to "wrong" expectations on the part of the parents, teachers, etc., and to their "wrong" treatment of girls, and it makes the girls themselves think that a (successful) career in mathematics contradicts femininity. In this framework it is assumed that females (and males) are compelled to comply with the gender-role expectations; they apparently have no choice. This assumption shows a background of thinking the sociologist Wilson (1981) called the "normative paradigm" of sociology. Within this paradigm the interactive behaviour is determined by the role expectations to which the individuals are exposed because the individuals feel the need to behave in the way in which they are expected.

To assume a gender-role socialisation law is problematic in two respects. First, girls get a specific status: they are the victims of the gender-role expectations determining behaviour in general and that towards mathematics in particular. What happens in teacher-girl interactions is described by terms such as prejudice against girls, discrimination of girls, and so on. Feminists and critical psychologists (Haug, 1980, Thürmer-Rohr, 1992) point out the problem that arises from the victim-model: namely, how victims – or objects – of an action can change into autonomous active subjects of an action. A second problematic aspect is that analyses or interpretations of empirical results based on the conception presented above show a tendency to remain within the framework built by the gender-role stereotypes. If one starts with the general law of the gender-stereotyped socialisation and considers the world determined by this law then there is the risk that gaining insights is limited by these stereotypes.

The Alternative Interpretation

Obviously Wilma is about to introduce her procedure. She starts by stating what is unknown (13). In that she refers to the problem solved just before; one doesn't know the denominator of the formula for the purchasing power drawn up a short time ago. The teacher, however, does not give her the opportunity to do so. He immediately presents his alternative ("yes or I can say I call the whole fraction q", 16-17). This does not happen by chance: at the very beginning the teacher gives a hint at his approach to the problem by the utterance "how shall we" (1); it might be continued with "call (the unknown quantity)". Wilma has not paid attention to this hint. By the linguistic construction "or" Wilma's approach remains a correct one. Nevertheless, it is rejected by the teacher without giving reasons. It might be that he has not yet understood it when he changes to the way he himself has figured out. Wilma obviously hesitates to continue writing (17). By that she looks as though she could not follow the teacher's suggestion. Simultaneously, she forces the teacher to refer to the shared knowledge with regard to abbreviations (17-18). Her following "yes" (20) accompanied by a shrugging of her shoulders shows that she agrees with the teacher as to the abbreviation but does not know how to solve the problem from this starting point. The teacher now has his way by strictly directing the solution process (21-22). He does not succeed, however, in dissuading her from her original procedure. Her answer "the n I know" (23) to his question shows that she still adheres to her concept: an n appears only in the formula for the purchasing power, not in the teacher's utterance. The teacher undertakes the necessary calculation (31-34) because Wilma, as she stated before this transcript starts, does not have a pocket calculator. At this moment of the solution process it is impossible to read off the value of the rate of price increase. Thus, Wilma cannot answer the questions of the teacher referring to this (34-35, 37), but she does not try to do so either (i.e., to guess the value). She still muses over her original procedure; both the teacher and the student go their own ways. She looks as if she developed her concept to get a term for the value of the rate she could draw from ("well this is p divided by a hundred", 40-41, "and this is then", 43). Alone she does not succeed however, and the teacher doesn't take notice of her attempts. Finally, the teacher himself carries out the necessary transformation. The fact that he begins to write could be a reaction to her hesitating – he wants to finish this task now. But his reaction could be interpreted in another way too: it could be that he has seen that he was wrong when he asked her to read off the value of the rate and now wants to correct this mistake without the students perceiving this.

In this analysis situative aspects play a much more important role than in the first one. It is essential from the point of view underlying this analysis to pay attention to the interactive development of the dialogue. As can be seen in this second interpretation, I did not derive her or his behaviour from general laws concerning the gender relation or the relation of women and mathematics. This means that from this point of view she is no victim of his gender-role guided behaviour of internalised female role-expectations. For this analysis the following considerations are important: The teacher has got his solution in his mind, the girl Wilma has her's. Both of them adhere to their approaches figured out at the beginning – not only the teacher, but Wilma also. She does not take notice of the teacher's hint, but persists in her concept. The teacher does not succeed in sidetracking the girl. Thus, teacher and girl fail to adjust to each other.

This scene illustrates the "emerging of failure" and the "authoritative insistence on the desired answer" that I have reconstructed as teacher-girl-specific modifications of the interaction in the mathematics classroom (Jungwirth, 1991).

The interactionist and ethnomethodological approach ...

I want to present now the theoretical background for this analysis. In order to make it clear I will go far back. The bases are microsociological theoretical approaches stemming from constructivist scientific theory: I mentioned them in the introduction in my paper: symbolic interactionism and ethnomethodology (Blumer, 1969, Garfinkel, 1967, Goffman, 1974, Wilson, 1981). Symbolic interactionism is based on the assumption that people construct subjective meanings for things, but nevertheless arrive at a common reality and at a knowledge that is universally valid. Social interaction is where this happens. Blumer sums up the starting point as follows:

> The first premise is that human beings act toward things on the basis of the meanings that the things have for them. .... The second premise is that the meaning of such things is derived from, or arises out of, the social interaction that one has with one's fellows. The third premise is that these meanings are handled in, and modified through, an interpretative process used by the person in dealing with the things he encounters. (p. 2)

In other words, people show each other the meanings of the objects, adjust these meanings according to one another's reactions, stabilising or modifying them at the same time. As a consequence, all that appears as an objective social reality is just constructed in the mutual perceiving and acting. Social interaction establishes what we call our reality. This is also what ethnomethodology says, but ethnomethodologists say something else – that social reality appears as a given reality is a result of the describing of everyday affairs while managing them:

> The activities whereby members produce and manage settings of organized everyday affairs are identical with members' procedures for making those settings 'accountable'... By his accounting practices the member makes familiar, commonplace activities of everyday life recognisable as familiar, commonplace activities (Garfinkel, 1967, p. 1ff).

The research interest of the ethnomethodology is directed to the methods the members of society use to do so.

... and its relevance for teacher-student-interaction

What is the meaning of this general framework with regard to the issue of this paper? Teachers and students construct subjective meanings of the themes and situations in the mathematics classroom. It might be, and probably is, that they misunderstand each other, as the teacher and the students interpret classroom situations under different perspectives because of their differing life experiences. For example, a teacher turns to a mathematical subject from the official mathematical perspective, the students do not. The action of the students, as well as that of the teacher, is the one which is most reasonable in the given context from their perspective. Of course, teachers and students are not empty vessels with regard to nonmathematical issues. They have built up their conceptions of what is typical for a teacher, for a student, for a mathematics classroom interaction, and for females and for males.

Let me turn to the last aspect first. Are these not the gender-role stereotypes from the first analysis? There is a great difference in this respect compared with the approach I presented before. Gender, too, is seen from an ethnomethodological perspective (Kessler & McKenna, 1978). This means "that a person's gender is what they are regarded and treated as, that is, the gender of someone is the same as the gender attribution which is made about them" (Kessler & McKenna, 1978, p. 39). Gender attribution to a person is a complex interactive process in which this person presents her – or himself in a specific way and the other interprets the presentation. This ethnomethodological perspective does not provide gender-roles forcing female and male behaviour. It means that gender roles are not, or at most partly, the cause of gender-specific behaviour; they are, on the contrary, the result of females and males behaving in specific ways, and they can only exist if they are established and confirmed in everyday interaction.

In my second analysis I do not need to go back to conceptions about gender-specific behaviour in order to explain what is going on. From the interactionist and ethnomethodological perspective another aspect is much more important.

As I already mentioned, teacher and students might misunderstand each other because they construct different subjective meanings of the things. There is the risk that the interaction breaks down. But in general it runs rather smoothly: Teacher and students act routinely in a way that the common interaction is established (Jungwirth, 1993b, Voigt, 1984, 1989). One could say that teacher and students implicitly know what to do in order to maintain the classroom discourse. For example, it goes without saying that the teacher gives hints in order to direct the students towards the correct solution of a task, and that the students pay attention to these hints. Both – students as well as the teacher – contribute to the actual discourse. At least in the German-speaking countries, the discourse common in the mathematics classroom can be described by the the tripartite scheme of teacher question – answer of the students – evaluation of the answer of the students by the teacher. In detail, several interaction patterns have been reconstructed (Bauersfeld, 1978, Voigt, 1984) – for example, the "funnel pattern" in episodes where a failure of a student is corrected; in the funnel pattern the scope of possible actions of the student and of the teacher is restricted step by step.

Girls tend to use "improper" participation methods

The salient point in the interactionist and ethnomethodological argumentation is the action of the girls in the common classroom discourse. In the scene presented here there is not such a smooth running interaction. Teacher and girls go their own ways, they do not come together. The girl Wilma plays an active role in this – she is not flexible enough to turn to the teacher's problem-solving approach. If we assume that her behaviour is the most reasonable from her perspective, this means that she has got a different conception of how to engage in problem solving at the blackboard, different from that fitting in the common classroom discourse.

The findings of my research on interaction in the mathematics classroom (Jungwirth, 1991) show that Wilma is not a single case. I have reconstructed a tendency of girls to use deviant methods of participation. Teacher and girls as well as teacher and boys can modify the common running of the interaction; the modifications established in the interaction between teacher and boys can be regarded as variants of the common patterns because the basic structure "one adjusting to the other" of the tripartite sequences is maintained. Modifications established in the interaction between teacher and girls, however, can be interpreted as clear deviations from these patterns because the adjusting fails; the smooth running of the interaction process is obstructed or interrupted. The deviant acting of the girls becomes a problem when the teacher – and fellow-students – as a consequence get the impression that girls are less competent in mathematics than boys because usually a student's contribution to a smooth running interaction process is a token of mathematical competence.

As the modifications depend on the gender of the students, the question might arise why it is the girls who show a tendency to deviant conceptions and methods of participation. An explanation for this can hardly recur to the gender-role stereotypes because girls use practices which obstruct or interrupt the smooth discourse. The result that it is the girls who do so is inconsistent with the expectations arising from the female stereotype. According to this, the ability to adjust is typical whereas it is typical for males to resist and to be obstinate.

The social world of the girls - an explanation for their specific way of participating in the common mathematics classroom

I want to present an explanation for this that is a sociolinguistic one. The main premise is that girls and boys have specific practices in talking situations. This means that they are accustomed to doing specific things with language and, corresponding to this, are used to treating subjects of conversation in specific ways. The development of these practices is assumed to take place in the peer groups of girls and boys. These have different goals and different structures (Maltz & Borker, 1982). To summarise: The starting point is a sociolinguistic subculture of the girls and a sociolinguistic subculture of the boys; in their social worlds they learn different speaking practices, different conceptions of how subjects are treated as a rule, and, therefore, how topics in a classroom interaction should be treated.

The social world of girls may be characterized in terms of closeness and intimacy. Girls play in small groups or in pairs. In their world above all girls learn to establish and to maintain relationships of equality and closeness. For this it is necessary to ponder the thoughts of the other group members, to cooperate, and to try to find the "true" meaning of the utterances of the others. In addition, it is necessary for every girl to consider accurately what she may tell the others. This means that an ability of thinking about problems quite alone is important. As Maltz and Borker (1982,) say: "Basically girls learn to do three things with words: (1) to create and maintain relationships of closeness and equality, (2) to criticise others in acceptable ways, and (3) to interpret accurately the speech of other girls" (p. 205).

In the social world of boys, posturing and counter-posturing are important. Boys play, in comparison with girls, in larger and more hierarchically organized groups. Above all, speech is used to do the following things: "to assert one's position of dominance, (2) to attract and maintain an audience, and (3) to assert oneself when other speakers have the floor" (Maltz & Borker, 1982, p. 207). This means that boys learn to present themselves and to meet challenges. In addition, they learn to make remarks and side comments by which they can attract the audience.

Having these social worlds in mind, it is not surprising that it's the girls-teacher interaction where a failure emerges – girls learn to ponder over a problem by themselves rather than to immediately react. To learn to interpret accurately the speech of others is not a real advantage of girls because they learn to do this in an intimate situation with which the classroom discourse has nothing in common.

In sum, the methods of successful participation in the common classroom discourse following the tripartite scheme – and this means guessing answers, immediately adjusting to the actual turns and new questions of the teacher, and so on – are more related to the conversational practices of the boys than those of the girls. Living in the world of boys is a better preparation for successful participation in classroom discourse than is living in the world of girls. This means, seen the other way round, that girls have more problems finding their way in the classroom, and this might have the consequence that they withdraw from mathematics as soon as they are no longer obliged to attend mathematics classes.

To summarise: The interactionist and ethnomethodological approach provides a further explanation of what happens in interaction in the mathematics classroom. From this point of view, the interaction processes, especially their girl-specific features, cannot be considered as a discrimination of girls – unless one would admit that girls discriminate against themselves. What can be said is that there is an incongruity in the world of girls and the common culture of teaching of mathematics. The observation that girls tend to fail to participate properly in the latter should be an additional motive to reflect the usual way of teaching mathematics – a claim various researchers have already set up.

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Appendix: Transcription Code

NBoy	not identified male student
'	very short pause (max. 1 second)
(a sec p)	pause lasting a seconds
´	raising the voice
-	maintaining the pitch
.	lowering the voice
<u>exact</u>	emphasizing
exact	drawling
(.)	unidentified utterance lasting max. 1 sec.
(one?)	inarticulate; supposed wording
(laughter)	characterizing environmental events and processes

GENDER DIFFERENCES IN THE SPECIAL HIGHER SECONDARY-SCHOOL MATHEMATICS CLASSES OF THE GYMNASIUM IN PRAGUE

Jiří Kadleček, Oldřich Odvárko, Jan Troják
Charles University, Prague

Introduction

Special mathematics classes were constituted in Czechoslovakia to support the development of mathematically gifted pupils and to prepare them for the further study of the technical or natural sciences.

The advantage of this form of fostering the pupil's mathematical talent is found primarily in the following:

- The pupils are subjected to a special kind of mathematics education mainly during their ordinary daytime school attendance, so that their extracurricular activity and normal development in other aspects need not be affected.
- A greater concentration of the gifted pupils in one class increases their learning efforts by provoking their natural competitiveness and thus speeding up the development of their respective talents.

In the mathematics classes that are established beginning in the fifth year of compulsory school attendance (i.e., from the age of 10), the number of hours weekly of mathematics, physics, and chemistry, as compared with the normal classes, has been increased considerably.

In the mathematics classes of the basic school (from the fifth till the eighth year of school attendance, pupils 10-14 years old), first of all the development of mathematical thinking is supported, together with the ability to solve the more complicated mathematical problems. The pupils are guided to search and process the information on their own and to work economically in teams.

The higher secondary school mathematics classes are organized within the framework of the so called "gymnasium" (of four years of duration) and attended by the basic school leavers (14 years old) after passing the talent entrance examination. The character of the education in these classes is determined by an overall orientation towards mathematics as a profile subject. The pupils are being pre-

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pared for university study of science and technical orientation, but, because the mathematics education contributes significantly to the development of the pupil's thinking in general, it is becoming a prerequisite to the study of any university specialization.

For the gymnasial mathematics classes, the separate programs and curricula used to be issued by the Ministry and special textbooks have always been published.

In the current school year, the situation is similar to that of recent years, except that the individual schools are much more free to alter the traditional organizational pattern, and it seems that in a few years not only the mathematics classes, but the whole school system, will undergo some transformations.

For a better understanding of the differences between the special mathematics classes and classes of all other types, we are introducing at least some important fragments of the programs valid at the present time. (For comparison with the programs formerly used, see Odvárko (1986) and Odvárko and Troják (1988).)

For normal gymnasial classes, liberal arts classes, science, and some other types (except for special mathematics classes), the education program has been determined by decree of the Ministry of Education, wherein the number of weekly hours of every subject in every year has been recommended with the possibility of changes within each of the subjects, provided that the total recommended hours during all the four years of study in that particular subject will not be decreased. In every year, a certain number of teaching hours has been given for the disposal of the headmaster to reinforce the recommended number of weekly hours in any subject (no matter whether the subject is a compulsory one or facultative) and so to increase the minimal quota given by the decree

	Year			
Subject	**1**	**2**	**3**	**4**
Mathematics	3	3	2	2
Physics	2	2	2	0
Chemistry	2	2	2	0
Computer Science	2	0	0	0
Facultative Subjects	0	0	2	6
Headmaster's Hours	3	5	7	9

Analogically also the program for the special mathematics classes was formed by the ministerial decree issued already in 1990 (the minimal quota of the mathematics hours is, understandably, higher). For comparison, we are presenting a part of the program formed at the gymnasium in Prague - Korunni St. (valid from the school-year 1992/93) in compliance with the above-mentioned decree of the Ministry of Education. The headmaster's hours were already redistributed and they are included in the numbers of hours below.

	Year			
Subject	**1**	**2**	**3**	**4**
Mathematics	6	6	6	6
Physics	4	4	4	0
Chemistry	2	3	2	0
Computer Science	2	2	0	0
Descriptive Geometry	0	0	2	2
Facultative Subjects	0	0	2	11

The curriculum of mathematics for special mathematics classes contains two parts in each of the four years. In the first part, the compulsory topics are determined; the second part contains further supplementary topics, with the recommendation that the teacher choose some in order to widen and/or deepen the compulsory part of the curriculum.

The compulsory part of the program differs only very little either in structure or in timing from the curriculum of the normal gymnasial classes. The differences can be found, though, in a certain widening of the content by which primarily the introduction of further methods used in the practical application of mathematics is pursued.

To illustrate how this is done in the present-day curriculum, let us mention at least the most important differences in contents:

- the Diophantine equations are added to linear equations;
- when dealing with the systems of the linear equations, pupils get acquainted with the matrix representation of the system, Gauss elimination method and Cramer's rule for the solution of the system of linear equations;
- in the framework of combinatorics much more attention than in normal classes is being paid to statistics;
- when learning about complex numbers, pupils encounter also the "funda-mental theorem of algebra" and simple examples of the functions of the complex variable;
- the composition of congruences is added to plane geometry;
- in analytic geometry, the notion of vector product is introduced.

The following supplementary topics are suggested by the present-day curriculum:

in the first year:	systems of more than 3 linear equations, special cases of the higher degree algebraic equations, elements of the group theory, Boolean algebras;
in the 2nd year:	functions of two real variables, non-Euclidean geometry, spherical geometry, introduction to graph theory;
in the 3rd year:	application of complex numbers, numerical solutions of selected kinds of equations, partial derivative and total differential, matrix algebra;
in the 4th year:	quadrics, differential equations, application of calculus in physics, elements of the axiomatic set theory.

In special mathematics classes, a much greater emphasis is placed on the axiomatic approach to the theories, on the understanding of basic ideas of the proofs, on the acquirement of more complex problem-solving methods, where the application of the formulae and/or procedures from several different theories are required and on the application of mathematics in the other natural sciences (physics in particular).

Study Achievements of Girls in the Mathematics Classes

At the end of the school year 1989/90, we carried out rather an extensive statistical investigation concerning the schoolwork results of the pupils of the mathematics classes at the gymnasium in Prague (Korunni St.). The data we had the opportunity to process, were all the results of the half-year classification of those students who actually finished the school, passing the final ("maturity") examination between 1973 and 1990, without taking into account the students who, for various reasons, left the school prematurely. We investigated the sample of 702 pupils, wherein there were 154 girls (i.e., 21.94%).

From the above-mentioned data, rather profound information has been gained on the fluctuation of the average result of the mathematics classification, the changes of the average of the overall classifications of all the pupils of one particular class as well as of all the classes in the appropriate period of the existence of special mathematics classes, changes and development of the classification in the run of the study from the first to the fourth year, the differences between the results achieved by boys and girls, etc.

To illustrate, we would like to present here only those results that we considered to be interesting from the point of view of the relation of girls towards mathematics.

In the table below, we scrutinize the distribution of the average results in mathematics on one side, and the average of the classification of all the subjects in the course of the entire four-year study in the mathematics class on the other side. The numbers are related to the whole period investigated, i.e., from 1969 until 1990.

Denotation:

Tm	the "threshold value" of the average classification in mathematics from the set of values {4; 3; 2.5; 2; 1.8; 1.6; 1.4; 1.2; 1.1; 1};
To	the "threshold value" of the overall average classification from the same set of values mentioned above;
G(Tm), B(Tm).	the number of girls and boys respectively the average mathemat ics classification of which is less or equal to Tm;
G(To), B(To).	the number of girls and boys respectively the over all average of which is less or equal to To;
P(Tm)	the percentage of girls in the total number of pupils, the average mathematics classification of which is less or equal to Tm;
P(To)	the percentage of girls in the total number of pupils, the overall average classification of which is less or equal to To.

Table 1

Tm=To	B(Tm)	G(Tm)	P(Tm)	B(To)	G(To)	P(To)
4.0	548	154	21.94%	548	154	21.94%
3.0	539	152	22.00%	548	154	21.94%
2.5	489	129	20.87%	539	150	21.77%
2.0	358	97	21.32%	436	132	23.24%
1.8	285	66	18.80%	343	116	25.27%
1.6	217	44	16.86%	237	82	25.71%
1.4	186	37	16.59%	126	48	27.59%
1.2	113	15	11.72%	50	15	23.08%
1.1	73	6	7.59%	28	5	15.15%
1.0	73	6	7.59%	2	0	0.00%

Table 1 displays clearly that the percentage of girls in the sample of all pupils, whose average classification in mathematics is less or equal to Tm decrease rather quickly when the value of Tm decreases, so that for Tm = 1.2 is the percentage of girls almost one half of the percentage for Tm = 2.

Observing the distribution of the average overall classification of girls in its dependence on the changes of To, the situation differs essentially. While starting with To = 2 till To = 1.2, the percentage of girls does not decrease any more, it is always higher than the percentage of girls in the sample of all pupils.

The next table shows this phenomenon from a still different point of view. The differences between boys and girls will appear even stronger, if we compare the value of the average overall classification with the value of the average classification of mathematics. As in Table 1, the numbers are related to the whole investigated period 1969-1990.

Denotation:

G'(To), B'(To). . . number of girls and of boys respectively the average over all classification of which is less or equal to To, while the average classification of mathematics is still smaller or equal to that value of the average overall classification - numbers of pupils who are "on average" better in mathematics than in all the other school-subjects.

P'(To) . . . . percentage of girls in the sample of all pupils, the average overall classification of which is less or equal to To, whiletheaverage classification of mathematics is still smaller or equal to that value of the average overall classification.

Table 2

To	B'(To)	G'(To)	P'(To)
4.0	218	33	13.15%
3.0	218	33	13.15%
2.4	216	31	12.55%
2.0	178	27	13.17%
1.8	149	26	14.86%
1.6	110	20	15.38%
1.4	64	13	16.88%
1.2	23	6	20.69%
1.1	13	2	13.33%
1.0	2	0	0.00%

For all the chosen values of To, the percentage of girls shown in the last column of the Table 2 is always smaller than the percentage of all the girls (21.94 %) in the whole sample of the 702 pupils under investigation.

Comparing the values B'(To) and G'(To) to the values B(To) and G(To) we can see that the expressions B(To) - B'(To) and G(To) - G'(To) have rather high values, very likely due to the fact that the averages of the overall classification can be strongly influenced by the classification of such subjects as physical training and some others that were formerly classified with a greater indulgence.

We would like to end this part of our information with expressing our conviction that the main contribution of our findings to the question of gender differences in mathematical giftedness lies in the fact that:

- our observation is not based on a particular test, but on the experienced teacher's assessment of the pupil's results in the course of his four-year study;
- the data used here are gathered from the rather long period of two decades.

The results presented here can be compared with those contained in Blum, Hensgen and Trest (1985), though the population under investigation is different; our observation concerns only a part of the secondary school population with well-established, above-average abilities in mathematics.

How the Girls and Boys View Mathematics

The results we have obtained from the above-mentioned statistical investigation have encouraged us to attempt to discover the causes of the differences displayed by the classification results of the girls and boys in the special mathematics classes.

For this purpose, we have used (almost without any change) the questionnaire for pupils from the publication of Wieczerkowski and Jansen (1989) with the kind consent of the authors, with whom we were involved in a consultative cooperation for many years. The aim of the questionnaire is to find out the personal attitudes of the pupils towards mathematics as such, as well as towards mathematics as a school-subject, and how this attitude is reflected by their respective relations to their peers and to the mathematics teacher.

We used the questionnaire in 1991 again at the special mathematics classes of the Prague gymnasium in Korunni Street. The number of respondents is shown synoptically by Table 3.

Table 3

Year	Number of Pupils	Number of Boys	Number of Girls
1	64	52	12
2	50	33	17
3	50	38	12
4	42	26	16
Total numbers	206	149	57

The first 15 questions of the questionnaire with the results in all the sample under investigation is presented in Figure 1, where we have used the same recording of the results as in Wieczerkowski and Jansen (1989). From the graph, we can read the information on the differences between girls and boys in the opinions in several items of the questionnaire.

a) For girls, in comparison with boys, mathematics means less as far as it concerns the results of the study of mathematics (ques. 2), their own future (ques. 3) or the significance of mathematics for their respective professions (ques. 6).

b) Girls find mathematics less fun as a school-subject than boys (ques. 1) and they are also much less interested in a mathematical extracurricular activity (ques. 11)

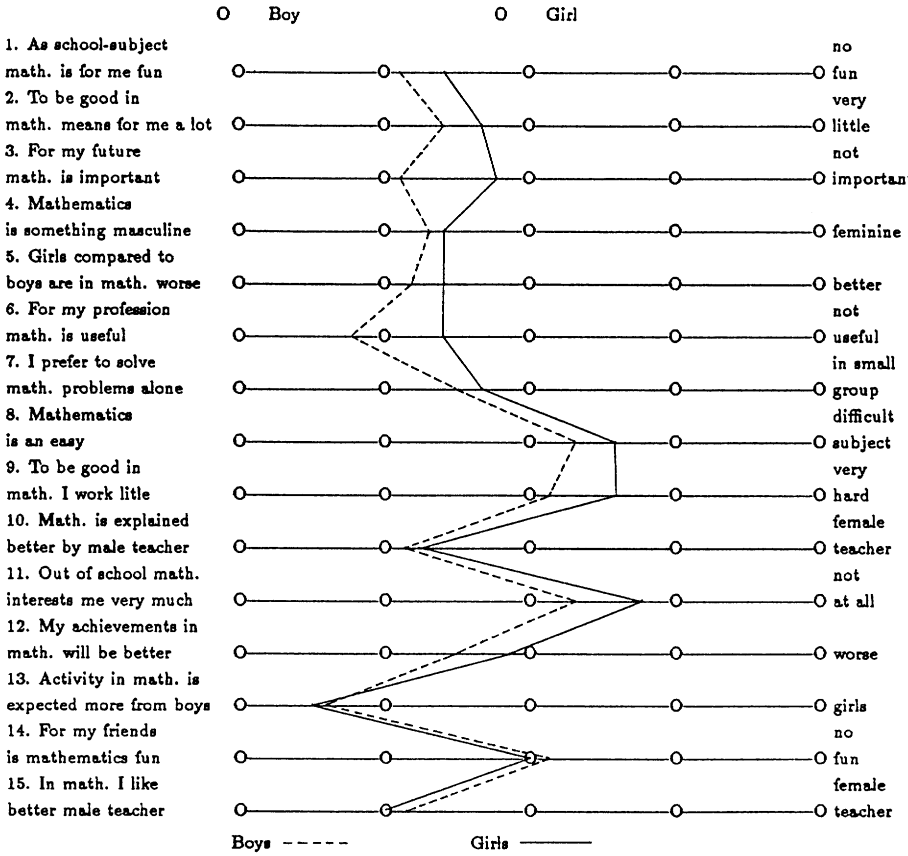

Figure 1

These results correspond also with the answers to the question 17 of the questionnaire, which had the following form:

If I had to decide now for a specialization I would choose:

History	English	German
Mathematics	Sport	Physics
Arts	Computer Sc	Religion

(The pupils were obliged to mark one of the items.)

In Table 4, we can see the answers in percentage (rounded to units) of the whole sample, as well as of the separate classes (B denotes the percentage of boys, G denotes the percentage of girls).

Table 4

	Total		1st year		2nd year		3rd year		4th year	
	B	G	B	G	B	G	B	G	B	G
Mathematics	42%	30%	64%	58%	36%	12%	32%	17%	19%	38%
Computer Sc.	14%	0%	8%	0%	15%	0%	18%	0%	19%	0%
Physics	17%	4%	8%	0%	21%	0%	18%	0%	31%	12%
Total	73%	34%	80%	58%	72%	12%	68%	17%	69%	50%
English	11%	32%	8%	17%	10%	59%	13%	33%	15%	12%
The others	12%	26%	10%	25%	15%	23%	11%	33%	16%	26%
No choice	4%	8%	2%	0%	3%	6%	8%	17%	0%	12%

The differences in the percentages of girls and boys on the choice of subjects like mathematics, physics, and computer science is noticeable on first sight. None of the 57 girls would like to specialize in computer science and also the percentage of girls choosing physics is much lower than that of boys. The girls prefer English and other liberal arts (58% of girls in total in comparison with 23% of boys).

c) Girls consider mathematics a more difficult subject than boys do (ques. 8) and to be good at it they have to work harder (ques. 9).

There are some items on the questionnaire to which the answers of boys and girls do not differ much;

d) Girls agree with boys that mathematics is a men's affair rather than a women's (ques. 4), that boys are better than girls in mathematics (ques. 5), and that boys develop more activity in mathematics than girls (ques. 13). Similarly, we can find the consent of boys and girls in the answers to the questions 7, 10, 14 and 15.

e) Girls are less than boys convinced that their results in mathematics will get better (ques. 12).

The certain lack of the girls' confidence in their abilities in mathematics is certified also by their answers to the question 16, which read:

> In the entrance examinations (to the mathematics class at the gymnasium) I succeeded, because
>
> I was lucky
> I worked hard to prepare myself
> I am mathematically gifted
>
> (The pupils were instructed to mark one of the offered items.)

The percentage (rounded to units) of the answers is presented in Table 5.

Table 5

	B+G	B	G
Lucky	54 %	49 %	68 %
Working hard	12 %	13 %	7 %
Gifted	28 %	35 %	12 %
No answer	6 %	3 %	13 %
Total numbers	206	149	57

In September 1993 we gave a similar questionnaire to all the pupils in the mathematics classes at the same gymnasium in Korunni st., Prague, intending to learn more about the differences between girls and boys in their changing attitudes to mathematics during their four years of study. The most interesting results were answers to question 8 of the questionnaire, presented in Table 6. As in Table 4, we can see in Table 6 the answers in percentage (rounded to units) of the whole sample, as well as of the separate classes (B denotes again the percentage of boys, G denotes the percentage of girls).

Table 6

	Total		1st year		2nd year		3rd year		4th year	
(%)	B	G	B	G	B	G	B	G	B	G
Mathematics	26	30	34	75	36	67	21	25	18	8
Computer Sc.	29	4	29	0	32	0	29	25	26	0
Physics	9	0	8	0	9	0	4	0	12	0
Total	64	34	71	75	77	67	54	50	56	8
English	12	22	4	25	5	0	21	50	18	17
Other	24	44	25	0	18	33	25	0	26	75

There is no "no choice" item in Table 6, because the questionnaire used now offered pupils many more possibilities to choose from than the one used before. Comparison of the results of both of the tables shows little difference;

- boys share an interest in both mathematics and computer science – a phenomenon which is rare amongst girls,
- interest in mathematics decreases in both groups, particularly during their third year of study, but the decrease in the girls' interest is greater.

The answers to questions 1 up to 7 yield no lead to finding the reason for the lesser interest in mathematics amongst girls;

- girls display a greater interest in mathematics than boys do in questions 1 and 6,
- only girls at the beginning of their third year consider mathematics more difficult than boys do, though it looks rather as if boys find mathematics far less difficult than girls (ques. 2),
- girls are generally highly satisfied with the tempo in which they are instructed in mathematics (ques. 3),

- girls' appreciation of the usefulness of mathematics grows from year to year (ques. 5),
- girls' expectations concerning the content of school mathematics is fulfilled much more than that of boys'(ques. 7).
- only in question 4 girls complain more than boys that mathematics is a time-consuming subject.

Hypothesis and Conclusions

Throughout the analysis of the results gained from the above-mentioned assessment of the 22-year period of the existence of special mathematics classes at Prague's gymnasium, supported by the results obtained using the Hamburg questionnaire, differences have been found between the attitude of boys and that of girls towards both – mathematics as well as learning of mathematics – while studying in the mathematics classes. Taking into account that all our conclusions are strongly related to the character of the data used, we are still convinced that the following assertions are justified.

We have investigated the population of 702 pupils, of which 22% were girls. If the hypothesis holds according to which

- boys and girls are equally talented in mathematics and
- the development of their talent as well as their interest in mathematics is devoid of the influence of their gender and
- the education of mathematics in special mathematics classes suits boys and girls equally well.

The same percentage of girls would be expected in each subgroup of pupils determined by the threshold value.

However, the data of Table 1 provide the ground for the rejection of such an expectancy and therefore for the rejection of the above-mentioned hypothesis itself. If accounting for the groups of pupils associated with the threshold values 1.2 and lower, the test by CHI-Square allows rejection of the hypothesis on the level of significance 0.005.

The teacher's classification (our main source of information), may not be considered reliable enough for such serious conclusions. We are of course aware of this fact. The thing we would like to draw the attention of the researchers to is that the validity of none of the three items of the hypothesis mentioned above can be taken for granted, that there is good reason to believe that at least one of these items is false and that, consequently, the following hypotheses are all worth further testing.

Hypothesis A

Mathematical talent is on average lower amongst girls than amongst boys. It would explain why when pupils face the learning of some difficult topics of mathematics, girls must expend too much effort to acquire it, they fall behind the boys, and their interest in learning mathematics recedes gradually.

Hypothesis B

The girls' concern is not so specifically concentrated in mathematics as is the boys'. The girls are interested in a much wider range of activity.
The cause of this phenomenon may lie in the following:

a) During attendance in the higher classes of the secondary school, girls undergo the process of adolescence, which in the girls' case evokes new specific orientations of interest.
b) Girls more than boys feel the need to be well classified in all the school subjects.
c) The technical applications (like computer science, physics etc.) by their extent and content are exceeding the girls' idea of mathematics as a school subject.

Hypothesis C

The school mathematics, primarily the school mathematics in the mathematics classes, is from the point of view of content, methods, and forms of presentation so constituted to better suit the motivational needs of boys.

Hypotheses B and C are supported by the ideas expressed in Stüven (1988) and Wieczerkovski and Jansen (1990). The verification of these mentioned hypotheses assumes, however, the realization of further systematic research, of the regular as well as the special mathematics classes during the entire period of study at secondary school, so that the different attitudes of girls and boys towards mathematics could be observed for a much longer period of time and the development compared in the samples where pupils display the interest and/or talent for mathematics with those where they do not. A rather extensive investigation of this type would require the cooperation of mathematicians and didacticians of mathematics with psychologists, pedagogues, and mathematics teachers.

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The Local Organising Committee in Sweden. Gerd Brandell, Bo Sjöström, Bengt Johansson, Barbro Grevholm, Bo Rosén, Göran Wanby and Lisbeth Lindberg. *Photo: Lennart Jonson ©*

BEYOND THE NUMBERS GAME[1]

Christine Keitel
Free University of Berlin

Three aspects will be addressed in the following paper: the image of mathematics in society, the virtue given to mathematics by media, common-sense speaking, and the general understanding of the use of mathematics; the image of mathematics as a discipline and as a social enterprise, provided by school mathematics through primary to secondary schools as well as at the university level; and, finally, the research on mathematics education, in particular in the field of psychological empirical research, and also in the section of gender and mathematics education. The question to be dealt with is about our obsession with numbers as a cultural and a social problem pervading nearly all aspects of our social life. In particular, whether the dominant reference to quantification is a misleading approach in studies on "gender and mathematics education" will be discussed, as it hinders the acceptance of qualitative studies.

In our culture, we are obsessed by numbers. That is, mathematics, it is said, is the mathematisation of our world. Is this so? In the public image of mathematics, mathematics is only about numbers, and numbers are everywhere, numbers are the data to which we refer, are the facts we use in argumentation (in her talk, although rejecting the empiricist approach in the psychology of mathematics education, even Elizabeth Fennema justified her arguments several times by comments such as: "There is no evidence in data that ..., clear data show that on average ..." or, "the majority of girls have deficiencies in spatial abilities ..."). Mathematisation of our society in most cases means an extended use of numbers. So the question of Philip Davis (1989) might well be justified: "Are we drunk on numbers?"

In primary school mathematics, the handling of the basic operations remains the dominant activity of teachers and pupils, numbers are in drill and practice tasks, in stupid (and rarely in enlightening) tasks from time to time, but hardly any other mathematics. The usual exercises as well as the so-called "applications" in word problems reduce mathematics to a numbers game only – find the operation and execute! Even when geometry is the focus, the calculating of perimeter or area is the major concern. In secondary mathematics, equations, or functions, or calculus are rarely analysed theoretically, but reduced to calculations, and linear algebra for most students at university level is just calculating matrices and so on. Most problems beyond the numbers games are excluded or neglected in the dominant practice of school mathematics.

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Most research in psychological studies in mathematics education as well as in gender-related investigations is data gathering, a kind of numbers game, the correlating of the numbers. The important step to an abstraction via formalizing, the step from the qualitative prolem to the quantitative representation of the problem in form of numbers and a structural relationship, is mostly taken for granted, the usual empirical approach. The transformation of problems into calculable problems and the possible change that had taken place by this transformation (Do we investigate the same problem when dealing with the calculable format?) is not questioned. Only results referring back to data are accepted. Is this the only kind of truth as research results? Do we notice that we might have got a solution for quite a different problem than that with which we started? Applied to the problem of "gender and mathematics education," this might have driven us in the wrong direction.

For a more detailed outline, let us begin with personal experiences that shape our perceptions in a deeper way than we might realize, but that also make the place and legitimation of our statements more visible and easier to follow. From Nel Noddings I have learnt that it is important to make explicit the subjective point of view that is the background of the argument. I must confess that when I was pushed into the mathematics and science branch of the German Gymnasium in the age of 15 because of my deficiencies in languages,[2] I did not see this as an advantage but as a restriction: I never had any difficulties in learning mathematics and science nor did I find it extremely interesting, exciting, or beautiful. School mathematics did not offer this excitement or show the beauty, but languages, philosophy, history, and sociology attracted me and had the image of contributing to a real humanistic education. This appeal remained during my school years until the final examination, the Abitur.

Making the decision about what kind of academic courses I wanted to take, the question arose: If you decide for mathematics and science, as a girl, you might encounter difficulties but you will get social appreciation by highly paid grants. I was persuaded to study mathematics and physics – by people who knew better than I what is socially acceptable – but I tried to keep on studying philosophy, sociology, history, and political science as well. The first years at university did not change the poor image that I held of mathematics and science. No insight into the creativity of mathematics was offered, just boring exercises, boring lectures and seminars. I also earned the hostile admiration of my student colleagues, who found it extremely interesting that a girl could succeed in university studies in mathematics and physics. In my first seminar after 1 1/2 years of university studies, I had to prepare a lecture in rather advanced applied mathematics, pushed into this task by a "caring" professor as my main "tutor." For the first time in my life I felt enthusiasm for the work I did. It had cost me several school years and nearly two years at university to acquire this experience, and, in my view, for most pupils it will never happen.

Girls do not specialize very early, but try to keep their choices open and get as many different approaches to knowledge and insight as possible. I found a lot of similarities in biographical data from my female colleagues. If interested in learning and studying, specializing in mathematics and science is often felt as a restriction, a reduction, a cutting off of very important questions they would like to follow as well. All this is caused by the two aspects I have already addressed: the very simplistic and poor image of mathematics in society and the superficial way it is used, considered, and analyzed; and the very poor image of mathematics one gets from school mathematics. If one decides to study mathematics only for career options, for the purpose of getting as quickly as possible into higher level academic positions, this restriction might be accepted. If the interest of university studies is to pursue certain interests and epistemological questions, the early school and university studies of mathematics offer a very reductive and restricted view and no answer to these kinds of questions.

It is obvious that mathematics is used nearly everywhere in our social life, to the good and to the bad. Mathematics is not only applied deliberately, its use is often taken for granted. Hardly anywhere in school mathematics is insight provided into the way mathematics can be or should be applied. No consciousness is developed to show that mathematics can be used in a restricted means, to cheat or trick, not as an empowering but as a dispowering means. The propaganda of mathematicians and mathematics teachers that mathematics is enlightening, creative, and beautiful, is rarely justified by the poor teaching they offer in all levels from school to university. Hardly anywhere is mathematics criticised, nowhere are applications of mathematics questioned. Mathematized arguments are true. Mathematization seems to be mainly quantification, in particular when going to the most modern tool we have developed: the computer. I would like to ask for going beyond the numbers game, focussing on qualitative arguments, not only on quantified ones, and to interpret quantified data by qualitative complementing studies. This heuristic view of mathematics has to be popularized in the media, and should be supported much more by mathematicians and mathematics teachers.

There is also a need to change the focus of mathematics teaching. There should be much more emphasis on the non-numerical aspects of mathematics, on the geometrical understanding, geometrical thinking, the arguing, the reasoning, and the trial-and-error approach that might be the base for many mathematizations: formalizing of action and language and creating abstractions. Pupils have to experience not only the "grammar" of mathematics but also the "literature" of mathematics. The literature of mathematics, its social use and its applicability, have to be discussed by social negotiation and contract (see Davis, 1989; Keitel, Kotzmann, & Skovsmose, 1993). Empowering pupils to participate in this democratic negotiation would be a much more reasonable goal for school mathematics than to just drill in and ask for admiration of mathematics, which, in fact, in school or university is experienced mainly as boring.

A further demand is to develop, or to add and integrate, new research paradigms into our research designs that do not rely only on empirical methods – statistical data and numbers should be complemented by qualitative analysis and hermeneutic interpretation of statements; qualitative arguments we find in comments or in protocols or in reflections of problemsolving procedures. There are already studies that make references to "telling stories": biographical stories, school stories about mathematics and about mathematics teaching that foster a critique of mathematics and mathematics teaching, provide an awareness of the way in which mathematics is applied and how to struggle with formalization.

Let me describe very briefly how I myself tried to pursue these three demands within a project concerning "girls with special interests in mathematical, scientific, and technological activities outside school." It was a project that took place in afternoon meetings at the Technical University of Berlin. A mixed group, consisting of 64 pupils from age 16 to 18 (mainly from the higher secondary, the Gymnasium branch), was assembled once a week in university seminar rooms for interdisciplinary work on "spacelab activities." Integrated mathematics, science, and technology teaching that centered on problems and constructional ideas was provided. At the end of the project, some technological machines – in analogy to "spacelab tools" – were developed. The teaching and learning were done in groups: of the 64 pupils, approximately 45 were girls, so that the boys were a minority; throughout the project the pupils were distributed into small working groups (every working group was a mixed group), and most pupils, for the first time in their school lives, worked together and discussed mathematics and science, as well as sharing ideas among boys and girls about technological constructions.

I do not intend to speak about the teaching and learning in these groups. Of course, we had specially interested girls and boys; of course, these extra-curricular activities were just an addition to the school curricula and an enrichment in the working style as well as in the learning environment when dealing with problems as the base for learning and teaching. What I want to report here is the part of my evaluation of this project in which I tried to find out by several different tools and means what image of mathematics and science is held by these pupils; which interests they want to pursue during their school, university, and future professional lives; why these pupils are specially interested in mathematics, science, and technology; and if there are other interests as well, and how to describe the difference in interest and achievement for those interested girls compared with others of the same age group. In particular, I tried to compare them with future teachers of primary schools. I used an extended questionnaire that included multiple-choice questions but left substantial space for additional comments, statements, biographical notes, reports, personal stories, and critiques.

After "working on the numbers" to create "the average girl" or "the average pupil" out of the group for this or that aspect of my evaluation, I concentrated on interpreting the comments, statements, and stories provided by the pupils. This I found most interesting and much more illustrative than the usual data gathering and calculating. First of all, I noticed that the stories of the girls were generally much richer than the stories of the boys. This might be due to the fact that all the participating boys had very early specialized in mathematics, science, and informatics and kept this special interest throughout the years without any special reflection or justification while reducing other possible interests; it might be also that boys hesitated to write "about" mathematics or about their thinking and feeling more than did girls. The difference was significant. To the end of their school years and even for their university studies and future professional lives, the girls did not like to specialize but tried to follow as long as possible a broad variety of interests.They described specializing in mathematics and science as a severe restriction and reduction of their possibilities and opportunities.

The participating girls as well as the boys had no difficulties with mathematics and science during their school lives, and they had positive, enriching, and, moreover, successful experiences with mathematics and science "beyond the numbers game." ("We talked about philosophies of mathematics and science; this was the most enlightening experience in my school life...") There is no time to give concrete examples, but these can be read in the study report (Keitel & Jablonka, 1992). When I then compared the stories and statements with the questionnaires taken from the future teachers in primary schools, their experiences were completely different and mostly discouraging. They hated mathematics, although they agreed about the everyday use of mathematics in their lives and took for granted and did not question at all what they disliked; they did not accept the numbers games but, nevertheless, aimed at following them obediently. Can they be good teachers in primary mathematics? How could we provide them with additional, different experiences that allow them to question, to criticise, and to reinterpret their learning of mathematics, and in this way to change their views and ways of teaching. By comparing and analysing the positive remarks of the "interested" girls and the negative remarks of the "disappointed" teacher students we might acquire more new ideas than just by collecting data on differences in achievement in mathematics and choices of mathematics.

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Notes

[1]The title of this presentation refers to the book by David Hamilton et al. (1977),"Beyond the Numbers Game: A Reader in Educational Evaluation," which is a strong plea for more qualitative approaches in educational studies. I use the same title to provoke considerations about the predominant approaches and paradigms when dealing with the question "gender and mathematics education": empirical methods, comparisons in terms of numbers looking for differences, and the predominant ways of conceiving mathematics education as "handling just numbers."

[2] This was due to the fact that I grew up in Eastern Germany, and the only foreign language I was able to learn was Russian.

IS THERE A FEMALE MATHEMATICS? A VIEW FROM THE NEW ZEALAND SUPERMARKET AND GARDEN

Gordon Knight and Gillian Thornley
Massey University

Introduction

A recent research study in New Zealand (Knight, Arnold, Carter, Kelly, & Thornley, 1992) was aimed at determining the mathematical needs of school leavers in everyday life, in employment, and in further education. In this study, samples of people were interviewed concerning the mathematical skills and processes they used in everyday life and employment tasks.

Although gender differences were not the focus of the study, the samples were balanced with respect to gender and, as might be expected, significant gender differences in the use of mathematics were found.

The majority of these differences seem to be the result of men and women undertaking different domestic tasks, having different hobbies, and being employed in different sections of the workforce.

However, there seems to be some evidence, particularly in the everyday life survey, that men and women tend to approach the same task involving mathematics in different ways. If this is the case, and the different approaches can be identified, the information would be helpful in the current debate concerning "female mathematics"and could influence the way in which mathematics is taught.

In this paper, the question of gender differences in approach to mathematical tasks is discussed in the context of the completed New Zealand study, and an approach to proposed further research in this area is outlined.

Gender Differences in the Use of Mathematics in Everyday Life

In the everyday-life survey of the New Zealand study the interviews were task-based rather than skills-based. That is, the interviewees were asked to identify tasks in their everyday life that might entail mathematics. The mathematics used in each task was then identified.

The table below shows the percentage of females and males who stated that they used mathematics in the tasks indicated.

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Task	% using mathematics	
	Female	Male
Supermarket Shopping	87	65
Gardening	46	74
Cooking	89	35
Knitting and Sewing	78	9
Painting and Wallpapering	22	84
Making Curtains	39	9
Building, Carpentry, etc.	7	67
Running a Car	39	79
Holidays	70	74
Sport	35	58
Gambling	9	26
Hobbies	48	60
Cheque Account	37	40
Tax Return	41	63
Mortgage, Investments, Superannuation	26	47
Other Financial Tasks	61	77

The gender differences are clear and reflect the gender role nature of New Zealand society.

The differences in task participation were reflected, inevitably, in the data relating to the mathematics used. The following gender differences in the use of specific mathematical skills were statistically significant.

	% of those interviewed using the skill	
	Female	Male
Use of calculator	50	74
Arithmetic with decimals	78	95
Use of percentages	28	58
Statistical calculations	2	12

These skills are not unrelated. It is quite likely that someone doing arithmetic with decimals will use a calculator and that someone involved in statistical calculations will use percentages, decimals, and a calculator.

The relationship of these results to those in the previous table is also clear. The highest use of each of these skills was in the tasks related to finance, such as cheque account, tax return, etc., all of which were more likely to be undertaken by men than women.

Gender Differences in the Use of Mathematics in the Workplace

In the workplace survey the interviews were also task oriented. The interviewees were initially asked whether, in the course of their current jobs, they had needed to undertake various tasks such as measurement, handle money, use a formula, etc. They were then asked to elaborate. A further set of questions concentrated on skills, and interviewees were asked more specific questions, such as "do you summarise data?" or "do you use calculus?". These questions were again followed up with more detailed questions.

The questions which produced the more striking gender differences were:

	% positive response	
	Female	Male
Have you ever in the course of your present job needed to:		
measure anything?	59	81
take readings from dials?	27	58
do any sampling?	13	39
understand specifications?	57	89
use formula to calculate something?	49	66
optimize anything?	69	91
	Female	Male
Do you ever use:		
Symbols or letters to represent numbers?	17	40
Trigonometric functions?	1	21

The only question that produced a major gender difference with a greater percentage of females using the task was

	Female	Male
Have you ever in the course of your present job needed to receive or pay out money?	74	43

It seems that these figures also reflect the gender role situation in New Zealand. New Zealand Department of Statistics figures indicate that about 45% of females in the workforce are employed in occupations classified as clerical or sales and service. The corresponding figure for males is 14%. On the other hand, 53% of males are employed in occupations classified as technicians, trades persons, plant and machinery operators, etc. compared to 20% of females. It is inevitable that this distribution will be reflected in the use of mathematics data.

Do Females and Males Approach the Same Task Involving Mathematics in Different Ways?

Although this information, which relates the gender differences in mathematics usage to gender roles in society, is useful and interesting, a further, perhaps more interesting, question arises. Do females and males, when engaged in the **same** task that might involve mathematics, approach it differently?

There are at least two factors that need to be considered in trying to answer this question.

Firstly, it was evident in the responses to the everyday life survey that the tasks involving mathematics were of two types. There were those tasks, such as filling in an income tax form, in which the mathematics involved is quite closely prescribed by the task itself. There were other tasks, such as supermarket shopping, in which a great deal of freedom was available to the subject, both as to whether or not to use mathematics, and in relation to the mathematical concepts and procedures to use.

In filling in one's income tax form, the only choices available are whether to do it oneself or ask someone else, and whether or not to use a calculator. Innovative approaches, such as estimation or suggestions that this is not the best way to calculate income tax, are unlikely to be welcomed by the tax authorities.

However, in supermarket shopping there is plenty of opportunity for choice. Do I need to know how much my groceries will cost when I reach the checkout? If so, do I need an exact or an approximate total? If an approximate total will do, how will I find it? What are the factors associated with the choices I make between brands and sizes of products?

It seems that differences of approach, which might provide useful information for the debate concerning the existence of a "female mathematics," are much more likely to be found in the second kind of task.

The tasks identified in the employment survey can also be classified in the same way. However, more of the tasks in employment seem to be of the kind where the mathematics is closely prescribed than is the case in everyday life. In many cases, the decisions such as what is to be measured and how, and whether or not data should be collected, are made at a managerial level and carried out by others. Similarly, whether or not trigonometry or calculus are to be used in an employment task depends at least as much on the task itself as on the person performing it.

There is, however, a second factor that is more evident in the employment tasks than those of everyday life, that is, the influence of the educational background of the subject. If we wish to draw conclusions concerning differences in approaches to a task by different people, we need to be sure that it would have been possible for them to approach the task in the same way.

For example, suppose the task were to find the height of a building and one subject used trigonometry whereas the other went to the top of the building, lowered a piece of string and then measured it. The difference would only be significant, from the point of view of the question we are discussing, if the person using trigonometry had access to the top of the building and to a length of string and if the second person had done some trigonometry.

Consequently any gender differences we can be reasonably sure are due to a different "female" approach to mathematics are likely to be found in tasks in which there is a considerable freedom of approach, that are undertaken by a significant proportion of both women and men, and in which the mathematics required for a number of different approaches is likely to be available to most people undertaking the task.

Two tasks, in New Zealand society, that satisfy these criteria are supermarket shopping and gardening.

Mathematics in the Supermarket and the Garden

Mathematics in the supermarket has been the subject of a number of studies (Lave, Murtagh, & de la Roche, 1984; Lave, 1988; Capon & Kuhn, 1982; Murtagh, 1985). Lave's book is particularly helpful because it provides a very strong theoretical background to the topic. However, none of these studies, it seems, address the question of gender difference. Capon and Kuhn's subjects were all female. Murtagh's sample was stratified with respect to age, education, and family size, but not gender, and there were 32 female and 3 male subjects in Lave's Adult Math Project. In the New Zealand study, 40 females and 28 males reported using mathematics in the supermarket and 21 females and 32 males reported using mathematics in relation to gardening, so it does seem that there should be the potential for identifying any gender differences in approach. The activities also provided a wide variety of responses, all of them based on relatively simple mathematical ideas that were likely to be available to most of the subjects.

The objectives and range of the New Zealand study meant that the time spent on interviewing subjects in relation to any one task was quite short. However, there did seem to be some anecdotal evidence of gender differences. The impression was that females were more innovative in their approaches to these tasks. The males seemed more likely to follow predictable "school mathematics" procedures.

In the supermarket, for example, the mathematics used related either to some sort of "total- cost" calculation or estimate of the cost of the groceries in the trolley on arrival at the check-out, or some form of "best-buy" decision, particularly in relation to product sizes. It was in the second of these that the innovative approach of some women was apparent. All the males, and most of the females, based their "best-buy" judgements on comparisons, in a variety of ways, between weights or volumes of products and their prices. It is very likely that this would be the expected approach in a mathematics classroom. However, one woman, in a pilot study, rejected weight or volume as the important factor in "best-buy" calculations. She preferred meal size. For her, the best packet of meat to buy was the cheapest one that would provide a meal for her family. The fact that a larger packet had a lower unit cost was unimportant if it would only last one meal. Similarly, a large, low-unit price packet of biscuits might be uneconomic since "they will eat them all at one sitting anyway."

Another woman, in discussing this question, agreed that unit price was an important factor, but wanted to include some geometrical constraint as well. It was no use buying a large box of soap powder if it would not fit on the shelf or a large bag of rice if it would not fit in her container.

In the garden, the mathematics used related principally to measurement, either in relation to mixing of sprays or fertilizers or to the setting out of the garden. In mixing sprays or fertilizers, instructions were usually given on the packet and these were followed, either approximately or precisely, using a variety of measuring devices, such as spoons, measuring jugs, milk bottles, lids, etc. In garden or plant layout, however, much more freedom was available. The methods used can be classified into three groups. Some people used formal measurement with a tape or ruler, either directly or transferring the measurement onto a stick or piece of string. Some used estimation, judging distances by eye, and some used informal measures, such as hammers, trowels, hands, strides, etc. The same number of females and males reported being involved in setting out gardens or plants, and again there seem to be gender differences in approach.

	% of respondents	
	Female	Male
Formal measurement	0	35
Estimation	71	41
Informal measurement	29	24

Further anecdotal evidence, not from the study, is provided by the wife of one of the authors who has an intriguing informal measure in relation to gardening. When asked how much gardening she did today she will reply "two barrow-loads" rather than "three hours," preferring an achievement-based measure of the quantity of weeds removed to the more obvious time-on-task measure.

Conclusions

The important feature of these results is that women seem to be making choices that are less bound by conventional school mathematics than those made by men. This is not because they have not understood the school mathematics. The women who chose meal size and shelf or container size as a major factor in best-buy calculations were aware of weight or volume as another factor. Similarly, it is unlikely that the women who preferred informal measurement or estimation over formal measurement in the garden were incapable of using a tape or a ruler. Certainly the author's wife is quite capable of telling the time. A more plausible explanation seems to lie in the nature of school mathematics and perhaps in the nature of mathematics itself.

This should not surprise us. Leone Burton (1992) in her ICME 7 presentation on "Is There a Female Mathematics or a Feminist Style of Doing Mathematics?" stated that "the message of many of the anecdotes which are recounted by and about mathematicians is that a shared experience can trigger different mathematics in different people." Sue Rosser (1990) in discussing gender differences in science states that one of the differences that women scientists show in observation is "expansion of the kinds of observation beyond those traditionally carried out in scientific research to include various interactions, relationships, or events not seen or considered worthy of observation by traditional scientists operating from an androcentric perspective" (p. 38).

Both of these views are, it seems, reflected in the anecdotes of the supermarket and garden. If the mathematics that is triggered in males in these situation is school mathematics, which is the product of many centuries of male thinking, and that of the females mirrors much more the open views of women scientists, this is hardly surprising.

Further Research

The research that gave rise to this paper was not explicitly designed to explore gender differences, and the evidence from it is far from conclusive. However, interesting and important questions have arisen that the authors hope to pursue in a further research study. It is very likely that the study will rely quite heavily both in design and theoretical background on the work of Jean Lave (1988). However, the research questions will focus specifically on gender as a factor in the use of mathematics.

It is hoped, and expected, that such a study could contribute useful information to the debate concerning "female mathematics."

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GENDER AND MATHEMATICS PERFORMANCE: A QUESTION OF TESTING?

Gilah C. Leder
Monash University
Peter Taylor
University of Canberra

In this paper data from the large Australian Mathematics Competition are compared with student responses from a separately administered questionnaire to examine the effects of presenting items in multiple-choice format on females' and males' performance in mathematics. Because of the continued widespread use in many countries of tests with multiple-choice items, the findings have broad applicability.

Introduction

Dissatisfaction with traditional, supposedly objective, and typically norm-referenced, tests has led to the construction, trialing, and introduction of more diverse modes of assessment in mathematics (Leder, 1992; Romberg, 1992; Victorian Curriculum and Assessment Board, 1993). Yet inspection of the relevant literature and materials reveals a continued widespread reliance, in many countries, on tests with multiple-choice items – particularly in large-scale testings and examinations. Pragmatic explanations for the continued use of such tests include ease and apparent objectivity of scoring. When data from these testings are publicised, reported differences in performance of groups of females and males are commonly considered evidence for gender differences in the mathematics ability of females and males, rather than as possible artifacts of the *method* of assessment used. Likely measurement errors introduced by guessing are rarely discussed. That the effect of guessing should not be discounted is reinforced by the following excerpt.

> Multiple choice questions ... usually require the candidates to do nothing other than tick whichever one of (say) four options they believe to be right. But even one question of this sort can be answered in five different ways: one right, one for each of the three wrong alternatives, and one more for omitting to do the question at all. A multiple choice paper of only twenty questions, therefore can be answered in 5^{20} ways which give the candidates just over 9.5×10^{13} possible combinations of response – only one of which is all correct! (Bentley & Malvern, 1983, p. 3)

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Previous Research

More than 50 years ago Votaw (1936) reported that students described as dominant omitted fewer questions on a test in which they were instructed not to guess than students classed as submissive. Male-female differences were not investigated directly in the study. However, evidence that males often show higher levels of dominance than females (e.g., Maccoby & Jacklin, 1974) justifies reference to this early study. Using different experimental designs and settings, Crandell and Rabson (1960), Kass (1964), and Slovic (1966) also reported that females tended to be more cautious, males more prepared to take risks.

Greater use by females, compared with males, of the "I don't know" option, or omission of items on multiple-choice tests have been reported by Anderson (1989), Arzi (1985), and de Benedictus et al. (1982). After reviewing relevant work, Meyer (1992) concluded rather tentatively that there was "a greater tendency by males to guess answers [which] can result in higher test scores independent of ability" (Meyer, 1992, pp. 179-180). Atkins et al. (1991) noted that the occurrence and extent of gender differences in guessing or risk-taking behaviour seemed to depend on the method used to define and measure risk taking. They hypothesised that females engaged in a higher *absolute* measure of risk taking behaviour than males, though over a lower *fraction* of the questions about which they were uncertain.

The likely consequences for students of obeying instructions not to guess have been described by Rowley and Leder (1989):

> If you have zero knowledge on any question, it is a matter of indifference ... whether you guess or omit. If you have any degree of partial knowledge, you would be better to guess than to omit, since your likelihood of guessing is better than chance and the penalty for being wrong is based on chance. If you are persuaded by test instructions and the threat of penalty to omit, you are acting against your own best interests. (p. 24)

Thus students who conform to instructions on a test or competition paper not to guess are likely to penalise themselves.

In this paper the case of the Australian Mathematics Competition (AMC) is used to explore strategies adopted by students to answer questions written in multiple-choice format, and with a penalty imposed for a wrong answer. Whether males or females appeared more likely to take risks and answer questions when they were unsure of the answer was of particular interest.

The Australian Mathematics Competition

The AMC is held annually. In 1992 more than 400,000 Australian students from over 2000 schools entered the competition (Pederson, 1992). This represents approximately one third of the country's secondary school students.

Three different papers are set each year: the Junior paper for students in Grades 7 and 8, the Intermediate paper for those in Grades 9 and 10, and the Senior paper for students in Grades 11 and 12. Some questions appear on more than one paper and are thus attempted by students in at least four grade levels. The papers contain 30 multiple-choice questions, each with five alternatives. The first 10 questions are worth 3 points, the next 10 four points, and the last 10 five points. Students are advised "to avoid random guessing as one quarter of the marks assigned for that question will be deducted for an incorrect response." In this way it is hoped that students will omit questions rather than guess answers, if they are uncertain.

An important aim of the AMC is to provide the average student with an opportunity to succeed at mathematics. Accordingly, certificates of credit or higher are given to the top 45% of students in each age group within each region. Outstanding achievement is also recognised. The top students are presented with a medal at a national ceremony.

The AMC and Gender Differences in Mathematics Learning

Participation

In 1992, more females than males entered the Competition in Grades 7 to 10, but not in Grades 11 and 12. At each grade level proportionately more males than females were among the best performers, that is, those in the top 1%.

Longitudinal AMC participation data show an interesting trend. In the earlier years of the competition more females than males entered the AMC only in Grades 7 and 8. Since 1990, more females than males have entered the AMC in Grades 7, 8, 9, and 10. As well, the male:female participation ratios for the highest grades have steadily decreased – from 1.52:1 for Grade 12 in 1983 to 1.16:1 in 1992.

Given the voluntary aspect to participation in the AMC, it could be argued that the increased participation rates of females indicate their increasingly positive attitude towards mathematics.

Performance

On average, in 1992 males did better than females on at least 65% of the questions at Grade 7 and up to 90% of the questions at Grades 9, 10, and 12. More males than females also did very well in the AMC. However, the data in Table 1 reveal that there were no appreciable differences in the number of questions on which the best males (those in the top 1%) did better than the best females at the same grade level.

Whether females and males adopt different strategies when attempting the AMC, such as obeying or ignoring instructions to omit questions rather than guess, and the likely consequences of this behaviour, are described in this paper.

Table 1: M/F performance on individual questions: 1992

Grade	N(M>F)	Total	Group		Top 1%
Q no.	1-10	11-20	21-30	All	All
7	4	9	7	20	14^{1}
8	5	9	8	22	13^{2}
9	8	10	9	27	13^{2}
10	8	10	9	27	14
11	9	10	9	28	16^{3}
12	9	10	8	27	14^{3}

Notes: 1. N(M>F) denotes the number of questions on which males performed better than females.
2. 16^{3} denotes that males did better than females on 16 questions and that there was no difference in performance between the two groups on 3 questions.

Method

A sample of 70 schools in two states was randomly selected from those which entered the AMC in 1992 and asked to participate in a study designed to increase our knowledge of student strategies on multiple-choice questions. An advance copy of the questionnaire to be administered was sent to these schools. The procedures to be adopted to ensure that student performance on the competition paper was not affected by the additional demands made on them were outlined. Over 70% of schools approached responded to our request. Questionnaires were received from approximately 4800 students. A small number had to be discarded, leaving 4765 usable questionnaires.

The Questionnaire

The questionnaire to be completed by students immediately after they had attempted the AMC paper asked students to indicate their school, grade level, and sex. They were further asked to show, for each question on the competition paper, whether they had read the question, answered it, thought they had got the answer right, and, if so, which of the other alternatives *might* also have been correct and which they had ruled out as a possible answer.

Results

In this paper, particular emphasis is placed on the results of one group, the Grade 10 sample. Because students need to make choices about their future mathematics studies at the end of Grade 10, this group was selected for particular scrutiny. The findings are reported under various headings.

Table 2: Male(Female) responses (in %), Grade 10 sample

Item	Q was read	Q was answered	Diff^{ce}	Answer is right	% Guess?
	M(F)	M(F)	M(F)	M(F)	M(F)
1	97(100)	97(99)	-(1)	94(96)	3(3)
2	97(99)	95(97)	2(2)	87(82)	8(15)
3	97(99)	94(96)	3(3)	84(83)	10(13)
4	97(99)	89(92)	8(7)	73(74)	16(18)
5	96(98)	88(86)	8(12)	71(68)	17(18)
6	95(97)	95(95)	-(2)	77(78)	18(17)
7	94(97)	85(82)	9(15)	62(53)	23(29)
8	94(96)	74(60)	20(36)	48(32)	26(28)
9	93(97)	74(68)	19(29)	49(32)	25(36)
10	93(97)	89(87)	4(10)	70(68)	19(19)
11	94(94)	86(84)	8(10)	62(64)	24(20)
12	91(91)	75(64)	16(27)	49(38)	26(26)
13	92(93)	87(86)	5(7)	64(61)	23(25)
14	91(93)	87(85)	4(8)	63(64)	24(21)
15	90(92)	76(72)	14(20)	52(50)	24(22)
16	90(91)	84(74)	6(17)	59(50)	25(24)
17	87(90)	74(64)	13(26)	48(39)	26(25)
18	87(88)	69(51)	18(37)	46(31)	23(20)
19	86(89)	66(60)	20(29)	41(37)	25(23)
20	85(90)	80(76)	5(14)	54(53)	26(23)
21	84(87)	56(48)	28(39)	33(26)	23(22)
22	83(87)	56(43)	27(43)	30(21)	26(22)
23	78(82)	41(25)	37(57)	21(9)	20(16)
24	79(83)	50(36)	29(47)	27(17)	23(19)
25	80(86)	60(53)	20(33)	36(28)	24(25)
26	76(83)	55(46)	21(37)	30(23)	25(23)
27	72(80)	39(25)	33(55)	16(10)	23(15)
28	73(81)	50(37)	23(44)	24(19)	26(18)
29	75(80)	50(40)	25(40)	25(19)	25(21)
30	77(81)	47(37)	30(44)	23(15)	24(22)

1. *Did students answer the questionnaire seriously?*
The percentages of students who indicated that they had read the question should decrease with an increase in number of the test question if students answered the questionnaire thoughtfully. As well, the percentage of students who indicated that they had read a particular question should be greater than percentage of students who indicated that they had answered that question. Details for Grade 10 are shown in Table 2. Entries have been rounded to the nearest whole number so that minimal differences are ignored.

Inspection of these data confirmed that both assumptions held. For example, 99% of Grade 10 students indicated that they had read Question 1, 94% that they had read Question 11, 79% that they had read question 21, and 80% that they had read the last question on the paper.

2. *How many females and males read each question?*
From the entries in Table 2 it can be seen that more females than males indicated that they had read the question for 28 items, with no difference in the apparent reading rates on the remaining two items.

3. *How many females and males answered each question?*
For the Grade 10 sample more females than males claimed to have answered the question on only four occasions, all within the first few items on the paper. There was no difference in the response of the two groups on one question. Thus, females apparently had higher omission rates than males on 25 questions. As females claimed to have read more questions than males, the discrepancies are even greater than appears from the entries in column 3 in Table 2.

AMC statistics for the total group indicated that in Grade 10 females actually had higher omission rates than males on 25 questions, with no difference in the omission rates on another 4 questions. These data further confirm that students seem to have answered the questionnaire honestly and seriously and that the sample was representative of the population.

4. *What proportion of students answered the questions they had read?*
As foreshadowed in the previous section, and illustrated in Table 3, a higher proportion of males than females answered the questions they had read: 28 questions for the Grade 10 sample. Discrepancies were generally larger for later question than those earlier in the paper. *Are males more confident, more prepared to guess an answer even if they are not sure, particularly on difficult questions?*

5. *How confident were students about their answers?*
More males than females indicated that they believed that they had got the question correct for 25 questions (see Table 2). The population data show that the actual number was 27.

For two questions (Q.7 and Q.17) the difference in the percentages of males and females who *actually* answered the question correctly was at least 10%. Nine percent more males than females *thought* they had answered these two questions correctly. At least 10% more males than females *thought* they had answered another

Table 3: Confidence ratio: [%(ans)/%(read)]/question, Grade 10 sample

Item	M	F	Item	M	F	Item	M	F
1	1	.99	11	.92	.89	21	.67	.55
2	.98	.98	12	.82	.70	22	.68	.50
3	.97	.96	13	.95	.92	23	.52	.31
4	.92	.93	14	.95	.91	24	.63	.44
5	.92	.87	15	.84	.78	25	.75	.62
6	.99	.98	16	.94	.81	26	.73	.55
7	.90	.84	17	.85	.71	27	.54	.31
8	.79	.63	18	.79	.58	28	.69	.45
9	.79	.70	19	.77	.68	29	.67	.51
10	.95	.90	20	.93	.85	30	.62	.45

six questions correctly. These perceptions were not confirmed by the population results. *Apparently males are more likely than females to overestimate their performance.*

6. *Were students prepared to guess?*

To answer this question it is assumed that students guessed if they answered the question, yet had not indicated that they thought their answer to be correct, that is, N(% who answered) - N(% who believed their answer was correct).

Using this definition, males seemed more prepared than females to guess – on 18 questions (generally those in the second half of the paper) compared with 9 (mainly in the first half of the paper) on which females had been more prepared to guess. There was no difference between the two groups in the "guessing rate" on three questions (see Table 2).

7. *Were students prepared to take risks?*

A "risk indicator" was obtained by comparing the percentage of students who answered the question with those who believed they had not answered it correctly. Details are shown in Table 4.

These data suggest that females were more prepared to take risks than males on 19 questions, with no difference in the risk-taking rate on another 3 questions. *Are there two groups of females: one quite cautious about answering questions of which they are unsure, and a second group less likely to omit questions and quite prepared to guess an answer? Or is the apparent higher risk-taking rate of females another instance of lack of confidence, and an example of diffidence about claiming that an answer is probably correct?*

Table 4: Risk (Guess) ratio: [(% wrong/% ans)]/Question, Grade 10 sample

Item	M	F	Item	M	F	Item	M	F
1	.03	.02	11	.17	.17	21	.27	.44
2	.07	.12	12	.24	.30	22	.34	.42
3	.09	.10	13	.17	.17	23	.34	.52
4	.16	.14	14	.16	.15	24	.32	.42
5	.15	.16	15	.21	.19	25	.27	.34
6	.12	.12	16	.17	.24	26	.33	.35
7	.18	.27	17	.20	.28	27	.46	.44
8	.26	.32	18	.20	.25	28	.38	.35
9	.26	.28	19	.24	.27	29	.38	.35
10	.12	.14	20	.20	.18	30	.40	.46

8. *What are the implications of greater risk taking behaviour?*
Students are awarded 3 marks for a correct answer on the first 10 questions of the AMC paper, 4 marks on the next 10, and 5 marks for correct answers to Questions 21 to 30. Corresponding penalties for an incorrect answer are 3/4 of a point, 1 point, and 5/4 points. Several hypothetical examples of the consequences of giving an incorrect answer on the total marks likely to be obtained on the AMC paper are shown in Table 5.

It can be seen that students who guess "sensibly" are likely to obtain a higher score than those who tackle the paper conservatively.

Concluding Comments

Data from the AMC, and responses to a questionnaire administered immediately after students had attempted the competition papers, were used to investigate student strategies when answering multiple-choice questions with a penalty imposed for an incorrect answer. Various indicators confirmed that students had answered the questionnaire seriously.

Males appeared more confident than females about answering questions on the AMC papers. Even though most questions were read by more females than males, more males than females typically answered each question and thought they had obtained the correct answer. More males than females appeared prepared to guess an answer, if guessing was defined as the difference between those who had answered the question and those who thought they had done so correctly. Yet more females than males seemed prepared to take risks if risk-taking behaviour was defined as the percentage of students who answered a question despite believ-

Table 5: Marks likely to be obtained

Questions	N(attempted)	N(correct)	Score
1-10	10	10	30
11-20	5	5	20
21-30	5	5	25
Total			75
1-10	10	10	30
11-20	10	8	32 - 2
21-30	10	7	35 - 3.75
Total			91.25
1-10	10	10	30
11-20	10	7	28 - 3
21-30	10	6	30 - 5
Total			80
1-10	10	10	30
11-20	10	6	24 - 4
21-30	10	6	30 - 4
Total			76

ing their answer to be incorrect. Collectively the data confirm the importance of the choice of definition used to describe risk-taking behaviour, and the implications of this choice for apparent gender differences in such behaviour.

Consideration of hypothetical examples confirmed that students' scores on the AMC papers would be enhanced by sensible guessing or risk-taking behaviour. The data reported in this study suggest that at least some females, as well as many males, have recognised this. Appropriate discussions with students attempting mathematics questions of the AMC paper format may encourage more females to modify their strategy and improve their *apparent* mathematics performance.

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MATHEMATICS AND GENDER. THE SITUATION IN PORTUGAL

Elfrida Ralha
Universidade do Minho

Portugal is one of the (few) countries where women still choose, as they did in the past, mathematics as a career. Portuguese society in general, and parents in particular, would encourage a girl who shows some interest in mathematics to follow a mathematics degree and subsequently to become a mathematics teacher, whereas a boy with similar interest would be encouraged to follow an engineering degree and thus earn more money. For years there were these societal and parental pressures on girls to choose mathematics, together with the fact that most mathematics teachers in schools were, obviously, women. Recently things have changed. We now have many men teaching mathematics in our schools, and more boys undertaking mathematics as a degree. However, girls are not yet choosing engineering degrees (an exception, as in the past, is chemical engineering).

This study investigates the reasons for this evolution in the mathematics population and it explores girls' and boys' attitudes towards mathematics in our schools (1st cycle, 3rd cycle, secondary, and university levels). The main methods were classroom observations, informal talks with pupils and teachers and written questionnaires (open-ended and multiple choice). The results were also treated from a statistical point of view, but my main interest lay in diagnostic aspects towards awareness of people directly involved with issues of teaching and learning mathematics in Portugal.

Background

In Portugal the school leaving age was recently raised from 12 to 15 years. We now have nine years of compulsory education in a school system basically divided into five stages: 1st cycle (6 to 10 years old 4 years), 2nd cycle (11 to 12 years old, 2 years), 3rd cycle (13 to 15 years old, 3 years), secondary (16 to 19 years old, (2+1) years), university or polytechnic level (4/5 years or 3 years, respectively).

The mathematics curriculum is either nationally dictated by the Ministry of Education or, at the university level, it depends on each faculty's decision. One must add that:

In the first four grades... mathematics is subsumed under general education... however, there is a mathematics curriculum set out which should be covered.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

Mathematics is a separate time-tabled entity from grade 5 on (4 to 5 hours a week allocated to mathematics).... Mathematics is (in Portugal) a subject particularly associated with failure and repetition... (Howson, 1991).

1st cycle teachers teach basic general knowledge in writing, reading and arithmetic. They normally follow up their pupils' evolution from grade 1 to grade 4 as they usually keep the same group of children for the whole 1st cycle instruction, thus becoming, totally responsible for each year's academic results of their pupils; however, most of the 1st cycle teachers to whom I spoke acknowledged having felt, while students themselves, difficulties with their mathematics at school. These teachers now have a scientific preparation that is equivalent to a polytechnic degree (3 years after secondary school).

2nd cycle mathematics' teachers are expected to teach not only mathematics but also natural sciences to their pupils. They do not, as a rule, follow up the mathematical evolution of the pupils as they are not expected to keep the same group of children from one grade to the other. These teachers often complain about their pupils lacking basic knowledge of mathematics (namely, arithmetic). Recently more and more men, who are mostly engineers and economists and who may or may not have done some in-service training as teachers, are into teaching this level of mathematics. In some cases, the teachers at this level have a mathematics degree from the university equal, except for in-service training year, to that of the teachers in secondary schools.

3rd cycle and secondary mathematics teachers only teach mathematics to their pupils; they frequently keep their groups of pupils for one year. Many complain about the lack of previous mathematical knowledge of their pupils and also about the lack of time to follow up with the whole national mathematics program as it is dictated by the government. They often leave out the geometry chapters of the programs. Many are looking for the opportunity to teach only upper grades (10th, 11th and 12th) although they offer no specific reason for this choice. They are supposed to teach, on average, 22 hours a week; more and more men (engineers, economists, etc.), although not as much as in the previous level of instruction, are going into teaching mathematics at these levels. Generally, these teachers used to be women with five year[1] university degrees in mathematics.

University teachers who teach mathematics are, now more likely than in the past to be women. In 1976 one of our main and best known universities had only men as full-professors. No woman was, at that time, a member of a "waiting list"[2], whereas now (1993), at the same university, there is a woman full professor and a large majority on that "waiting list" are women as well as PhD(s), MSc(s) and assistants; although women have traditionally chosen mathematics as a career and they have always taught mathematics at our schools, nevertheless, most university mathematics teachers in Portugal were men; that is, only a small percentage of the mathematics students were men but women, either because of personal decisions (mainly related to their roles as mothers and wives) or for other reasons (such as selection) that are not yet completely clear to me, decided not to follow mathematics as a research subject such as it is done by university lecturers. These university teachers who may or may not, for obvious reasons related to each one's specialisation, meet a group of students more than once during the students' university studies, also complain about their students' poor mathematical background, especially since a *numerus clausulus* selection was brought into practice a few years ago. One now finds more and more students entering the university in general, and mathematics degrees in particular, with no motivation whatsoever for studying mathematics, only because they were forced, by the numerus clausulus official restrictions to divert from, for example, medicine degrees. Some of these teachers also acknowledged that the numerus clausulus policy is not selecting students. It is, instead, ordering the students and the real selection is being left to the university 1st year lecturers who, in some cases, have to lecture to more than one thousand students at a time.

Methods

1. At the university, two groups of students were involved in the research:
200 1st year social sciences students (Sociology and Social Communication) who were, as expected, mainly women (3:1) and who were doing a mathematics course with me;
200 1st year mathematics students (Teaching and Computer Science) who were, also as expected, mainly women (4:1) and were also doing a mathematics course with me. These older students were also submitted to classroom observations and interviews and they answered some open-ended questions, which gave me, ideas for the design of the multiplechoice questionnaires referred to earlier.
2. Four young mathematics teachers - three women and one man - very much interested in analysing learning processes within their classrooms were recruited to be involved with this specific research of analysing gender differences in attitudes towards mathematics in Portugal. These teachers (3rd cycle and secondary levels) had said to me, at the beginning of this study, that they were unaware of gender differences: to them both boys and girls were doing equally well, that is, there was no specific differentiation related to mathematics knowledge such as it was measured by them at school. The schools they worked in were situated in the suburban area of a small university town (Covilha) in Portugal. The teachers and myself undertook classroom observations at these schools for more than one year, we interviewed several pupils and we asked them to answer a multiple-choice questionnaire. We chose for each teacher to work with:
- one group of 7th grade pupils (compulsory education), which gave a total of 120 children (4x30), and
- one group of 10th grade pupils, which gave a total of 100 adolescents divided

into 50 pupils (2x25) who had chosen mathematics related options at this level[3] and 50 other pupils (2x25) who had not chosen mathematics related options but, in spite of that, are doing a compulsory mathematics course.[4]
3. Finally a group of 8 mathematics students who were doing 4th year courses at the university, and were doing with me a course on Mathematics Education, were also involved in interviewing, observing and analysing 22 3rd grade (1st cycle) children and talking with their teacher.

Results

A short summary of the most typical answers and attitudes to some of the questions and observations that were made of the students will be now presented in order to clarify the conclusions reached during this research study. They were as follows:

I) Open Questions (University 1st Year Students)
Question: What is mathematics, for you?
Mathematics Female Students – Typical Answer(s): Art, A very good degree,Truth, ...
Mathematics Male Students – Typical Answer(s): Game, Very powerful weapon, Mental exercise, ...
Social Sciences Female Students – Typical Answers: Obligation, Logical madness, ...
Social Sciences Male Students – Typical Answer(s): Least interesting aspect of life, Technical competence, ...
Common Answer(s) to all Students: Science, Philosophy, Numbers.

Question: Which are the factors that, in your opinion, interfere in your mathematics success?
Mathematics Female Students – Typical Answer(s): Teacher, Effort, ...
Mathematics Male Students – Typical Answer(s): Luck, Ease, ...
Social Sciences Female Students – Typical Answer(s): Teacher, Study, ...
Social Sciences Male Students – Typical Answer(s): Motivation, Luck, ...
Common Answer to all Students: Intelligence.

Question: Why did you choose mathematics as a degree?
Mathematics Female Students – Typical Answer(s): Professional interest, Teachers' influence, ...
Mathematics Male Students – Typical Answer(s): Utility, Social status, ...
Common Answer(s): Family's influence.
Question: Why did you choose Social Sciences as a degree?
Social Sciences Female Students – Typical Answer(s): To understand people better, No mathematics, ...

Social Sciences Male Students – Typical Answer(s): To communicate within this selfish society, Utility, ...
Common Answer(s): Professional interest, To get a (university) diploma.

Question: How important is it, for you, to achieve good results in mathematics?
Mathematics Female Students – Typical Answer(s): Self satisfaction, To finish soon, ...
Mathematics Male Students – Typical Answer(s): To get better opportunities in future, ...
Social Sciences Female Students – Typical Answer(s): To achieve the goals of the degree, ...
Social Sciences Male Students – Typical Answer(s): Not important at all, ...
Common Answer: Parents' satisfaction, ...

II) Multiple-choice Questions (7th Grade Pupils)
Possible Answers: No, Maybe, Yes.
Sentence: Mathematics is easy.
Boys' and Girls' Typical Answer: Maybe.
Sentence: I get better marks whenever I study more (mathematics).
Girls' Typical Answer: Yes.
Boys' Typical Answer: Maybe.
Sentence: I study mathematics by myself.[5]
Girls' Typical Answer: Yes.
Boys' Typical Answer: Maybe.
Sentence: Mathematics is related to life.
Girls' and Boys' Typical Answer: Yes.
Sentence: Mathematics is only for some special people.[6]
Girls' Typical Answer: Maybe.[7]
Boys' Typical Answer: Yes.
Sentence: Mathematics program is very long.
Boys' and Girls' Typical Answer: Maybe.

III) Multiple-choice Questions (10th Grade)
Possible Answers: Total Disagreement, Disagreement, Maybe, Agreement, Total Agreement.
Sentence: If I am lucky, I get good (mathematics) marks.
Girls' Typical Answer: Disagree.
Boys' Typical Answer: Maybe.
Sentence: Portuguese language is crucial for learning mathematics.
Girls' Typical Answer: Agree.[8]
Boys' Typical Answer: Maybe.

Sentence: I do not mind about the mathematics marks I get.
Girls' Typical Answer: Totally disagree.
Boys' Typical Answer: Disagree.

IV) Talks with 1st Grade Pupils
Question: Do you like mathematics?
Boy's Answer: I love it, doing calculations is my favourite thing, ... I like money, I've got a bank account with the money I got for my birthday and I like knowing how much it grows, ... that's why I like learning mathematics, ...
Girl's Answer: I always get my multiplication tables wrong at school, at home I use my sister's calculator and I enjoy mathematics more than at school, ... I can never remember multiples and sub multiples of weight units, it is very complicated, ... but some of my friends do it very well, they help me (not the boys), ...

V) Talks With Mathematics' Teachers
Woman with a Mathematics Degree: I like what I am doing. It is very difficult to teach mathematics (much more than what we are prepared for at the university) but I can't imagine myself doing anything else. Curricular changes are too many, I think.
Man with a Mathematics Degree: I like mathematics. For most of my students, mathematics is very difficult to learn.[9] I do private tutoring because I acknowledge the students' difficulties with learning mathematics. Curricular changes were not that necessary.
Woman with an Engineering Degree in Environment: I like mathematics but I do not like teaching very much. It is very difficult being a teacher. As long as I have to teach mathematics I'll do my best, I feel I did not get enough preparation at the university for teaching mathematics properly, but the Minister gave us this opportunity for working and I appreciate it very much. It is very hard to keep the pace with these curricular changes.
Man with an Engineering Degree in Mechanics: For me teaching mathematics is a job not work. I do not care about curricular changes, mathematics is always the same.

Conclusions

The first commandment for teaching mathematics is, as Polya (1954) says: "To be interested in the subject". In Portugal it seems to be generally true that: (a) women, more than men, are interested in mathematics (subject), and (b) women, more than men, are also interested in (motivated for) teaching; women, more than men, are socially encouraged to go into teaching.
This research showed me that:
1) Women are persistent in doing mathematics, if they are supposed to do so, even

acknowledging lack of mathematical ability, when men tend to give up (*Students from the mathematics course for Social Sciences*).

2) Women are willing to invest in study and to work hard to overcome the difficulties, when men would rather spend more time with the computers (*Students from Mathematics as a degree and talks with teachers*)

3) Most women who teach mathematics decided to do so many years before getting their mathematics degree and they wish, just as in the past, to achieve goals at their work, whereas most men who teach mathematics came into teaching for many other reasons, they look at their teaching profession as a job (*Talks with Teachers*).

4) "My mathematics' teacher can be tender and can also be implacable, but she is fair, ... last year I had an economist (man) who could always be convinced to postpone the test." (*From talks with a boy also confirmed in its general sense by some girls of the same class.*)

I also came to the conclusion that girls are curious. (*From talks and classroom observation of children attending 1st cycle*).

At school, I saw boys and girls being treated equally by their (male) teacher; I heard him telling me that boys may be doing a bit better than girls in mathematics, but with a different group, in other years, it might be the girls. I read their mathematics books, where most of the exercises seemed to me to be very far from reality.[10] I understood that memorisation was crucial for learning mathematics such as it is supposed to be taught.

But I also noticed some differences: girls spoke loudly to me (an intruder), girls were willing to discuss their ideas with any colleague, girls shared their opinions, asked questions, ... girls seemed to be more mature than boys.

Girls show their feelings. (*From talks and classroom observation of pupils attending 3rd cycle*). Most "maybe" answers belonged to boys, whereas girls seemed to have a more precise opinion about mathematics. Girls help their mothers at home and, at the same time, they are the ones who get their mathematics homework done more often. Girls get good marks in mathematics, the same as the boys. Girls are willing to learn, even if they cannot understand what "transformation geometry" is good for, whereas boys will not study it. Girls trust their mathematics teachers, whereas boys are confident about their own mathematics knowledge.

Girls are proud but not arrogant. (*From Talks and classroom observation of pupils attending secondary school.*) When girls took a decision they were proud of it, whether it means having to study mathematics in a compulsory way (Letters and Literature) or not (Chemistry), but boys are not very kind to the teacher if they think they are not supposed to be studying this or that mathematics.

In Portugal we have more and more students coming to schools in general, and to universities in particular; we also have more and more women staying at the universities to do mathematics as a research career. There are more and more opportunities for people to go into teaching of mathematics at schools no matter what their initial specialisation or interest. We are seeing private tutoring in mathematics grow as a plague. Nevertheless, girls and women are still doing and enjoying mathematics as well as boys and men.

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Notes

[1] The scientific preparation of these teachers used to be 5 years divided into 4 years plus 1 year in-service training at a secondary school where the student was supervised by three people: one mathematics' lecturer, one lecturer from a Faculty of Education and a third supervisor who was a mathematics teacher at the school.

[2] These lecturers are only waiting for a full-professor's vacancy to be brought into concurs since they have already passed successfully all the stages and examinations that, in Portugal, one has to go through after getting a PhD diploma.

[3] These students were registered for the so-called areas of "Health" and "Chemistry".

[4] These students were attending the so-called area of "Letters and Literature".

[5] This sentence was one of a set related to the problem of private tutoring in Mathematics that recently is becoming more and more present in Portugal.

[6] This sentence was one of a set related to the problem of understanding whether children felt themselves to be or not special gifted for studying Mathematics; some of their teachers said to believe that only some of the children could do Mathematics successfully at school.

[7] Some girls (the ones who were getting the best marks in Mathematics) added that they felt special for knowing Mathematics.

[8] These girls explained to me, later on, that most mathematics they were doing at school was problem solving and therefore, the Portuguese language was important for interpreting the situation. They added that most of these problem that they were supposed to solve had nothing to do with life, but they were fun.

[9] One should note that the woman refers to difficulties related to teaching where as the man refers to difficulty in learning, which shows a completely different attitude towards each one's

responsibility to the results they get with their students.

[10] As an example one may say that Portuguese school children are supposed to learn by heart units of weight such as "quintal" (one hundred Kilograms) that nobody seems to use any more. The word also means yard.

ACHIEVEMENT AND PARTICIPATION IN A NATIONWIDE CALCULUS EXAMINATION

Barbara Reilly, Margaret Morton and Alan Lee
University of Auckland

At the end of their secondary schooling (year 12) most New Zealand students, who plan to continue to tertiary education, sit a nationwide examination in the five subjects they have chosen to study to this level. One of the subjects candidates may choose is "Mathematics with Calculus"; this study presents an analysis of the marks, participation, and types of mathematics skills relevant to this examination over a period of several years. Total marks are discussed on the basis of four effects: gender, school type, school authority, and whether a student also studied the second possible mathematics paper, "Mathematics with Statistics", at this level. The findings indicate that number of mathematics papers taken, school type, and school authority are all highly significant and possibly explain the observed gender differences. It is noticeable that the percentage of females taking both mathematics papers is much lower than that for males. Also included is an attempt to identify measurable relationships between certain characteristics of the questions and gender differences in performance in them. This shows that questions that require more preliminary analysis suit males, whereas those questions more reliant on acquired knowledge favour females.

Introduction

The School System in New Zealand

New Zealand has a centralised education system. Schooling is compulsory for all children from ages 6 through 15, although in practice almost all 5-year-olds are enrolled. Internationally this corresponds to years or Grades 0 through 10. After this there are two further years of optional schooling, years 11 and 12. Usually a pupil will attend three levels of schools; primary (years 0 through 5) for those aged 5 to 10 years; intermediate (years 6 and 7), for 11- and 12-year olds; and secondary (years 8 through 12), for those aged 13 through 18. A typical student will spend one year at each grade level and be taught with others of a similar age. Prior to 1986 less than 30% of pupils completed five years of secondary schooling but this percentage now seems to be holding at around 50%.

Three different types of school authority exist in NZ; state, integrated, and

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

private. The largest number of children by far attend a neighbourhood school controlled and financed by the state. Private schools receive some funding from the state and are attended by students whose parents can afford to pay the fees. Integrated schools are schools of special character (usually religious or philosophical) that were once private and are now state funded. A census count in mid 1989 indicated that at the secondary level 86% of the students attended state schools, 9% went to integrated schools and the remaining 5% to private schools. In contrast to countries such as the USA, there are a large number of state (including integrated) secondary schools that are single sex; of the 380 secondary schools 284 are coeducational, 44 are for males only, 50 for females only, and the remaining 2 are male only at the junior level but coeducational at the senior level.

Education in NZ schools is based on a general curriculum with a common core of subjects until the end of year 9. School Certificate, at the end of the fifth form (year 10), is the first national examination encountered by students; with a few minor exceptions it is externally assessed. Most candidates will take examinations in the five or six subjects they have chosen. English is the only compulsory subject but most schools insist on some science and mathematics; consequently the subjects with the most entries are English, mathematics, and science.

Students who continue to the sixth form (year 11) enter for Sixth Form Certificate, which is awarded on the basis of their year's progress in five or six subjects; English remains compulsory. In 1989 the subjects taken by the greatest number of students were in order: English, mathematics, biology, physics, accounting, geography, chemistry, and economics.

In the final year of high school, seventh form (year 12), students who complete a satisfactory year's work are awarded a Higher School Certificate. Many also enter the national examination, commonly known as Bursary, for which candidates usually choose five subjects. Although this examination is not compulsory, tertiary institutions take students' results into consideration when they apply for admission into a faculty (e.g., law, medicine, engineering, commerce) with limited enrolment. Students entering the Bursary examination are charged a fee and are examined in a given subject at the same time throughout the country in specially organised locations under strictly supervised conditions. English is no longer compulsory, and there are two possible mathematics papers: Mathematics with Calculus (MWC) and Mathematics with Statistics (MWS).

Over the five years from 1986 to 1990 there has been a 67% increase in the number of Bursary entries over all subjects. The number of MWS candidates has kept pace with this, increasing by 64%, whilst entries for MWC rose only by 40%. In 1990, 89% of year 12 students took the Bursary examination. Of these 19,243 candidates, 7,984 attempted MWC. Currently the three Bursary examination papers with the highest candidate numbers are English, MWS, and MWC.

Details of this Study

This study considers the participation and performance of students in the NZ Bursary MWC national examination. Marks of all MWC students for whom full data was available were included and treated as if they were a sample from the population of all eligible potential candidates. Scripts were marked anonymously with no indication to the marker of the name, gender, school, or residential location of any candidate.

The MWC syllabus is designed for those students who want to continue with mathematics, engineering, or the physical sciences at the tertiary level, whereas the MWS paper is less specialized and intended more for those students wishing to pursue other careers that use mathematics.

All students who elect to study Bursary mathematics at year 12 will have studied the same amount and type of mathematics until the end of year 11. This material, together with the Bursary mathematics at year 12, will have been first encountered in the classroom. The Bursary mathematics examinations are three hour written tests, tables of formulae are provided, and calculators are permitted.

The first part of this study discusses two aspects: students' performance in the 1987, 1988, and 1990 MWC examinations on the basis of total marks obtained and the differing participation rates of females and males. We consider four variables: gender, whether the student also took the MWS paper, the type of school a student attended (single sex or coeducational), and the authority governing that school (private, state or integrated).

The second part of this study examines how the gender difference varies according to the type of mathematical activity being examined. For this analysis we restrict ourselves to just the 1988 and 1990 examinations because they had the same examiners and question format. As a model, we used previous research by Pattison, Holton, and Gordon (1984), which discusses performance over three successive years in the Victorian Higher School Certificate Mathematics Examination in Australia.

Participation and Achievement

Participation

Most NZ schools insist on a pupil taking mathematics until the end of year 10. For the past few years the percentages of males and females taking mathematics in the sixth form (year 11) have differed by around 6% in favour of the males. For those females who continue to the seventh form (year 12) the bias in the self selection process is perpetuated when 39% decide not to take any Bursary mathematics papers. The comparable figure for males is 22%. Failure to obtain a good Sixth Form Certificate grade in mathematics seems to deter more females than males from continuing with Bursary mathematics (Blithe, Forbes, Clark, Robinson, & Whitwell, in press).

In NZ in 1990 there were 9799 females and 9444 males in year 12 at high school. Ten percent of females took just MWC, 29% took just MWS, and 22% took both. Comparable percentages for males were 10% (just MWC), 27% (just MWS), and 41% (both). For those students who choose to do only one Bursary mathematics paper at year 12, it is clear that MWS is the more popular choice. To some extent this is to be expected as it was designed as a possibly terminating practical course for the majority of year 12 students.

The disparity between the percentages of males and females doing both Bursary mathematics papers and the larger proportion of females than males avoiding MWC both cause concern. MWC is generally recognised as the preferable mathematical choice for those students who intend to pursue university studies of a mathematically based subject. In Australia, a detailed analysis of enrolment patterns in mathematics in upper secondary schools showed the same trends (Dekkers, De Laeter, & Malone, 1991).

Just over one third of students who sit both Bursary mathematics papers are women, and this corresponds to the percentage of women doing first year mathematics at university. Studies in NZ (Forbes, Blithe, Clark, &Robinson, 1990) and Australia (Watson, 1989) demonstrate that at this level there is little difference (usually favouring females) in the average performance of males and females. The lesser presence of women in higher level mathematics papers and in mathematics related occupations cannot be explained by the small gender differences in mathematics performance.

Achievement

MWC results were analysed for 6363 candidates in 1987, 7159 candidates in 1988, and 7912 candidates in 1990. Respective mean percentage marks were 48.0 (sd 24.3), 50.7 (sd 22.5) and 45.4 (sd 18.3).

Appropriate statistics for the Bursary MWC results grouped by gender indicate that when the number of papers taken, school type and school authority are controlled for, the difference between the means of the results for male and female students is not statistically significant. Regardless of gender, MWC students who also sat MWS scored significantly higher marks (averaging from 5 to 9 marks more) than those sitting just MWC.

There is a clear advantage in this type of examination to students attending single-sex schools. In NZ many of the private and state schools recognised for academic excellence are single sex and in medium to high socio-economic areas, thus we are hindered by the possibility of nonequivalent group comparisons due to the more selective student intake by many single-sex schools.

Results were also grouped according to school authority: state, integrated, or private. (A description of these types of schools in NZ is included in the introductory section of this paper.) In all three years students in private schools average 3–

4 marks more than students in state schools; both these groups outperform students in integrated schools by an even greater margin. The differences in means between the three authorities do not always show the same advantage for males and females (Morton, Reilly, Forbes, & Robinson, in press).

Type of Mathematical Skills

Data Analysis

For our questionnaire we decided on six general characteristics (Real World Analysis, Preliminary Analysis, Spatial Processing, using Acquired Knowledge and Skills, Conceptual Demands, and Manipulative Skills) and four content areas (Algebra, Co-ordinate Geometry, Trigonometry, and Calculus). Each question in the 1988 and 1990 examination papers was assessed by 14 raters, all of whom were experienced teachers at secondary or tertiary level. Most had marked the MWC examination, five were current or past examiners and four were female. The assessment was carried out in the rater's own time and was generally reported to take several hours.

The examination for MWC is divided into two sections: Section A comprising seven routine questions (each worth 5 marks) examining a broad range of syllabus topics, and Section B, consisting of seven more demanding questions, which often require a mixture of mathematical skills to answer. Each of these longer questions is worth 13 marks and students choose to do five of the seven. There was no clearcut gender difference in this choice.

Our first task was to assess the degree of uniformity between the raters, in order to examine if there was a uniform agreement regarding the meaning of the 10 characteristics. There is a reasonable degree of agreement between the two years. The characteristics relating to subject matter all have high inter-rater correlations, indicating a strong agreement between raters. This means that these concepts are understood in the same manner by each rater. On the other hand, Manipulative Skills and Conceptual Demands are not understood in the same manner by each rater, and the degree of agreement is poor.

Regression and Correlation Analyses

We attempted to relate the gender differences from each question to the average score of each characteristic. We first completed regression analyses for each year and then for both years combined. The results are only presented for the combined data in this paper. The significance of the coefficients in the fitted model is a measure of how well a variable (i.e., characteristic) helps explain the difference in performance, over and above the explanatory power of the other variables in the model. Secondly, we computed correlation matrices. Each correlation coefficient, on the other hand, measures the strength of the relationship between two variables, ignoring all the others.

The data was pooled for the two years, and we ran a regression with all 28 questions. With so many variables to use, there were a number of possible models that might be considered. To reduce the number of variables, we subjected the data to an "all possible regressions" analysis. Experimenting with various fits indicated that the most important variables are Preliminary Analysis, Acquired Knowledge, Geometry Amount, and Calculus Amount. With these variables in the regression model, the addition of the other variables led to no significant improvement in the fit. In particular, the addition of the variable Spatial Processing did not seem to be important in any of the models fitted. This may be due to the fact that this variable is correlated with Preliminary Analysis and Acquired Knowledge. The conclusions are the same as for the separate analyses for each year: Preliminary Analysis and Geometry Amount favour males, and Acquired Knowledge and Calculus Amount suit females.

The correlation analysis confirms this result. Variables favouring males include Preliminary Analysis, Conceptual Demands, and Geometry Amount. Those favouring females include Manipulative Skills, Algebra Amount, and Calculus Amount. However, in view of the small number of questions, these results should be interpreted with caution.

Discussion

Candidates who enter Bursary MWC form a small proportion of their age cohort, consequently they can be regarded as a fairly selective group. The meta-analysis by Hyde, Fennema and Lamon (1990) indicates that the more select the student sample the greater is the gender difference in achievement. Our results show that when the effects of number of papers sat, school type and school authority are allowed for, the gender factor is not significant. The fact that the MWC candidates have similar mathematical backgrounds and the examination items cover material learnt almost exclusively in the classroom probably has a moderating influence.

The fact that the number of Bursary mathematics papers taken is the dominant effect is hardly surprising. Although less pronounced, the school type and school authority effects cannot be ignored. These two effects are almost certainly interrelated, since so many of the academically acclaimed single sex schools are either private or located in a high socio-economic area. At present we do not have access to the additional information for schools or individuals that would be required to do an analysis of the impact of socio-economic status on academic achievement.

The more traditional mathematics paper for students intending to study mathematics or mathematically related subjects to a higher level at university is MWC. Over the past few years there have been approximately equal numbers of females and males enrolling for year 12 schooling. From these, only 30% of the females, compared to 50% of the males, have attempted MWC. This lower participation rate by females clearly restricts the number with the background to pursue a mathematically related career. Moreover, without MWC, they cannot be doing both mathematics papers and gaining the performance advantage of such students.

Our attempt to characterize mathematics skills on a gender basis indicated that questions requiring more preliminary analysis suited males, whereas females were more at ease with those reliant on acquired knowledge. Spatial ability was not an important factor in its own right, and this suggested that it did not provide any additional explanation of gender difference to those inherent in the other variables. The two content areas for which the most significant gender differences occurred were co-ordinate geometry and calculus. The gender difference favoured males in the former case and females in the latter. It may well be that this section on type of mathematical skills was limited by the fact that we had to work with existing examination questions instead of questions designed specifically for our purposes.

Overall our results match those of other researchers in NZ and overseas. Given the small size of NZ and the fact that there is a national curriculum and strong links between educators, one of the primary values of an analysis of this type is that it causes people to reflect on their own teaching and examining techniques, the learning styles of their pupils, and the classroom environment.

Female students need to be made aware of the opportunities for them in the professional world centred around mathematics and science. It is important to have the views and contributions of all parts of society in order to give less biased direction to our scientific and technological developments and research. Differences in achievement between females and males are small and cannot account for the reason that there are so few women participating in tertiary training with the intention of pursuing a mathematically related career. Perhaps it is indeed true that "many more women than men see mathematics as neither relevant to their interests and experiences nor useful to them in their future careers and lives" (Barnes & Coupland, 1990, p. 73).

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A GENDER SPECIFIC VIEW ON GEOMETRY LEARNING

Kristina Reiss och Anke Albrecht
Flensburg University

During the past few decades numerous studies have investigated whether boys and girls differ in their mathematical abilities. Only slight differences were found between the two genders in primary school, but boys performed better than did girls when they reached adolescence (Fennema, 1987). Recent investigations and meta-analyses shed new light on these studies. On the one hand, there is discussion on whether there was bias in the testing materials which favored boys, and how it worked (Klieme, 1993). On the other hand, the observed sex-related differences have decreased during the last decade (the extensive analysis by Beermann, Heller, & Menacher, 1992; in particular Stumpf & Klieme, 1989). There might be the exception of gifted students (Hyde, Fennema & Lamon, 1990).

The observation that gender specific differences become smaller principally refers to all cognitive factors of mathematical performance. But even recent investigations demonstrate that there might be slight differences between boys and girls in their spatial abilities. This has been a result in a carefully planned international study which compared mathematical abilities of boys and girls in 20 different countries (Hanna, 1989). Gender specific differences independent of the culture or country were only observed in geometry and measuring. Other results show that these differences are also decreasing (Stumpf & Klieme, 1989). Moreover, they are not equally well established in all factors of spatial ability (Klieme, 1986; Linn & Peterson, 1985).

Spatial ability has been studied extensively (Rost, 1977). Although there is no empirical evidence for the relation between performance in spatial problems and performance in mathematics, it is nevertheless common sense that this factor covers at least crucial aspects of mathematical problem solving. A sound understanding of spatial aspects is particularly helpful for solving geometrical tasks. For this reason, investigation has repeatedly centred on whether the ability to form an adequate spatial perception or a mental representation of spatial relations can be regarded as a fixed component of a person or whether it is possible to train this ability. There is no definite answer but a study by Connor and Serbin (1985) gives evidence for training effects. Moreover, Lohmann (1988) has shown that the ability to mentally rotate geometrical entities can be improved by training. In

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particular, it is in this aspect where test results of boys are significantly better than those of girls (Linn & Hyde, 1989). Obviously a trainable component cannot be regarded as a gender specific difference.

Although in most aspects mathematical abilities of boys and girls seem to be the same, there are still differences insofar as the attitude towards mathematics, science, and engineering is concerned. At the high school or university level, mathematics, science, and engineering are still preferred by male students and less frequently chosen by female students. Nonetheless, it must be noted that things are changing. In summer 1989, about 40% of the freshmen in German universities were female. The percentage of mathematics beginners is slightly lower, namely about one third (Beerman, Heller, & Menacher, 1992, according to data of the German Federal Office of Statistics). The results are different in nearly all other science and engineering subjects, with the exception of biology (52.8% female students and 47.9% female freshmen). There are still only a few female students majoring in physics (9.6% female students and 10.6% female freshmen), chemistry (28.6% female students and 31.0% female freshmen), or construction engineering (8.7% female students and 15.1% female freshmen).

Similar results may be found in computer science at the university level (14.7% female students and 16.7% female freshmen), although there is an affinity to mathematics in the theoretical part of this subject. It has been stated that boys have more confidence in their mathematical ability than girls (Linn & Hyde, 1989). This is also true for computer science and the differences are even more striking. Empirical results can be interpreted to demonstrate that girls in girls' schools have a better performance in computer science than girls in mixed schools. One reason is that computer science in school is usually regarded equivalent with computer use and programming by students. Most students believe that extensive experience in the use of computers is essential for successfully mastering this subject. Needless to say, boys have more experience with computers and more frequently own a computer than do girls (Funken, 1992).

Goals of the study

Two of the discussed aspects have been investigated in the study: spatial performance and its trainability; and the use of computers by boys and girls. The hypotheses underlying the investigation are the following:

(1) There is no difference between boys and girls in spatial ability as tested in a standardized test.

(2) Spatial ability can be trained by adequate geometry instruction.

(3) There is no difference in the test results whether instruction is performed by compass and ruler or by the aid of a computer and adequate software. In particular, there are no sex-related differences with respect to computer use.

Design of the study

Forty-six boys and girls from two parallel courses of grade 8 took part in the investigation. One course consisted of 12 girls and 9 boys, the other of 13 girls and 12 boys. The subject matter, specific points and lines in a triangle, was covered in both courses in 10 lessons. One course (21 students) used computers and the geometry software Cabri Géomètre, the other course (25 students) was instructed traditionally, using compass and ruler. Students were assigned to one or the other of the courses using the criterion of simple feasibility of school organization. Instruction was performed in both courses by a college student nearing completion of her studies.

The boys and girls who participated in this study were between 13 and 14 years of age. This age group was selected for two reasons. On the one hand, numerous studies demonstrate that possible gender specific differences in mathematical performance usually do not start before the onset of puberty; on the other hand, an adequate level of cognitive development is connected with the meaningful use of computers and relatively complex software. A test was taken by both courses before and after the instruction. It was part of the *Berufsorientierungstests für achte bis zehnte Klassen*, a test of vocational orientation designed for grades 8 to 10. The subtasks chosen covered special aspects of spatial ability, spatial orientation, and measuring. The first part *Figuren erkennen* (shape recognizing) is supposed to reveal the ability to quickly discriminate between different shapes. Spatial perception and visualization is a necessary prerequisite for solving this task. The subtask *Quader* (cuboid) is designed to test the ability to restructure or reorient a perceived object. The last part *Spiegelbilder* (mirror images) is an indicator of a student's spatial orientation and his or her ability to estimate lengths.

One of the courses used computers and two to four persons formed a group. Protocols of these working sessions were transcribed; additionally we asked for the students' grades in mathematics during the last term. The median of these grades was calculated for each course separately. In each course two groups were formed, consisting, respectively, of students in the upper half and students in the lower half of classroom performance.

Results

The mathematics teachers described the performance level of both classes as different. The course using computers was regarded as superior to the one which was instructed traditionally. This difference is not reflected in the grades. There is no statistical difference between the two groups insofar as the grades are concerned. There is no significant difference in this dimension, either when we compare boys (n=21) and girls (n=25) in both courses or when we compare boys and girls in their specific courses.

Comparing the results for spatial abilities in the pre-test and post-test there is a highly significant difference (p=0.001, df=45). This difference is due to the two subtasks *Figuren erkennen* (p=0.0001) and *Quader* (p=0.0001). There is no difference in the subtask *Spiegelbilder* between pre-test and post-test, which means that there is no increase in learning for this dimension. Figure 1 shows the differences for the entire group between their pre-test and post-test scores. The box indicates the 50% of the students around the median, the line in the middle stands for the median, the small horizontal lines on top and bottom encompass 75% of the sample. Outside this area individual scores are drawn as small circles.

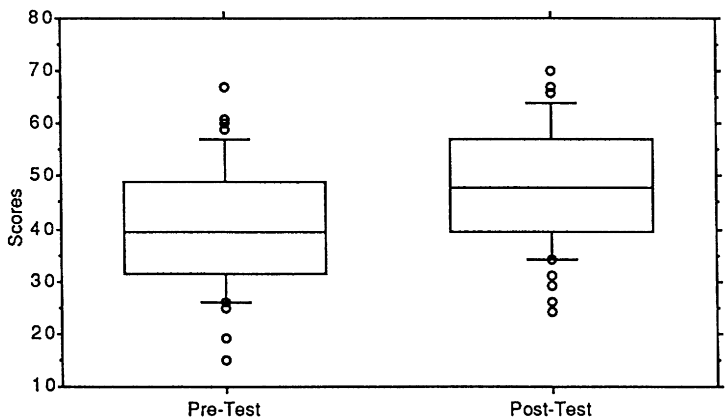

Figure 1 Differences between pre-test and post-test scores in spatial ability for the total sample (n=46)

There is a highly significant difference between pre-test and post-test scores not only for the total sample (n=46) but also for the two courses separately. Even if we differentiate smaller subgroups, the result is essentially the same. If we consider the course which was instructed traditionally, girls (p=0.0023, df=12) as well as boys (p=0.0020, df=11) perform significantly better in the post-test than in the pre-test. In the course using computers this is only true for the girls (p=0.0006, df=11). The boys show higher scores in the post-test but this difference is not statistically significant (p=0.1124, df=8).

There is no difference between boys and girls in all subtests of the total sample (n=46). This is true for the total test-score (p=0.6569 in the pre-test, p=0.8676 in the post-test) and we find the same results for each of the three sub-tests *Figuren erkennen*, *Quader*, and *Spiegelbilder.*

Figure 2 shows the results for the boys (pre-test and post-test scores). There is evidence of an increase in learning, with the variation of scores much larger in range for the lower 50% of the group in the pre-test and larger for the upper half in the post-test. Figure 3 shows the results for the girls.

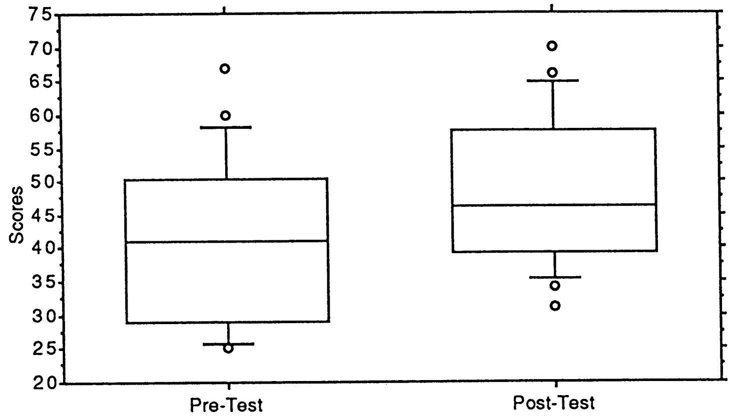

Figure 2 Results for the boys in the pre-test and post-test.

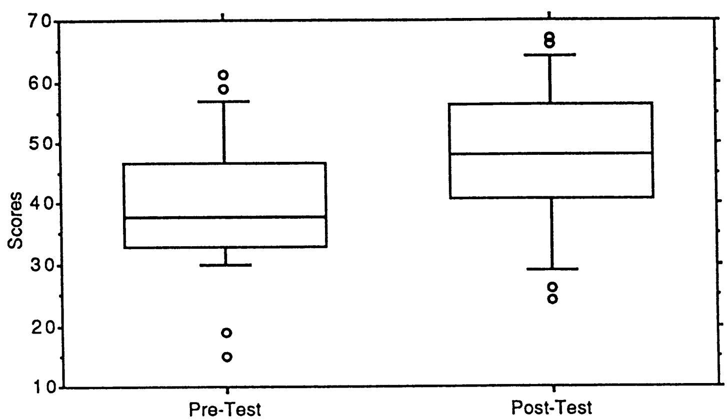

Figure 3 Results for the girls in pre-test and post-test.

If we distinguish between the course which was instructed traditionally and the computer-instructed course we find some interesting results. There are no differences between boys and girls of the course instructed traditionally. Girls perform slightly better in the sub-tests *Figuren erkennen* and *Spiegelbilder* in both pre-test and post-test. But these differences are not significant. The results are different for the course which was instructed with the aid of the computer, especially in the pre-test. There was a significant difference between boys and girls for the total score (p=0.0337) and for the sub-test *Quader* (p=0.0730) and *Spiegelbilder* (p=0.0574). Here the boys' scores were better than those of the girls. These differences have vanished in the post-test (p=0.2896 for the total test, p=0.2625 for the sub-test *Quader* and p=0.3928 for the sub-test *Spiegelbilder*). There are no differences between the two genders in the sub-test *Figuren erkennen* in either the pre-test or the post-test.

In figure 4 the results for the pre-test and the post-test are described for the boys of the computer group; the respective results for the girls are shown in figure 5. A striking change of scores is evident, particularly for the girls. Their range of scores in the pre-test is larger for the lower half of the sample than for the upper half, whereas in the post-test the variation of scores is larger in the upper half. In figure 5 the box on the right-hand side starts where the box on the left-hand side ends. This indicates a significant increase in test scores and also a definite improvement of the performance level in the girls' group.

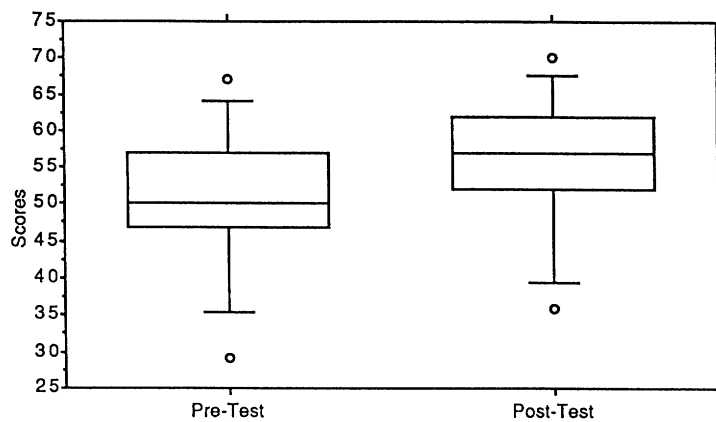

Figure 4 Pre-test and post-test performance of the boys in the compter-instructed course

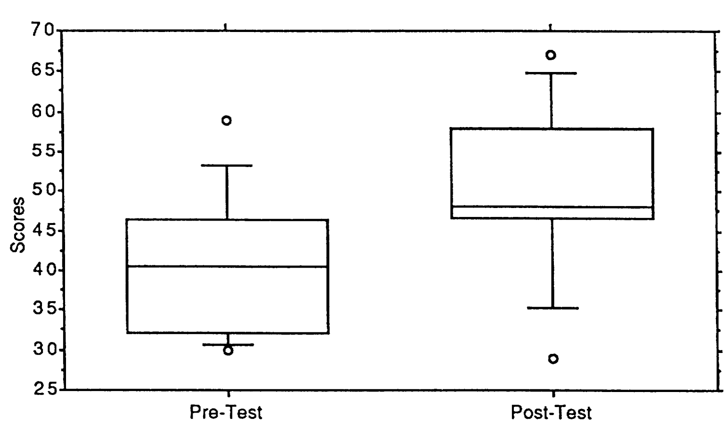

Figure 5 Pre-test and post-test performance of the girls in the compter-instructed course

Discussion

The study gives evidence that scores improved from pre-test to post-test. This is particularly true for the subtests *Figuren erkennen* and *Quader*. These subtests are especially concerned with spatial visualization and spatial orientation. The scores did not improve in the subtest *Spiegelbilder*. Here spatial orientation is of minor importance whereas the estimation of lengths is crucial. Thus, the students' better performance is restricted to a defined range of abilities. According to Anastasi (1981), the improvement of test scores can be attributed to three factors. These are (a) test-taking orientation, which is an elaborated way to cope with a testing situation or the test itself; (b) coaching that is a training in the investigated domain of performance; and (c) instruction in broad cognitive skills, pre-test and post-test, which generally improves problem solving behavior.

Test-taking orientation is a general ability of students to successfully take part in a test. This is independent of special intervention between pre-test and post-test. This factor for improving test scores can be excluded in the described study. On the one hand, a training item has been discussed before every subtest, which has a positive influence on the subsequent test performance (Wahlstrom & Boersman, 1968). On the other hand, the different results for the three subtests cannot be explained by such a training effect.

It is also unlikely that coaching had an effect on the test results because such training could only be applicable to very restricted domains of achievement. The professional orientation test used in this study is not an achievement test but a test of broad cognitive skills.

The approved ability levels between pre-test and post-test can only be attributed to the instruction. Thus it seems possible that the ability of spatial perception can be improved by appropriate geometry instruction and, in particular, this ability can be trained. It is of minor importance whether the instruction is given in a traditional way with compass and ruler or is done with the aid of a computer and appropriate software. It seems obvious that both methods teach general problem-solving strategies and working techniques. The differences in performance cannot be attributed to the method.

Additionally, the empirical results show that differences in performance between boys and girls cannot be found in the domain of spatial visualization. If we consider the total group, that is the students who were instructed traditionally and the students who were instructed with the aid of acomputer, both genders have essentially the same results in the pre-test and approximately the same results in the post-test. We find only one exception when we compare the pre-test results of boys and girls in the computer group. One must be cautious not to overinterpret these results because the group consisted of only 12 girls and 9 boys. Nevertheless, possible explanations have to be taken into consideration.

First, a methodological remark is necessary: the level of significance should have been adjusted by the Bonferoni method because a maximum of five tests of significance were performed for the same hypothesis. That is why the level of significance should not have a probability of .05 but a probability of .05/5 = .01. The pre-test difference between boys and girls of the computer group is the only result that is not significant after p has been adjusted by the Bonferoni method.

The different test motivations of boys and girls could be one reason why the test results are different. Girls generally show less interest in computers and their use during instruction. They were informed about the next steps in instruction, especially that they had been assigned to the computer group, so anxieties could have developed. An argument against this hypothesis is that there were no such differences in the parallel course where the boys were not well motivated because they would have preferred to work in the computer group. If we stick to this line of thought or if we generally attribute lower test scores to differences in achievement, then it is particularly interesting how the girls improve between pre-test and post-test. The number of items solved has increased to the same amount as in the boys group but also to the same amount as in the course where the students had been instructed traditionally.

Once more this study gives evidence for the trend that achievement differences between boys and girls in general mathematics education have virtually vanished. Very little difference could be found between the group that used the computer and the one that used compass and ruler. Although there were beginners' problems in using the computer at school, especially with the girls, the effect of instruction as far as it can be tested was not influenced significantly in either a positive or a negative direction.

Obviously, the product of learning, namely the test score, is the same for girls and boys. This is not necessarily true in the same way for the learning process. Observations not only in this class demonstrate that often boys and girls use different working techniques, they approach a mathematical domain in a different way, and they may evaluate a given situation differently. Possibly this implies that one should not ask whether boys or girls perform better in mathematics, whether they have the better spatial visualization, or whether they make better use of the computer. It might be much more relevant to ask whether and how the two genders differ in their specific approaches and their problem-solving strategies.

There is increasing evidence that one can find extremely different individual problem-solving strategies while students are working with the computer (Goebel & Vorberg, 1992; Haussmann & Reiss, 1989, 1990). But these strategies are not fixed, they can change over time and specifically they can become more elaborated (Haussmann & Reiss, 1993). These different modes are not gender specific but there are studies that find different preferences in problem-solving styles of boys and girls. According to Schiersmann (1987) women have an attitude towards technology and especially towards computers that is determined by applications and goals. Additionally, they prefer theoretical approaches (Funken, 1992).

Schwank (1990, 1992) identified two different cognitive structures and two different strategies, the predicative and functional structure, respectively, the problem-solving strategy based on concepts or on sequences. According to her hypothesis, girls prefer a predicative conceptual approach to tasks whereas boys prefer a functional predicative approach for the solution of mathematical problems.

In our investigation we essentially described an analysis of test results but we also have other data from the investigation, especially protocols of the group which has been instructed with the aid of a computer. Analyzing the protocols gives evidence that boys and girls have quite different approaches and a different motivation when they try to work with a computer and software. This may be partly due to their pre-experiences which may have an effect on the specific use of the program Cabri géomètre. On the other hand, the results of other investigations have shown that girls approach a task in a more thoughtful and less spontaneous way. This may be a disadvantage when working with computer software. If girls have an elaborated plan for the solution of a problem, they may have difficulties implementing it in a specific program which has a specific kind of logic. Similarly, a cautious approach together with a lack of practical experience often is a disadvantage in the first years of working with a computer. But if there is indeed a deficit, girls obviously reach the same level after some time. They succeed in the learning process in approximately the same way as boys do.

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GIRLS AND MATHEMATICS

Kristine Revak
College of St. Catherine

There is something missing from America's high schools. High-school-aged girls are not electing to take math classes, especially upper division math classes. Girls are under-represented in trigonometry, pre-calculus, and calculus classes. Furthermore, if girls do decide to enroll in these courses, their male peers often out-perform them, especially at tasks that involve critical thinking and complex problem-solving skills. Many theories address the possible causes of this deficiency, ranging from a student's internal belief system to teacher behavior to the environment in which the student was raised. I will consider each of these in this paper and then address suggested actions for encouraging girls to choose to study math, possibly resulting in successful outcomes for these girls.

First, I will discuss the present relationship between girls and mathematics. If we were to visit a classroom in almost any elementary school, we would discover that girls and boys each believe their respective sex is more skilled in the area of math. Were we to poll these same students again in junior high or high school, we would find that both sexes now hold the opinion that boys are generally better at math. Girls and boys perform equally in math until the time that math courses are no longer required. This is the time when girls begin their decline with respect to this study. In 12th grade, boys may outnumber girls in calculus classes by a 2:1 ratio (Skolnick, Langbort, & Day, 1982). The results from a California annual achievement test showed that girls were better at computational and one-step word problems whereas boys excelled at problems involving multiple steps, application, and advanced logical reasoning (Skolnick, Langbort, & Day, 1982). These skills are critical to advanced mathematics. Further, a study done by Crandall, Katkovsky, and Preston in 1962 found that girls underestimate their math problem-solving skills (Fennema, 1980).

Fennema (1990a) believes that girls must have equality with respect to three areas. First, girls must receive equality of educational opportunities. Although girls are free to take upper level math courses, the reality is that they do not take advantage of this opportunity. Therefore, Fennema agrees that legal equality of educational opportunity exists, but there is not equality in actual practice. Next, Fennema calls for equality of educational treatment. In the classroom, evidence shows that teachers interact and initiate contact more often with their male students. Also, males receive more discipline as well as more praise from teachers

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

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than do girls. This treatment does not constitute equality. Finally, Fennema cites equality in educational outcomes as the third goal. She claims that males learn more and different math in high school, that males are more able to transfer learning to the solution of complex problems, and that males leave high school with a personal belief system that enables them to pursue jobs in the math field.

The next section of this paper is devoted to a discussion of the causes behind girls' lack of participation in mathematics. To begin, let us consider the early environment of children and how certain environments may put girls at a disadvantage when studying math. As children, females tend to engage in activities such as playing with dolls and playing house. These activities involve verbal, interpersonal, and fine-motor skills, generally thought to be feminine traits. In addition, drawing and sewing do not require independent problem solving and are more structured (Skolnick, Langbort, & Day, 1982). On the other hand, boys, who discuss batting averages and the speeds of traveling balls or bikes, are creating a base upon which math and physics can be built. Furthermore, other pasttimes for little boys, such as constructing model airplanes, using microscopes, and fixing things, involve manipulating objects. Boys spend their time finding relationships between objects, taking objects apart, grouping them, and visualizing (Skolnick, Langbort, & Day, 1982). These skills prove to be quite useful in the study of mathematics and put boys at an advantage over girls, who have not developed such skills.

Next, let us consider a model of influences that may affect girls' ability to engage themselves in high-level cognitive, activities. The Autonomous Learning Behavior Model from Fennema and Peterson (1985) explains that external and societal forces influence the development of girls' internal belief system. This system includes factors of confidence, perception of the usefulness of math, perceptions concerning math's congruency to sex-roles, and also girls' attributional style. These factors in turn influence students' autonomous learning behaviors, which determine if a girl will succeed at tasks that challenge her cognitive ability. All of these areas will be discussed individually below.

First, I will address the issue of confidence and how a student's confidence level can affect her math achievement. A student's view of herself and how skilled she believes she is at mathematical tasks directly influences her success in math. If a student works from a strong self-concept, the student will tackle new material willingly and will not become discouraged in the face of a challenge. Rather, she will persist because she believes she can succeed. In a Fennema and Sherman study, math achievement tests and scales of confidence levels were administered to a group of students. The results showed gender differences in achievement and confidence, both in favour of the male subjects. In fact, gender differences in confidence were found even when there was no difference in achievement levels. Confidence was found to be strongly linked to achievement (Meyer & Koehler, 1990, p. 61).

Similar to confidence, perceived usefulness can determine whether a student will continue in her study when faced with difficult material. Once a student has the ability to make choices about whether to continue her study of mathematics, if she does not consider math to be of use to her, then surely she will discontinue her study in order to pursue other subject areas. A study by Fennema and Sherman uncovered gender differences in math achievement which favoured males and were accompanied by a greater perception of math's usefulness by males (Meyer & Koehler, 1990, p. 63). Again, female students are at a disadvantage to males with respect to perceived usefulness of math. Females are still not encouraged to enter math and science related professions, therefore, math skills are not considered a necessity for many girls.

This leads to a discussion of the influences of female students' conception of the congruency of mathematics to their perceived sex-roles. During the adolescent years, young adults are changing in physical and emotional ways and are also experimenting with adult roles for the first time. There is great need for social approval, which causes pressure to conform to traditional sex roles as they test new relationships with members of the opposite sex (Skolnick, Langbort, & Day, 1982). When girls enter junior and senior high school they encounter more male teachers, a less intimate atmosphere, and a shortage of female peers in the classroom (Skolnick, Langbort, & Day, 1982). Unless a girl has an incredible amount of self-confidence, she may attempt to become inconspicuous in the classroom, especially in the math classroom. She will not want to compete with boys from whom she is attempting to gain social approval. These feelings may develop into a fear of success on the part of girls. Fear of success may manifest itself as conflict, fear, or decreased performance as the teen-aged girl is torn between goals of success and fulfilling what she perceives to be her role as a female. Horner states that fear of success results in either a fear of losing femininity and self-esteem or fear of social rejection because of her success (Meyer & Koehler, 1990 , pp. 64-65). If a female student is rejected by her peers because of her success, she may purposely do poorly despite her ability.

The final category of the internal belief system is attributional style. A student's attributional style is determined by the factors to which he or she attributes success. A study by Wolleat, Pedro, Becker, and Fennema concluded that males attribute their successes to ability more often than do females, whereas females attribute success to effort. In the instance of failure, females were found to credit their lack of ability or the difficulty of the task (Meyer & Koehler, 1990, p. 67). The significance of this finding is that external or unstable factors of success or failure will cause students to believe that they will not succeed or fail in the future. Conversely, internal and stable factors encourage students to expect similar successes or failures (Meyer & Koehler, 1990). To explain further, effort is an unstable factor of success or failure. Therefore, when girls tend to attribute their successes to effort, they do not have an unchanging force on which to base their success. They may believe that if their effort is not as high on the next task, they might fail. On the other hand, when failures are attributed to ability, which is a stable and internal force, the girls expect more failures in the future.

Let us next consider the relation of autonomous learning behaviors (ALBs) to math performance. The Autonomous Learning Behavior Model introduced earlier shows these ALBs being influenced by the internal belief system of which I just concluded an in-depth examination. The possession of ALBs allows students to perform highly complex math problems through the ability to work independently, persist in difficulty, and ultimately succeed (Fennema & Peterson, 1985). The autonomous learner assumes control of her learning and chooses to study high cognitive level mathematics. A cycle is created when the learner succeeds and, as a result, her internal belief system is strengthened, which helps bring about further success, etc (Meyer & Koehler, 1990). As a student uses ALBs, these skills are strengthened and become more developed. Fennema and Peterson (1985) suggest that ALBs may serve as a mediator between classroom processes, internal motivation, and math achievement.

Perhaps the lack of ALBs is a factor in the lack of girls engaging in the study of mathematics. The success of autonomous learners depends upon students' ability to work independently, and many researchers have found that girls are not encouraged to be independent in the classroom. Fennema and Peterson discovered that teachers allow their male students to be more creative than their female students in problem solving, encouraging the boys to stray from rules and algorithms (Fabricant, Svitak, & Kenschaft, 1990). The girls would continue to depend upon set rules in order to solve problems and would find the most frustration when faced with difficult problems that do not fit into previously known algorithms. When dependence upon algorithms is combined with a female's need to seek approval from others, girls will attempt to solve a challenging problem, be unable to locate an effective algorithm, and will be too afraid to venture away from known methods for a solution for fear of failure. This dependence may result in intellectual self-doubting, learning anxiety, and eventually a distaste for math (Skolnick, Langbort, & Day, 1982).

The following study will illustrate how teachers can help or hinder the development of autonomous learning behaviors. In her study, Mary Schatz Koehler, Assistant Professor of Mathematical Science at San Diego University, examined two honors math classes, each with about the same number of male and female students. The two teachers used similar teaching styles, employing strong classroom management and clear presentation of material. At the end of the study, Koehler found that, although the male students performed equally well in each class, the female students of one class performed above those in the other. By examining the teacher's treatment of their students in each of the classes, we can gain insight into this discrepancy. Let us name the class with the higher achieving girls Class A and the other Class B. Now, the teacher in Class B provided more time for seatwork, and was more available for student questions as this teacher circulated the room during seatwork. Teacher B encouraged students to work together on their homework. In the end, Class B had more student-initiated questions as well as more teacher-initiated offers of assistance. In Class A, the students were given less time for seatwork. Teacher A sat at her desk and did not directly encourage questions from the students. Teacher A received less student-initiated questions and offered help less often than Teacher B. Why did the girls in Class A achieve at a higher level? The students in Class A had more opportunity to develop ALBs whereas the students in Class B became dependent upon their teacher and upon each other (Koehler, 1990). From this study we can see that ALBs may have a significant effect upon girls' math achievement.

In the final section of this paper, let us address the actions teachers and parents can take in order to combat their female student's or daughter's lack of interest in mathematics. To begin with, let us consider the advantages or disadvantages of co-ed and single-sex learning environments. A study by MacDonald considered all-women's math classes at the University of Missouri-Kansas City and found that the students in the class received higher grades, had better attitudes toward math, and were more likely to continue their math education than their co-ed peers (Fabricant, Svitak, & Kenschaft, 1990). According to the American Federation of Teachers, women in co-ed science labs assume more passive, subservient roles than their male lab partners. Generally, the women recorded the results of the experiments that the men performed (Fabricant, Svitak, Kenschaft, 1990). A study by Webb and colleagues reported that in co-ed groups of eighth graders, the girls were less likely than the boys to have their requests responded to, especially when the requests were directed at the boys (AWM Newsletter 1992).

Another factor that may encourage girls to take up the study of math is providing young girls with influential female role models. If women who are involved in the math and science fields become involved in young girls' lives, the girls will learn that science and technology affect women's lives. Hopefully, they will begin to consider math and science as useful to them. Researchers Fox and Cohn suggest providing these role models for girls before that age of 12 in order to combat sex-role differentiation and to provide them the security to pursue traditional male professions.

Yet another influence on girls in the math classroom is the belief systems of their teachers. All the categories we discussed before including usefulness, sex-role congruency, and attributional style are part of the teacher's belief system as well as the students, and they can affect both teaching and students (Fennema, 1990b). For example, if a certain teacher believed that math was an inappropriate course of study for girls, then that teacher would probably focus his/her attention on the male members of the class. Obviously, this teaching style would have adverse affects on the female students in the class. If an instructor feels that girls succeed because of the effort that they put into their coursework and not their ability and skill, then the females that interact with this teacher may learn to attribute their successes to the same factors.

The final course of action is found in the teaching strategies of math teachers. When studying fourth grade students, Peterson and Fennema found that the girls performed better in a cooperative environment versus a competitive one. On the other hand, boys' achievement was worse in a cooperative environment (AWM Newsletter, 1992). Cooperative work involves more social interaction, which is considered a feminine characteristic. This atmosphere has less anxiety for females when they begin to view boys as partners as they see them share and participate. Further, this environment reduces the pressures when asking a question and allows for discussion, sharing of information, and problem-solving techniques (Skolnick, Langbort, & Day, 1982). Conversely, independent study requires students to rely on their own thinking and ability to problem solve. In my earlier discussion of autonomous learning behaviors, independent study was valued over cooperative work because it discouraged the student from becoming dependent on other people for answers or strategies. Perhaps a teacher would seek to achieve a balance of independence and cooperation in the classroom. This would show that they valued both skills and would develop both of these skills in their students.

In this paper, I have attempted to fully consider the dilemma of the lack of participation of girls and women in the study of math. I began by considering the actual problem as it now exists and then entered into lengthy discussion of the probable causes for this deficit. Finally, I explained the various ways that this problem can be combatted. There are many boundaries that have been set for women, many of which we are still working to overcome. Math is one of the tools that can assist women in improving their place in society. I chose to research this topic in the hope that one day every woman will recognize the relevance of mathematics to her life and will feel that she has the option of studying this valuable subject if she so chooses.

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GENDER ISSUES IN MATHEMATICS IN KUWAIT

Khariya Saif
University of Kuwait

The aim of the present study is to show some gender issues in Kuwait and to find out what factors affect girls' performance in mathematics. Four factors will be discussed here, but these are not the only factors. There are other factors that play an important role, such as the social background of the students, the curriculum, and the beliefs of teachers. Each of these factors requires a separate study.

Background of the present study

Before 1988, there was no research in the field of gender issues in mathematics education in Kuwait. This may be because teachers, parents, and the educational society in Kuwait did not feel that they were facing any important problem.

A study on gender differences in Kuwait began on 1989 (Saif, 1990). The study discussed the variables that may affect the achievement of boys and girls in mathematics in the Western countries in general and in Kuwait in particular. The study also considered the difference in the attainment of boys and girls within specific areas of mathematics, such as Geometry, Algebra and Arithmetic, and the reasons for such differences e.g., students' attitudes, teachers' behaviour and expectations, and the tasks on which teachers place more stress in the classroom.

The present study

Historically, in many countries women have been seriously under-represented in mathematics. Hanna (1991) says "that girls cut themselves off from further educational opportunities and career choices because they avoid mathematics courses when they are no longer compulsory".

In Kuwait it is not the case. Kuwaiti girls choose or like to study mathematics more than Kuwaiti boys. Also non-Kuwaiti females who are from Egypt and Syria and live in Kuwait choose mathematics more than do males. It is not the case in England and U.S.A., the countries which generate much research on gender. This makes the work represented in this paper of particular interest especially because it takes place in an educational system that has very marked differences from those of the U.K. and U.S.A. Kuwait has 12 years of compulsory schooling. Children start school when they are six and leave school when they are eighteen. The com-

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Gender and Mathematics Education, an ICMI Study

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pulsory 12 years of school is divided into three stages:

4 years - the primary stage
4 years - the intermediate stage
4 years - the secondary stage

Two years of kindergarten precede the primary stage. All the students have to take mathematics when they are 6 years old until they reach the age of 16. After that they can choose to take mathematics or not, because at that age students are usually streamed by choice into two channels: science and arts. Students who choose science must continue studying mathematics until they finish secondary school. Successful candidates in the public examination at the end of the courses, are awarded the General Secondary School Certificate, which makes them eligible for university education. Kuwaiti males and females study separately. Male students are taught by men and female students by women.

Higher Education

The University of Kuwait opened in 1966 with two faculties (Science and Arts and Education) and the Girls' College. The University continued its policy of establishing new faculties until the number of faculties reached nine, these faculties are: Faculty of Arts, Faculty of Science, Faculty of Commerce, Faculty of Law, Faculty of Engineering and Petroleum, Faculty of Medicine, Faculty of Allied Health, Faculty of Education, and Faculty of Sharai and Islamic Studies.

The Girls' College has ceased to exist as a separate entity and women students are now admitted to all faculties. After this brief description of the system of education in Kuwait, I will return to the point with which I began. The proportion of females in mathematics education society is very large.

Gender of Students Who Major in Mathematics

Women are in a large majority in Kuwait in undergraduate mathematics and it seems that mathematics is thought a female subject, and because of that it seems not to be masculine to study mathematics in the University and the College of Basic Education (see table1).

Table 1

The Number of Mathematics Degrees Awarded to Male and Female Students in Kuwait University (1973 -1991).

Year	Kuwaiti		Non-Kuwaiti	
	M	F	M	F
1973	2	4	6	3
1973	2	5	4	3
1975	4	8	6	6
1976	3	6	14	15
1977	2	6	13	17
1978	4	12	21	18
1979	5	18	12	17
1980	3	15	15	17
1981	6	18	11	15
1982	4	21	8	9
1983	3	19	6	11
1984	4	17	7	13
1985	5	21	7	17
1986	6	22	13	23
1987	7	21	12	29
1988	15	40	29	39
1989	14	57	49	61
1990	5	55	19	39
1991	13	76	13	10

Looking at this table, one can conclude that the number of girls awarded mathematics degrees is greater than that of boys for all the years from 1973 to 1991. Also, the ratio of male to female students in the statistics departments in the College of Basic Education and the University is 1:4 (see table 2).

Table 2

Percentages of Students who Major in Mathematics

Type of institute	Males	Females
University	240	704
	25%	75%
College of basic education	184	404
	30%	70%

Gender of Mathematics Teachers

The percentage of male teachers is very low compared with that of female teachers (see table 3).

Table 3

Numbers of Male and Female Maths Teachers

Stage	Males		Females	
	Kuwaiti	Non-Kuwaiti	Kuwaiti	Non-Kuwaiti
Primary	284	180	923	72
Intermediate	28	421	271	225
Secondary	14	394	172	256
Total	326	995	1366	553

Ministry of Education, 1993

From this table we notice that :

(1) The percentage of Kuwaiti mathematics female teachers is approxi mately 71% of the total number of female mathematics teachers in the primary, intermediate and secondary schools in Kuwait.

(2) The number of Kuwaiti male mathematics teachers is very small; they constitute only 24% of the total number of mathematics teachers in Kuwait.

(3) Kuwaiti females choose teaching as a career more frequently than males.

Gender Differences In Students' Achievement and Attitudes Toward Mathematics

Some results of a study on student attainment in Kuwait (Saif, 1990) shows that in grade 9 (the beginning of the secondary stage) there are no marked overall differences in the mathematics performance of boys and girls in Kuwait. However, there are marked differences in performance within specific areas of mathematics, such as Algebra, Arithmetic and Geometry. Girls outperformed boys in Algebra and Arithmetic items, while boys outperformed girls in Geometry. These results reinforce those of SIMS (1987).

A study of students' attitudes toward mathematics was carried out by Saif (1988, 1990) on 7th grade and 9th grade students in Kuwait. A questionnaire was administered to 7th grade students in 1988. To allow comparisons to be made the same questionnaire was given to 9th grade students in 1990. The main results obtained from that questionnaire were:

1. Girls in Kuwait strongly reject the idea that mathematics is a male domain at both levels (intermediate and secondary). One result is that mathematics is not a masculine subject in the University of Kuwait (see Table 1).
2. Girls' anxiety toward mathematics decreases at the secondary level to the same degree that they reject the idea that they were frightened of the subject, whereas boys show no significant change in their feelings of fear toward mathematics in this stage.

3. Girls' confidence in their mathematical ability increases at the secondary level more than does boys; this could be a reason for the relatively good performance of girls in mathematics in that stage.
4. Boys in Kuwait think that geometry is important to get good jobs and they wish to have jobs in which they can use mathematics in the future.

Although many girls believe that knowledge of Geometry is important to get a job, they also know that there are many jobs in which one does not use Geometry and they would like to have jobs in which they will not use Geometry.

In general the results of the study on students' attainment in mathematics were not as expected by the researcher. The performance of girls was on the same level as that of boys in spite of the different attitudes of boys and girls toward mathematics and of many factors that could badly affect the girls' attainment, in particular, the social attitudes and the traditional views, which play a large role in the Gulf countries. Many parents are in favour of educating their sons more than their daughters; their attitude is that girls were created only for bearing and raising children. This leads us to think that there must be other factors that have great influence on girls and which serve females to advantage in mathematics in Kuwait, as was noted from the previous tables. I believe that some of these factors are :

1. Single sex schooling.
2. Female and male teachers.
3. Feeling of fear toward mathematics.
4. Girls confidence in their mathematical ability.

1. Single-Sex Schooling

Kuwait, like other Gulf countries, has separate single-sex schools for the Primary, Intermediate, and Secondary levels. Only Kindergarten and University are co-educational.

Some developing countries provide single-sex education. Girls schools are of lower standard than boys schools. The governments of traditional societies that question the value of education for women are less willing to invest in education exclusively for females. In Kenya, most government schools are single-sex and girls' schools receive lower funding (Lee & Lockheed, 1990). This is not the case in Kuwait, where both boys' and girls' schools, are of the same standard. The government offers the same funding for boys' and girls' schools. Some researchers (Cockcroft, 1982; Ormerod, 1981; Harding, 1981) have argued that girls studying mathematics and science seem to be disadvantaged in a mixed school setting. Others (Dale, 1974; Bone, 1983; Steedman, 1983; and Smith, 1986) have argued that girls in mixed schools performed at least as well in mathematics as those in single-sex schools. We believe that single sex schooling provides an advantage to girls in Kuwait as it encourages girls to ask questions in the mathematics class without being embarrassed, as all the students in the class are girls.

2. Female And Male Teachers

When teachers enter the classrooms, they carry with them not only their books, papers, and chalk, but also many socially influenced beliefs, thoughts, and expectations about suitable behaviour, values, and careers for boys and girls. These expectations have a great effect on the behaviour of the students. The main problem here is to measure these expectations and thoughts and to see whether there are differences between male and female teachers. It was a problem for a woman researcher to enter a boys' school to study teachers' behaviour in the classrooms. Accordingly, other ways to study these behaviours were attempted. One approach was to study male and female teachers' expectations concerning their students' attainment in mathematics and to find out whether there are differences between their expectations towards female and male students. Before carrying out the study, a preliminary meeting was held with the mathematics advisers of the schools involved (10 male and 10 female schools, 2 schools from each educational district). The study was explained to them and they were told not to mention the study to the teachers involved.

The study was divided into two stages:

In the first stage the adviser gave the question papers to 10 male and 10 female secondary school mathematics teachers and asked them to give their expectations for their classes to each question, that is, the teacher was to write the percentage of students in each class expected to answer each question correctly. The advisers then collected the teachers' expectations papers. The second stage was on the second day when the advisers gave the same teachers the same question papers to give to their classes (9th grade). Upon completion, the students' scripts were marked by the researcher. The percentage of correct responses has been tabulated for each question in each class. The resulting data were analyzed statistically using a t-test in order to compare teachers' expectations with students' results. This was done for each teacher, averaging over all questions, as shown in the following table 4. The analysis suggests that overall there is no significant difference between the expectations of male teachers and the student' results ($t = 1.16$ not significant). However, there was a significant difference between the expectations and the students' results of female teachers ($t = 2.18467$), i.e. the data suggest that female teachers tend to overestimate in their expectations. This contradicts the results obtained by Jacobsen (1985).

Table 4

Male teachers				**Female teachers**			
Teacher No	**Teachers' Expect.**	**Students' Results**	**Difference**	**Teacher No**	**Teachers' Expect.**	**Students' Results**	**Difference**
1	60.00	48.54	11.55	1	57.00	57.95	-0.95
2	42.75	51.85	-9.10	2	51.65	49.85	1.80
3	63.40	64.45	-1.05	3	51.50	46.00	5.50
4	50.25	51.85	-1.60	4	59.10	63.60	-4.50
5	69.30	37.20	32.10	5	48.00	46.20	1.80
6	41.60	50.50	-8.90	6	58.50	41.90	16.60
7	52.50	41.60	10.90	7	70.50	41.55	28.95
8	38.25	35.60	2.65	8	58.00	48.06	9.95
9	42.50	34.65	7.85	9	49.50	42.05	-17.35
10	31.75	31.65	0.10	10	39.50	42.05	-2.55
Average	49.23	44.78	4.45	Average	54.35	46.93	7.395

3. Feeling of Fear Toward Mathematics

Anxiety and lack of confidence are closely related, and this seems to have a particular implication for pupils' mathematical performance. In recent years considerable attention has been paid to this relationship. As Fennema (1980) puts it, "one tends to do those things that one feels confident to do and to avoid activities that arouse anxiety. For people who have always been good at mathematics it is somtimes difficult to understand the feeling of those who have always felt at best uneasy and inadequate and at worst totally numbed when confronted with mathematical problems or tests." In Kuwait the feeling of fear of mathematics decreases very much in the case of secondary girls as was mentioned before (in the results of the attitude questionnaire) and this may have caused girls' achievement in mathematics to increase, as high anxiety is often associated with lower achievement in mathematics.

4. Girls' Confidence in Their Mathematics Ability

Confidence is one of the affective variables that helps in explaining sex-related differences in mathematics learning. Confidence in learning mathematics has to do with how sure a student is of her or his ability to learn and perform well in the subject. It is an important factor because it has a significant positive correlation with mathematics achievement (see Fennema & Sherman, 1977). Many females show a lack of confidence in mathematics and believe that to be successful in it is unfeminine. In Kuwait results of a study on students' attitudes (Saif, 1990) showed that girls' confidence in their mathematical ability increased at the secondary level supporting findings of better performance of girls in mathematics at that level. Finally, we can say that all the previous factors could give an advantage to girls to do well in mathematics in Kuwait.

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FROM A FEMINIST TO AN INCLUSIVE PEDAGOGY IN MATHEMATICS

Claudie Solar
University of Ottawa

"Feminist pedagogy" is an emerging concept and has become a subject of interest and analysis, at least in America. In Canada, the *Canadian Journal of Education / Revue canadienne de l'éducation* produced a special issue on feminist pedagogy in 1992; the Women's Studies Programme of the University of Saskatchewan sponsored a conference on this theme in September 1993. I am doing the same for the francophone scientific community of Canada at the May 1994 ACFAS[1] Conference. More closely linked to mathematics, MOIFEM,[2] which is the French Canadian section of IOWME,[3] has always structured some of its activities around this issue, starting with the conference on women and mathematics education held in 1989 (see Davis, Steiger, & Tennenhouse, 1989). In 1991-1992, members of MOIFEM met twice to discuss feminist pedagogy in order to develop a group perspective on it (Solar et al., 1992). In this presentation, I will summarize the contribution of feminist pedagogies as developed in women's studies and feminist studies programmes, present the different dimensions of inequitable teaching practices as analysed in the production by Concordia University's Status of Women Office, *Inequity in the Classroom* (Solar, 1992b), and then combine these data in order to design a frame of reference for an inclusive pedagogy in mathematics education. But first, let us remind ourselves why should we bother and how we should do it.

Why bother and how?

Even though, in many countries now, women do succeed in mathematics as well as men (Hanna, 1989; Baudelot & Establet, 1992; IAEP, 1992; Lafortune & Kayler, 1992), they still generally do not choose to embrace scientific careers. For example, in Canada only 6% of mathematics professors at the university level are women although women make up 18% of the overall university professoriate (Mura, 1990, p. 1). Hence, to succeed in mathematics it is not enough to pursue mathematics. To do so, one has to believe one can succeed, and women do not seem to have this internal belief system (Meyer & Koehler, 1990). In fact, both boys and girls believe that men are superior in mathematics (Lafortune, 1989, p. 152). Girls more than boys are not confident of their capacities (Lemoyne, 1989, p.

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Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

56). Fifty-nine per cent of boys, as compared to 19% of girls, attribute their success in mathematics to their abilities or the easiness of the task whereas 69% of girls and 37% of boys think they succeed because they work hard (Mura, Cloutier, & Kimball, 1986). These Canadian data corroborate similar findings to those presented by Lynn Joffe and Derek Foxman (1986). Hence, neither boys nor girls have a realistic perception of their capacities in mathematics.

Internal influences are presently part of the explanation for why women do not persevere in mathematics. For example, in a recent research conducted throughout Canada, Holmes and Silverman (1992) found that, among girls in grades 8, 9, and 10, there are 15% more girls than boys who do not feel good about themselves, and who have less self-confidence as well as confidence in their capacities (pp. 11-14). Confronted with a mathematical problem that cannot be solved immediately, girls experience the situation as a threat but boys do not (Lemoyne, 1989, p. 58). Apparently, there is a close link between a girl's self-image and her image of a scientist. This link might play a significant role in the choice of a career. Holmes and Silverman found that one Canadian girl out of five is not satisfied with the way she looks (pp. 16-17) and almost all of them believe appearance is part of success in a career. So, if girls put a lot of emphasis on their appearance and if the image of scientists they have is the stereotypical one of a neglected and solitary male, then they will not embrace a scientific carreer.

What I would like to point out here is that promoting women in mathematics is a global matter; it does not deal only with mathematics, it deals with the environment, the socio-cultural and the learning environment. It deals also with boys and men for, as Baudelot and Establet (1992) write, the school progress of girls does not translate into the promotion of women. Even with the same diploma, women experience salary inequities, job segregation, and promotion difficulties (p. 15).

A pedagogy that would deal with gender and mathematics education should take into account the above-mentioned aspects. Such a pedagogy can draw from feminist pedagogy the dimensions that deal with the promotion of women and that can be applied to a co-educational mathematics class. It can also draw from an analysis of discriminatory classroom practices. I will call this resulting pedagogy an inclusive pedagogy, for it should encompass women not only as a social class but also as diversified socio-economic and ethnocultural classes. It should also take into account the diversity of men and break away from the stereotyped male norm. This leads me to a movement from a feminist to an inclusive pedagogy in mathematics.

Feminist pedagogy

In a recent publication, I defined "feminist pedagogy as the science that studies the teaching, the learning, the knowledge and the educational environment from a feminist perspective" (Solar, 1992a, p. 267, translated by the author) and I ana-

lysed the feminist pedagogies as developed in the women's studies or feminist studies classroom practice. These pedagogies are influenced in particular by consciousness-raising pedagogy or liberation pedagogy, of which they retain the following dialectical aspects:
- silence / speech
- passivity / active participation
- powerlessness / empowerment
- omission / inclusion (Solar, 1992a, p. 273, translated by the author)

Critical thinking, critical pedagogy, humanistic education and obviously different feminist theories have also contributed to construct feminist pedagogies that have the following characteristics:

1) breaking the silence and giving all women the right to speak
2) creating an appropriate learning climate for women, that is a climate where competition is reduced and cooperation installed
3) changing the power distribution in the classroom in order to counteract domination and hierarchy
4) sharing feminist knowledge which ties in with women's lives
5) valuing intuition and emotions as opposed to rationality and objectivity
6) taking experience as a source of knowledge
7) demystifying the construction of knowledge, its political value and the way women relate to it
8) revealing women's omission and constructing a women's collective memory
9) working towards social change and giving women the means to do so
10) transmitting the necessary intellectual tools to build up a feminist critique
11) using a verbal and written language which respects women's experience and diversity
12) working towards the transformation of education (Solar, 1992b, p. 277, translated by the author)

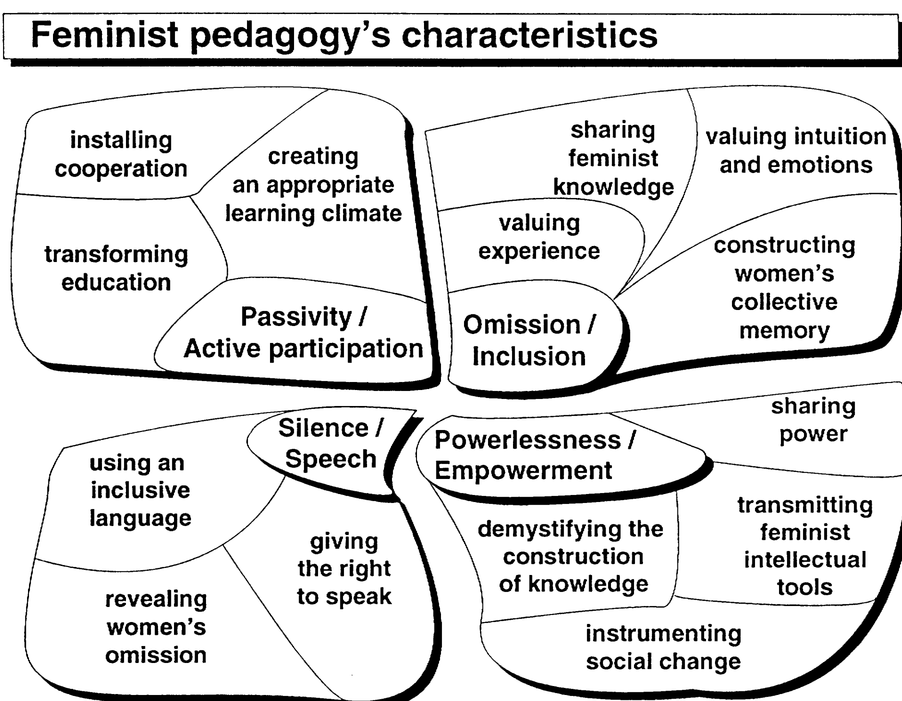
Those criteria came from a set of publications dealing mainly with feminist teaching in the social sciences, literature, and arts. Few were taken from natural sciences and even fewer from mathematics and physical science. Furthermore, these characteristics emerged from college and university settings and need adaptation for elementary and secondary schooling (Tite, 1986; Weiler, 1988).

The MOIFEM discussion group on feminist pedagogy followed the same lines, starting with a discussion of the expression "feminist pedagogy." The feminist perspective is not shared by all in the same way nor with the same intensity, and the co-educational settings of schools challenge the apparent exclusion of boys and men in feminism. No expression is as yet acceptable to all even though we all shared the same goal, that is, to see women use, work in, and/or contribute to the development of mathematics. As presented in the latest publication of MOIFEM (Lafortune & Kayler, 1992), there was consensus on some of the dimensions of our teaching practices. These are:

a) denouncing women's omission in mathematics and its teaching;
b) valuing emotions and intuition in mathematics as counterparts to rationality and objectivity;
c) using experience as a source of knowledge;
d) demythicizing mathematics and demystifying its social and political uses;
e) breaking women's silence and making women speak up in their learning of mathematics;
f) creating a favourable climate for learning using cooperative rather than competitive settings;
g) providing intellectual tools for women to pursue the task of developing the perspective of women and mathematics. (Solar et al., 1992, pp. 38-39, translated by the author)

As we can see, these characteristics on which MOIFEM reached consensus spread among the four dialectical aspects retained from liberatory pedagogy. And if the analysis of the characteristics of feminist pedagogy is pushed further and juxtaposed to these dialectical aspects, then it is possible to classify these characteristics with respect to the dialectical dimensions. Even though certain characteristics may belong to more than one dimension, Figure 1 provides a new schema of feminist pedagogy's characteristics that is easier to grasp and to work with.

The feminist pedagogy's characteristics table offers a frame of reference for constructing feminist pedagogical approaches, and, before applying it to mathematics education, let us consider what a favourable climate for learning math-



ematics entails. For that, we will turn to non-discriminatory classroom practices.

Inequity in the classroom

Inequity in the Classroom is a multimedia production[4] developed at the Office on the Status of Women at Concordia University[5] when I was working there. The video and manual are designed to sensitize college and university professors as well as adult educators to the manifestations of, effects of, and alternatives to classroom practices that lead to discrimination towards women because of their sex, race, ethnicity, class, religion, sexual orientation, age, or disability. This understanding of discrimination against women in teaching, which eventually prevents them from pursuing further education, has been developed with a grid specifically designed to reveal certain aspects of classroom practices. This grid includes discriminatory classroom interaction (sexism, ethnocultural discrimination and racism, sexual orientation, and other forms of discrimination), stereotypes, nonverbal behaviors, language, and curriculum.

Here is a sample of discrimatory classroom practices[6] that may occur in a mathematics class:

- letting boys and men monopolize class time and take more than their share of the teacher's attention, or choosing to ask girls and women factbased questions while asking boys and men to use analytical and synthetical processes;
- sounding surprised by altering one's tone of voice or intonation when girls and women respond well;
- commenting on students' mathematics difficulties by referring to their age or disability;
- expecting Asians to succeed in mathematics or having low achievement expectations for other racial or ethnic groups;
- using course material that depicts males as doing scientific experiments and a male perspective on life; or course material where females are omitted;
- adopting a closed and indifferent posture when interacting with girls or women or persons of racial or ethnocultural minorities but being open and alert with white boys and men;
- gravitating towards the area of the classroom where most men and boys are sitting;
- using a generic masculine language and always referring to a mathematician as a man;
- using a language that universalizes experience and ignores the differences between people and cultures;
- making racist and sexist puns and jokes;
- omitting important achievements, issues, contributions of women, people with disabilities, minorities, and different ethnocultural groups in course content;
- underestimating the assumptions and premises on which theories are con-

structed, so as to imply that knowledge, here mathematics, is neutral;
- never using real-life mathematics situations that refer to women's main activities and responsibilities in society.

Let us remember that most books on the history of mathematics do not mention any women mathematicians, at least in the Western accounts of the history of mathematics that I know. And among specific points, I would like to emphasize the dimension of sexist humour sometimes used by professors to create a so-called more relaxed climate. In fact, humour discriminates against women (Bouchard, 1989). Most sexist jokes relate to women's lack of intelligence, in the same way most racist jokes relate to the lack of intelligence of a specific group of people. These jokes are particularly damaging when we know that mathematics is considered a science for intelligent people.

Towards an inclusive pedagogy in mathematics education

At the beginning of this article, I spoke of feminist pedagogy as dealing with the teaching, the learning, the curriculum, and the educational environment in a feminist perspective. Here, the context is mathematics education. Let us now see how feminist pedagogy and non-discriminatory classroom practices can be combined to provide guidelines for an inclusive pedagogy in mathematics education.

Teaching

There is abundant evidence that professors interact differently with their female and male students, in mathematics education as well as in other subject matter (see Fennema & Leder, 1990; Lafortune & Kayler, 1992; Solar, 1992a). Discrimination in teaching mathematics is pervasive, and, from the above discussion on non-discriminatory practices, we can say that when a mathematics professor manages to respect all students

- by paying attention to all of them,
- by having high achievement expectations for them,
- by using an inclusive language,
- by avoiding stereotypes,
- by forbidding sexist and racist humour,
- by referring to the contributions of women in the ancient and contemporary history of mathematics,
- by using real life situations related to women's lives,
- by using problems and pedagogical settings that require students to speak of the process they follow to solve them and bring them to realize thatthere is neither one solution to most problems nor one way to solve them, then this professor is working toward breaking the gap between women and mathematics.

Learning

There is also evidence that girls and women do not share "equal opportunity to learn mathematics" with their male counterparts (Fennema, 1990, p. 2). The literature on feminist pedagogy and classroom discriminatory practices provides indications of how the situation can be improved. Hence,

- when boys and girls, men and women are able to share their thinking and their understanding of mathematics,
- when the problems they have to solve also include mathematical situations that women confront in real life,
- when stereotypes, sexist language and humour are banished from the class by the professor,
- when mistakes or faults are taken as a springboard to learning,
- when cooperation is valued as a pedagogical setting,
- when the construction of mathematics is part of the curriculum as well as the use of mathematics in society,
- when women mathematicians and scientists are made visible,

then students may learn that mathematics is accessible to all of them, and they might develop a positive attitude towards it. This type of learning should help girls and women to be autonomous and independent in their learning of mathematics and in the use of mathematics, as Elizabeth Fennema advocates.

Curriculum

Less has been said on the mathematics curriculum, but, if "mathematics is a unique product of human culture" (Fennema, 1990, p. 2), and if mathematics has been until now a male construct (Mura, 1986), then mathematics curriculum, just like any curriculum, is a male curriculum and needs to be reviewed. It is time to examine the mathematical school curriculum to see how it builds into a male science in the mind of both males and females, how it serves men more than women, and how it should be changed to be balanced. Whatever the present situation, we already know that when the mathematics curriculum

- includes women in its history, ancient and contemporary,
- includes mathematical situations experienced mainly by women,
- includes material which is not stereotypical, sexist or racist,
- includes aspects of mathematics which are more related to women's lives,
- when mathematics is presented as a process rather than a set of rules (Allen, 1992),

then mathematics is more inclusive of women.

Educational environment

The educational environment is a difficult one to grasp and to change. It encompasses the school climate and the school project, the teachers' beliefs and expectations as well as the ones of the learners, the peer values and pressures as well as the family values and pressures, and, finally, it encompasses the culture of a society in which women are still second-class citizens even though tremendous changes have already taken place in many countries. This is why changing the educational environment requires everyone, from whatever discipline and whatever their role in the education of women and men, to contribute to the eradication of discrimination. I believe that the above-given directions in teaching, learning, and the curriculum are conducive to an educational environment that is inclusive of women and minorities.

Conclusions

Discrimination in mathematics education is a complex situation and a complex problem to solve. For more than 20 years now, researchers have been studying its diverse manifestations. Its complexity has often imposed the fragmentation of the problem into smaller subproblems or subissues. With respect to this aspect, Gilah Leder (1990) writes: "The tendency to concentrate on one or perhaps a small set of variables, and to ascribe differences obtained to these variables alone, has at times given rise to unproductive and largely artificial controversies" (p. 14). However, as she points out, the synthesis of the findings offers an integrative view of gender differences. In this article, I tried also to reunite different variables influencing women's learning in the mathematics classroom. This is done from a pedagogical perspective in which feminist pedagogy and nondiscrimatory classroom practices are combined to define a frame of reference for an inclusive pedagogy in mathematics education. The inclusive pedagogy in mathematics education needs to take into account the complexity of the different manifestations of discrimination against women in their learning of mathematics, and its framework offers an alternative to this discrimination so that, in the future, women will be equal "definers" of the mathematical domain.

Note: I would like to thank Martine Clément, consultant in educational technology, for the artwork of the figure.

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[1] Association canadienne-francaise pour l´lavancement des sciences.

[2] MOuvement International pour les Femmes et l'Enseignement des Mathematiques.

[3] International Organisation for Women and Mathematics Education.

[4] *Inequity in the classroom* is a multimedia package that includes a video, focusing mainly on sex and race discrimination, and a book, containing a training guide, fact sheets related to different forms of discrimination, an annotated bibliography, and a selected bibliography.

[5] The video and the manual can be purchased for $ 40 CDN each, plus mailing and handling costs, from: Visual Media Resources, Concordia university, H-341, 1455 de Maisonneuve West, Montréal, Québec, canada. H3G1M8 (FAX: 1-514-848-3441). Both are aviable i French under the title: *En toute égalité.*

[6] Most of these examples are taken from the Fact Sheets of *Inequity in the Class room* (Solar, 1992, pp. 53-68). More examples can be found in these Fact Sheets or in the bibliography provided.

WE WANT TO UNDERSTAND!

Else-Marie Staberg
University of Umeå

This paper reviews some results from my study *Different Worlds, Different Values: How Girls and Boys Meet Chemistry, Physics and Technology at the Upper Level of Compulsory School* (Staberg 1992). The study, written in Swedish but with an extensive summary in English, explores girls' and boys' actions in and thoughts about these subjects. The study has a feminist perspective focusing on girls. I followed two teaching groups from the start in Grade 7, when the pupils were 13 years old, until they made their choice of study programmes in upper secondary school in Grade 9. The main methods were classroom observations and taped interviews. Here I have chosen to give some glimpses of the results. I will concentrate on what girls and boys say about understanding and my interpretation of their sayings.

Do I Really Understand?

"In Grade 6 one thought it would be awfully difficult," says Maria in Grade 7. She is talking about physics and chemistry. From where did she get the notion that these subjects should be so difficult? She is one of the conscientious girls. Often unsure, her anxiety is hampering. She always finds fault with herself. As in the following episode in Grade 7:

> The pupils are heating water measuring the temperature. When the water starts boiling the temperature remains constant. The teaching idea is that the pupils shall "discover" this phenomenon, get excited, and draw conclusions. But Maria doesn't react according to thenorm. When her group's thermometer remains on 100° she turns to me startled and says, "Our thermometer is broken!"

The consequence of her anxiety about making mistakes is that she cannot reflect over her observations. Later her anxiety lessens and she declares that she likes the science subjects because she understands. Another girl, Eva, is more irritated than anxious in Grade 7. She says, "It is only in biology I understand something." Eva likes to explain to others and wants to become a teacher, but is doubting her ability. In Grade 9, she declares that

> All the teachers say that "Yes, you make too big demands ...." Yes, like that, but I don't think I make too big demands because I do not understand, OK, it is wrong to say that I do not understand at all, I understand of course a little, but not, that is, ... as I would like ... When you shall explain, you explain it totally wrong ... you understand yourself ... but it is so extremly difficult to explain.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

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Already in Grade 7 the girls talk about the importance of understanding. This doesn't fit with the common view that for girls it is enough to learn by heart. Nor do my observations confirm girls as rote learners in contrast to boys who have real understanding. These notions in the "girls and mathematics" discourse have also been deconstructed by Valerie Walkerdine & the Girls and Mathematics Unit (1989). My informants Anna and Sara are, instead, of the opinion that it is girls who want to understand, but are prevented by boys who might seem to understand. Sara exemplifies:

> That is what bothers me. They like to make an impression ... they like to seem to know a lot, like, ... if you say during the lesson, "We don't understand this," then, like... ,"Eehhh, don't you understand? But Jesus, we can't keep talking about that now, we must go further," they say. "But we want to understand this," we keep on saying.

Klara has similar experiences and tells me in Grade 7: "Then I ask ... and some boys just ... 'Ah, didn't you understand that'... like that, you know." To me it is noteworthy that these girls, who in many respects are self-confident, are so sensitive to the reactions of boys. None of these clever girls thinks that the boys have a better understanding. Klara says, "No I don't think so! Cause sometimes they seem quite nullified when somebody asks them." Sara even thinks that boys are the ones who learn mechanically and that this is enough for them.

> They can understand as far as they rattle off mechanically, but ... it doesn't matter to them, but it does to us if we don't understand. We want to understand.

The observations confirm the girls' statements. Several boys who through their actions show that they do not understand do not confess this openly in the classroom, nor during interviews. It might be, as Sara says, that it doesn't matter as much to them if they do not understand.

It seems as if the girls seek more than the atomistic knowledge that often is the result of studies in natural sciences. The details are clear to them, but it is irritating to them not to understand the whole. This is one of the reasons for girls gnawing insecurity about their real understanding. Sara, in Grade 8, says:

> Yes ... small ... I understand precisely all small components of the electric motor but I do not understand ... why it starts rotating ... you understand to a certain limit.

Johanna, in Grade 9, talking about chemistry, wants to read more about what is going to happen before she makes an experiment "cause otherwise you just see ... yes, a little brown here and a little blue ... but you don't get it ... *really*."

Anna, in Grade 8, finds it peculiar that she, during every lesson in physis, is thinking, "No, I do not know any physics, I do not understand anything." But when she comes home "then I get it, like ... but I always think that I don't understand."

I have chosen these quotations in order to show how preoccupied the girls are with questions about their understanding. In spite of the claims from the girls, observations and interviews show that several of the quoted girls have a good understanding. These girls probably ask questions during lessons because they have understood enough to know what to ask about and are confident enough to ask in public. Still they question their own comprehension.

But far from every girl dares to ask. Britta, who hardly puts up her hand when teachers ask questions in fear of giving the wrong answer and therefore be laughed at by boys, says in Grade 9 : "Like chemistry ... I hardly understand anything ... then it is no fun either."

Britta belongs to the really invisible girls in the classroom. Always friendly, sometimes a bit sad but not reacting outwardly to her situation. She would never ask in front of the whole class and very seldom asks for help during lab work. She never makes trouble and consequently gets very little attention. As an observer I could see that her self-esteem was very low and that she got very little response from teachers. But according to my opinion she was able to understand more than she imagined and was given the opportunity to.

The will and the courage to ask decreases over the years. When you have studied a subject a long time, there are more and more gaps in the knowledge that can become visible. It is overwhelming and instead "You just sit there and agree," as Carola says, responding to what Lotta says in Grade 9:

> I think it is rather difficult.... Because ... there are many new things ... and now we have studied it [natural sciences] for almost three years and ... if somebody asks and you still don't understand and you sit there ... then it is difficult to ask about the same thing ... it is no use asking.

Now let us look at boys' points of view. I will quote four boys who are doing well in school. None of them, however, is as clever as Anna, Sara, or Johanna.

> I missed all that with ... I was sick ... that with density, so it has been a bit more difficult than normally.... But, I will solve it. (Karl, Grade 7)
>
> Some maybe think it is difficult. I sometimes find it a little difficult. To sit there and try to learn nucleuses and so on, it is a bit boring ... but it is fun as well to know. (Roger, Grade 9)
>
> You have to think for a while to understand what it [physics] really is about. But if you do that, then you know. (David, Grade 9)
>
> It is easy for me to learn ... if I have read something once I know it fairly well ... there are those who have to read 15 times and still don't understand what it is about ... it depends on how effectively you read. (Urban, Grade 9)

The attitudes of these boys are different from those of the girls. They say that if you make an effort you will know. They do not worry about any lack of understanding. Not even boys with obvious difficulties in school talk about any difficulties understanding. Kurt, who has been given the lowest possible mark in physics, sticks to the idea that physics "is not so bloody difficult."

The majority of the boys display similar attitudes during all three years. There are, of course, boys who say that they are no good and will get low marks. But they keep talking more about the marks than about the understanding. Formulas and laws are awkward, they think, but, in contrast to the girls, it is very seldom a boy says he doesn't understand. They put the lack of success outside themselves.

The different reactions of girls and boys are similar to the different attitudes to success (or lack of) in mathematics discussed by Elizabeth Fennema (1979).

Are the Pupils Expected to Understand?

In reality, I am sure there are many – boys and girls – who have difficulties. A short description of a lesson illustrates how difficult it can be to understand.

> The goal of the lesson is to introduce Ohm's law. The unit ohm and the person Ohm are mentioned early in the lesson and the teacher demonstrates how to measure voltage, current, and resistance. Pupils in the front row read the instruments and the results are written on the black board. Eva and Pernilla are, like me, sitting in the back row. It is difficult to follow even if the instruments are designed for demonstrations. Eva asks, "Is this in the book?" When that proves to be true, she stops trying to follow the lesson and starts talking to Pernilla. But at the end of the presentation she suddenly asks: "What is ohm?."
>
> The message of the lesson was lost for Eva, she knows that and is bothered. After the introduction of Ohm's law the pupils, working in pairs, are measuring voltage and current in order to verify the law. Anna and Sara are doing very well. They have understood the introduction, they can manage the apparatus, know what results they ought to get, and adjust their measurements accordingly. Pernilla and Eva are getting peculiar results, partly depending on their unfamiliarity with the apparatus. As they did not understand the introduction of Ohm's law they are stuck until the teacher comes and "pilots" them to the correct results.

The concept "pilot" (Lundgren, 1979) implies that the teacher guides the pupil past the difficulties to a correct solution rather than helping her to a further understanding. However, the pupils meet lots of facts, concepts, rules, and laws during science lessons in lower secondary school, and one cannot imagine that more than a minority of the pupils can digest and understand them. On many of the observed lessons the theories seemed to be too much for most pupils.

But it is the girls who express a desire to understand. Girls who so often are accused of being swots, lacking real understanding, and only learning by rote. Maybe the pupils are not supposed to understand. The subjects they are trying to learn are grouped in the curriculum under the concept "orientation of nature." The curriculum has expressions like "orientation about," "getting aquainted with," and "made conscious of," aims that do not necessarily imply understanding. But at least the girls' interpretation of teaching and school books is that they are supposed to understand. The question is, what does "understand" mean? Do girls and boys interpret "understanding" in different ways?

Differences Among Girls

The active and successful girls create an impression that girls have the same possibilities as boys to make themselves heard in the classroom. But this is not the case. Many girls are very quiet, they "just sit and agree," as Carola said. These silent losers in the classroom are girls with a failing or at least ceasing interest in the natural sciences and who also accept and find it natural that boys have power over these school subjects.

The interactions in the classroom are such that a majority of the girls are learning that they do not understand. This meta-learning can contribute to the girls rejection of physics and chemistry as unfeminine and also promote a decreasing self-confidence. Among the three possible forms of learning (Lundgren, 1979) in the classroom – subject knowledge, linguistic competence, and meta-learning – only the latter becomes effective. The girls' interest or disinterest in physics and chemistry are closely bound to their notions about their own understanding or lack of it. Another factor is the masculinity of these subjects. Three ideal types emerged among the girls in the study: *Interested* girls, who, in spite of success, often feel that they would like to understand more comprehensively; *disinterested* girls, who, in spite of some success, do not think they understand and also reject physics and chemistry as unfeminine subjects, and *unsure* girls, who find the subjects unfeminine but also look upon themselves as failures.

I found it useful to relate the girls' attitudes to their own knowledge to the different ways of knowing, described by Mary Field Belenky and her co-workers (1986). The unsure girls represent partly the form called "silence," meaning also an inner silence, a lack of an inner voice. "Received knowledge" is also common among the unsure girls. The girls receive knowledge from authorities with whom they do not identify. The authorities have the right answers, but are "they." The uninterested but more self-confident listen to their own voice, listen to it finding it as good as the voices of the authorities. They tend to "subjectivity." Among them and among the interested girls "received knowledge" still persists. Both, but especially the interested, have accquired "procedural knowledge," and the latter also show signs of "constructed knowledge," built with their own voice and the "procedural knowledge." All girls, except the most silent, strive for a "connected knowledge" (in contrast to "separated"), a knowledge that connects their own and others lives and values with the subjects studied.

The girls have different attitudes to the strive for power in the classroom during lessons in natural sciences, shown by the boys. Some *ignored* the boys, usually the ones I defined as interested, others were *irritated* and could be found among the interested and the disinterested. Those who *accepted* boys' domination in the classroom belong to the disinterested or the unsure. Almost all the accepting girls have parents without a higher education, in contrast to the interested girls, who come from families where, especially the father, has a higher education, often in science.

What I have said concerns the subjects physics and chemistry. Technology, which nowadays is part of the "orientation of nature," is disliked by all girls. They are bored by the activities and define technology as a domain for boys.

The Construction of Gender

The pupils come from different worlds determined by gender and social background. The data show that the construction of gender continues in the classroom and is connected with the background of the children. I agree with Sandra Harding (1986) that this shaping occurs on three levels – individual, structural, and symbolic. The data also show that the gender construction in the classroom is connected with a social construction of chemistry, physics, and technology as masculine domains. These mutual processes, concerning gender and technology, have been pointed out by Cynthia Cockburn (1991). Alison Kelly (1985) has earlier shown that boys masculinise school science.

Already the cold and hard surroundings, above all in physics and technology, signal a symbolic masculinity. The content, and especially some practical tasks, have strong connections with boys' activities and toys. The boys' construction of the subjects as masculine manifests itself as a desire to control content, apparatus, and experiments. They compete with each other but often also criticize girls. Girls get criticized if they give wrong answers, but also if they show too much interest or knowledge. Boys ridicule or they ignore girls, two known techniques used by members of a more powerful group towards members of a subordinated group. The female teachers are a counter-force, but their authority is questioned by boys, at least in physics and technology.

On the individual level there are successful girls who, depending on their family backgrounds, can break the pattern and refuse to see chemistry and physics as structurally masculine domains. The majority of the girls, however, construct these subjects, like technology, as masculine. Through their construction of femininity they gradually distance themselves from the experiments, the boys' plays. They see maturity and femininity in contrast to the more childish boys and the masculinity of the subjects. The girls' accentuation of a holistic understanding and a connected knowledge can also be interpreted as a rejection of masculine and separate knowledge.

All the girls are prone to question their understanding. To question your understanding is a way of expressing doubts of your intellectual ability. That we women often lack confidence concerning our intellectual ability is shown for example by Belenky, Clinchy, Goldberger, and Tarule (1986). I therefore suggest that girls' questions about understanding might be linked not only to the girls' craving for a connected knowledge but also to the social constructions of gender and science on the symbolic level.

To the existing "thought figures" about femininity belong ideas about women as illogical, irrational, and emotional. Women have also been excluded from science with the argument that people doing science have to be logical, rational, and objective, traits that "by nature" are given to men, not to women. These notions still exist, at least unconsciously, even if women formally have access to science.

Valerie Walkerdine and the Girls and Mathematics Unit (1989) have shown that teachers explain away girls' success in mathematics, and their interpretation is that in the discourse of girls and mathematics it is against nature that girls have mathematical ability, ability to reason. My interpretation is that also girls are aware of these gender dichotomies. They have a conscious or unconscious knowledge of symbolic gender, making them question their own ability for logical thinking, their reasoning.

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GENDER AND MATHEMATICS EDUCATION IN PAPUA NEW GUINEA

Neela Sukthankar
University of Technology
Papua New Guinea

"Educate a boy and you educate one person. Educate a girl and you educate a nation" (UNESCO, 1980).

Papua New Guinea among many other developing countries in the world today, faces the problem of illiteracy and women's education. This is a global issue. By educating a girl, we can say that we educate not only the future families, but we are also educating to provide the most basic human needs. This is a worldwide concern, one that raises questions about overall human survival and the future of mankind.

Women of Papua New Guinea, like many elsewhere, do not as yet play a significant role in making the major decisions that affect the economic, political, and social direction of the country. Very few women have been elected to the national parliament or provincial governments, or hold senior positions in the public service and private enterprise. Because of this, women have little direct influence on the decisions being made about the future development of Papua New Guinea. Women's education can play a special role in this country to prepare women for greater participation in decision making.

To talk about the need for a policy to integrate women more fully in the society in general, and in employment in particular, strikes nobody as odd, although in most countries there are slightly more women than men. They present the only example of a majority relegated to the fringe of society, where they are kept firmly in place by concerted and highly effective mechanisms of a male-dominated system. A characteristic of this status is a fundamental ambiguity between theoretical equality of rights and effective discrimination in their application.

In the first part of this paper we deal with the mathematics education of girls in Papua New Guinea, referring to the social and cultural context. Giving details about the education system in PNG, we look for various reasons why females are reluctant to enrol for higher university education. We also discuss the causes for women's low and unequal participation in the work force, especially in the highly professional jobs, and what would be the possible reasons and remedies.

The second part of this paper describes our research project currently being

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carried out at the University of Technology to determine the mathematical performance of female students in comparison with their male counterparts. In the Department of Business Studies, the results indicated that in the first year at Unitech the relative performance of male students was significantly better compared to female students, but in the second year the female students performed better in comparison to males. We would like to find possible reasons for such a change. Are there social factors involved that affected the transfer from national high schools to universities for the female students? We are carrying out this research in various other departments at Unitech and also at UPNG in National Capital District. The detailed research will be published elsewhere.

Education of girls in Papua New Guinea

Traditionally, the girls in Papua New Guinea are not encouraged to pursue any education. To educate a son is considered an investment. Consider the case of a girl in her household: she is a secondary mother figure, cooking, cleaning, and looking after her younger brothers and sisters. She has hardly enough time to go to school, let alone to study enough to make the going worthwhile. This may sound extreme but it is in no way out of the ordinary in a country where the average family consists of six or seven children. But cases like this only serve to widen the rift between the male and female students in Papua New Guinea. It is only too obvious that, if we are to improve the quality of education the **girl child** receives, we must change prevailing attitudes towards women. This social attitude, which perceives women as housewives to be, is unfortunately institutionalized by the society in its education system.

In general, PNG culture puts male interests first. One direct result of this is that families tend to favour their sons for educational opportunities, especially if there are tight financial circumstances. Many Papua New Guinean parents see education as an investment leading to modern-sector employment and they prefer to invest in their son's education rather than their daughter's as they do not benefit from their daughter's income after her marriage. Throughout Papua New Guinea, there is a system of "bride price," where the family and the clan of the bridegroom are supposed to pay a certain amount of money and valuables to the bride's family in a negotiated deal. The bride price symbolizes the transfer or exchange of rights over a woman from her own family to her husband's. The bride price that she would receive does not necessarily depend on the education she receives. This custom is so ingrained in the society that the persisting rule, even among well-educated families, is to send their male child to an expensive quality international school or maybe a school overseas, whereas the female children are sent to a smaller community school that costs a fraction of the fee and does not have such good facilities.

Education system in Pua New Guinea

In Papua New Guinea the education system follows 6+2+2+2 system. Community schools cater for Grades 1-6, the provincial high schools from 7-10, and the four national high schools teach Grades 11 and 12. The Grade 12 school leavers join the two universities for their university education or the tertiary institutions for their vocational training.

Due to limited places available at the secondary and high school level, not every child who enters the primary or community school is able to go up to Grade 12. There are Grade 6, Grade 8, and Grade 10 drop-outs. From the records of the Statistics Division, Department of Education, only 10% of Grade 10 students go to Grade 11, and of these 10% only a third are females. From the enrolment records of the two universities we observe that not more than 17% of the entrants are females, which shows that only a small number of females who complete Grade 12 opt for university education. As 95% of our female students continue through Grade 12, why are only 17% of the total university entrants females?

We should look into the various reasons why females are reluctant to enrol for higher university education. The case study of female students by Penny Warner Smith at the Goroka teachers' College shows that the female secondary trainees at GTC were more or less pressured by their parents to find employment to support their families rather than going for higher education at the universities (Crossley & Wormald, 1988). To increase the graduate rates of women and encourage higher participation, the universities should invest more funding for women. The universities have a dual responsibility: firstly, to retain the female students by taking care of their special needs, thus ensuring there are no further drop-outs and, secondly, to make a special effort to encourage female enrolment at the university. There is a high rate of female drop-outs from the registered courses. It is therefore recommended that the balance of investment should be positively biased in favour of women until the number of women seeking university training approaches parity with men.

We will now discuss the **factors affecting enrolment and retention** at school level. Increasing the percentage of females in the primary schools will reduce the imbalance in education and employment opportunities between males and females. This cannot be achieved without affecting other components of the education system. If we increase the number of girls in the primary school without proportionately expanding the number of places available in the secondary school, we simply increase the number of Grade 6 school leavers who are denied entry to high school. It is therefore necessary that girls who enter the primary schools should be assured of their admission to the high schools and the university. Since the number of females entering the university is so low, it would be a good idea if some reserved quota system is introduced to encourage more female participation in the education system. In fact, there should be an increase in the number of primary and secondary schools so that every child can have a chance to complete at least Grade 10 education.

Education of girls in Papua New Guinea should be considered as a priority; access to community school should be the right of every child. The locations of community schools in villages are some of the constraints faced by girls in attending schools, especially if they have to walk long distances in order to go to school. In certain areas of low population, an informal boarding school may be an option. This should be monitored by school inspectors so that parents can feel confident in allowing their daughters leave home for education at such a young age. All sectors of the community need to be aware of the need for educated Papua New Guinean women.

Male jealousy of women in responsible senior positions or with more educational qualifications or better wages is another constraint on female achievement. Such attitudes have a serious effect on women in employment and in tertiary education. There is a need for **female role models**. In universities or teachers' colleges barely 15% of the staff are females. Some believe that **distance education** may be a better mode of study for Papua New Guinean women, so they do not come in contact with the negative effects of male students' attitude or male teachers low expectation of female students. However, some have pointed out the lack of proper study facilities for women as distance learners and the constant interruption of their study by their families and other community demands. In this respect, the women in rural areas are more disadvantaged than those in towns.

The **self-concept** of many female Papua New Guineans is also a stumbling block. Most girls are much more likely to see themselves in the role of either a wife or a mother, which generally does not command the respect they deserve. **Low self-esteem and low aspirations** are major constraints in educational achievements for girls. In short, the pace of social and economic development in Papua New Guinea will in part depend on effective measures to improve the status and productivity of women.

Mathematics education of girls in Papua New Guinea

Girls and women's low or unequal participation is deeply rooted in cultural values and **sex stereotyping**. There are certain subjects favoured by men such as woodwork, metal work, physics, and other sciences, etc., whereas home economics, arts, textiles, child care, etc. are favoured by women. Furthermore, female students are expected to be docile and retiring in an atmosphere which generally demands that boldness and flair be equated with success. The effects of this are especially obvious in mathematics and science subjects, particularly at the university level. Often the dichotomy is so great that female students opt to drop out of these courses in favour of arts-related subjects. It is no use women being taught the sciences and technical subjects and men being trained in domestic skills if these skills are not utilized once the individuals leave school due to stereotyping in urban and rural employment sectors. If this cultural gulf could be bridged, then the basis of sex stereotyping would be significantly reduced. But this is a change that cannot be legislated or planned for, educational policies and practices are thus invaluable as a way of changing people's attitudes.

Our aim is to investigate the cross-cultural issues affecting the mathematical education of females in Papua New Guinea and find ways of improving the quality of mathematical education. In PNG, as in any other country, the culture, social beliefs, and tradition play an important role in the education of females. The behaviour towards women is reflected through these social patterns. The expectations are different. What would seem a natural expectation may turn out to be the opposite due to the social norms. There is a difference between the life styles of men and women. Women are considered as subordinate to men, though this concept is slowly vanishing and therefore the sex-related differences are slowly diminishing. Women are officially encouraged to play an active role in the development of Papua New Guinea, but the majority of them do not even have an opportunity to obtain the secondary education.

Women pursuing higher level mathematics in Papua New Guinea today are inevitably pioneers, to a large extent, the first generation to do so. Naturally this means there are several problems concerning mathematics education for women. (a) predominantly male educators; (b) lack of female role models, and (c) tertiary educators (especially at university level) are generally expatriates, therefore creating the problem of cultural differences. It is therefore very important to fight for gender equity in mathematics education. Gender equity must focus on recruiting more female educators as well as on encouraging more female students to take mathematics.

If we consider the social context in which mathematics takes place in Papua New Guinea, the use of mathematics in society is often restricted to the most elementary use of arithmetic. In 1988, the National Statistics Bureau calculated that 86% of the population lived in rural areas. In such regions, far from the urban necessity for accounting, measuring, and calculating, the people have little need for advanced mathematics. It is little wonder that children from these backgrounds find it hard to accept the value of abstract mathematical procedures, and the mathematical education of women is therefore less than a priority. Peer pressures are naturally evident in the pre-tertiary education, before children have developed a strong sense of individuality. Mathematics is a keystone of PNG society, the basis of many other male-dominated fields at the University level (e.g. engineering, applied sciences, architecture, accountancy, etc.). At the present time, very few Papua New Guinean women work in these areas. This is often due to the fact that these are still considered as male oriented subjects, and unless women take special initiative, they are not particularly encouraged to fulfil the prerequisites aimed at securing admission to the university courses in these subjects. These professions are still tacitly designated as male provinces, and encouraging gender equity in mathematics will lead to women having a chance to work in mathematics-related fields as well.

Mathematics and sciences are still considered as male subjects in many developed countries. It is also observed that boys perform better compared to girls in these subjects and are expected to do so due to these social beliefs. Various remedial measures are being taken to change this attitude. To improve the performance of girls, some countries are trying sex-segregated schools and observing how this influences the performance of girls in these subjects. Sex-segregated schools are not at all common in Papua New Guinea. However, many provincial schools have segregated classes for certain subjects on a regular basis. It is usual to have girls doing home sciences while boys take woodwork. Although it has been observed that girls consistently perform better in these classes, it is difficult to say how much of this is a result of the segregation itself, and how much of this is due to the fact that teachers expect them to do well in these so called "female" subjects. In any case, there is probably not enough evidence of improved female performance to opt for widespread segregation.

There are very few women who have managed to secure top jobs such as managers, doctors, lawyers, or secondary or high school teachers and lecturers at the university. The women who have found their way into the workforce, mainly do the following types of jobs: (a) typists, receptionist, domestic, or menial workers; (b) preschool, lower primary teachers, or (c) nurses or orderlies in the hospitals. The types of jobs these women are doing do not require any high-level mathematics. The teaching jobs in the primary schools do require some elementary arithmetic and geometry. The secretarial jobs require some knowledge of word processing, operating fax machines, and jobs in the bank and the travel industry require some skills with specialized computer packages. Nevertheless, although these jobs require some familiarity with technology, the tasks are generally routine and require little in-depth understanding of the mechanisms involved. It is therefore very important that the female students aspiring to take up high positions in the society should take the prerequisite mathematical courses in national high schools. This can be achieved if their teachers and parents give them enough encouragement and support. This is slowly happening now; women are being made aware of this through media. There is a special weekly feature for women, published in the daily newspaper *Post Courier*, devoted to various problems faced by women in PNG. It also includes articles by women in high positions, to serve as role models for the women in Papua New Guinea. So changes are taking place, though slowly.

Many politicians and educators have found hopeful signs that women might actually be starting to achieve equal educational opportunities. Increasing tertiary enrolments and an increasing number of women in the work force have been cited as some of these signs. But there are no indications that rural areas are progressing past the restrictive cultural boundaries. As the number of schools increase in the rural areas, the progress will be a natural outcome. Nevertheless, there is a frightening phenomenon that cuts across rural-urban differences, a reaction against women. Law enforcement officials have noted a rising level of violence against women. What is worse, traditionalists have blamed this violence on reforms that have taken place since independence in 1975. The reforms have made it possible for women to have a greater say in the running of their lives, and the right to demand an education. The traditionalists claim that this has resulted in a loss of male authority, and that the violence is merely a way for men to protest against the disruption of their lives. In conclusion, it seems that the key is extensive legal and social reforms that must try to heal the cause, the harmful traditional attitudes, instead of the symptoms.

Mathematical performance of students in the Papua New Guinea university of technology

This study investigates the progressive mathematical performance of students in the Department of Business Studies from Grade 12 in the national high schools up to the second year of their education in The University of Technology. We are mainly interested in the comparison between the male and the female students as they go from national high schools to the university. We have used t-tests to determine whether there is any significant difference in the relative performance. The results indicate that in the first year at Unitech, the male students performed better in mathematics than the female students in the Departments of Business Studies. But this situation changed in the second year, when we found that the female students performed significantly better in comparison to their male counterparts. Comparing the mean marks of both male and female students in their second year, it shows that although the female students did not progress so well in comparison to the males from Grade 12 to the first year, the transition from first year to the second year shows that they made up for their losses of the first year.

The University of Technology caters to students seeking degrees in engineering, business studies, applied sciences, architecture, etc. The number of female students at the Unitech is about 17% of the total number and a large majority of them take either business studies or Applied sciences. In this paper we will restrict ourselves to the students from the Department of Business Studies, which consists of a high percentage of female students compared to the other departments. A detailed study of students in applied sciences and other departments, and also for the students in UPNG, will be published separately.

There are a few female students who have shown outstanding performance, but it has also been observed that many of them came from a privileged background. There is not much research done in this area in relation to the female students in Unitech. Colwell and Gilmour (1990) have discussed the comparison between females and males in physics and have shown that there is no significant difference between the performance of the male and female students in the first year of the university. However, their performance in mathematics and physics is highly correlated to their Grade 12 maths results. Robertson (1993) has also shown that majority of failures in Businesses studies are directly related to the Grade 12 results of these students.

Presently, we are interested in investigating how the Grade 12 results have a recurring influence on the progress of these students, and in comparing the mathematical performance of female students with their male counterparts. We have considered the continuous progress for three years of the students from business studies. Our sample consisted of 83 students of business studies, and their progress over the three years 1989, 1990, and 1992 has been closely monitored. In 1991, there were students' strikes and consequently they boycotted the exams. The same students rejoined this year and we continued to record their progress this year. Business studies students take courses in mathematics computing, accounting, economics, etc. During their second year at the university, they take advanced courses in these subjects in addition to others. The university keeps a record of these students' Grade 12 marks from the national high school. The Grade 12 students take either the "Major" or the "Minor" maths course. The first-year marks of these students are their final end-of-year marks in mathematics and consist of continuous assessment and the final examination marks. In the present study, only the national students are considered. Although the main aim of this work is related to the mathematics performance of the female students, we also wish to investigate how this affects their general performance in their chosen degree courses.

Table 1

Students Performance in the years 1989, 1990, 1992

	Males	Females	All
	50	33	83
1989 Mean	54.230	55.970	54.928
Std.dev.	9.709	8.053	9.075
1990 Mean	63.220	59.121	61.590
Std.dev.	14.437	11.973	13.585
1992 Mean	63.180	64.424	63.675
Std.dev.	9.009	8.474	8.770

The results of the first t-test are given in the following:

Table 2

Sex	N	Mean	Std Dev	SE Mean
Male	50	9.0	12.3	1.7
Female	33	3.2	11.1	1.9

$p = 0.028$

A close look at the marks in Table 1 shows that there is an increase in the mean marks of both groups during the last three years. We also note that the standard deviation is comparatively high. T-test is performed on these samples to learn more about the behaviour between the female and male students' progress. The first t-test performed is to determine whether there is a significant difference between the relative performance of the male and the female students in the first year of their university exams and their Grade 12 marks in the Business Studies Department. The second t-test is done to see whether there is any significant difference in the relative performance between the second-year and the first-year male and female students in the Business Studies Department.

The results of the second t-test are as follows:

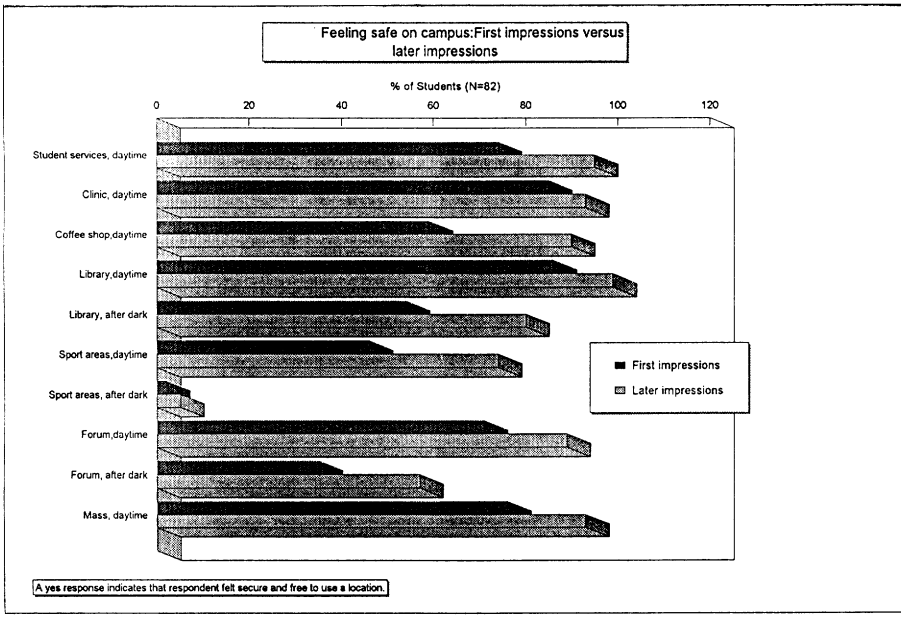
Table 3

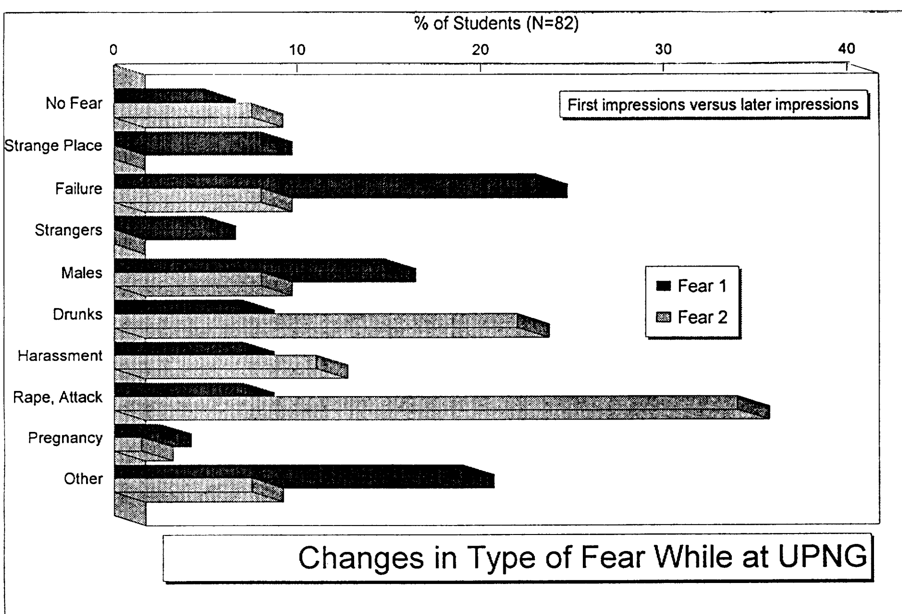
Sex	N	Mean	Std.Dev	SE Mean
Male	50	-0.0	11.4	1.6
Female	33	5.3	11.8	2.1

$p = 0.045$

Both these tests show that the results are significant at 0.05 level. There is a very significant difference in the relative performance of the male and female students studying in the Department of Business Studies at the University of Technology.

Table 2 shows that the mean of the male students increased by 9% compared to a 3.2% mean increase of the females. This shows that in the first year of the university, given the same conditions, the male students performed better in mathematics than the females in business studies. The picture in the second year is completely different, as can be seen from the Table 3. There is no increase in the mean of male students going from first year to second year, however, female students have a 5.3% increase in their mean over the first year. In the second year, there is a difference in the relative performance between the male and female students in the Department of Business Studies at the university. In fact the girls seem to be performing better in the second year. We note here that the mathematical topics are different for both the years. They do more statistics in the second year compared to business mathematics in the first year.





We do not know the reasons for such a disparity between the two performances. However, one cannot deny that the results of females in the second year are astonishing. There could be several reasons why females performed so badly in the first year. From the graphs above we observe that females might have faced lots of problems while settling down in the first year, which affected their performance. Most of the girls join the Unitech after year 12 in the national high schools. The sudden change from a small secure high school environment to a large university campus has been frightening for a majority of the female students. If we study the graphs carefully, we note that the female students were more relaxed after their first year on campus. We tried to investigate this by going through the responses to our questionnaire to 100 female students on the Unitech campus. We may comment that if the female students had the potential to perform as well as their male fellow students, then they tried to make up in the second year for what they lost in the first year. Certainly, there is a trend to make us believe this possibility. There are not many courses at Unitech where the percentage of female students is as high as in the case of the Business Studies Department, but we would like to investigate whether a similar trend exists in UPNG, where there are courses attracting a good percentage of female students, to compare these results in a more general situation.

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GENDER AND MATHEMATICS EDUCATION: THE VIEWS FROM CHINA

Tang, Rui Fen
East China Normal University

China has a long history and a large population. Chinese mathematics and mathematics education also have an age-old tradition and have exerted great influence upon some regions and throughout East Asia. Foreign thoughts on mathematics education entered China, first from Japan (1910 – 1920) and then from the United States (1921–1949). In the period 1949 – 1960, the Soviet Union exerted a great influence on China. However, in the early 1960s, China established a complete system in mathematics education and began to shape her own character.

China is a developing country. In the past few years things have changed considerably. There is now a national policy of nine years of compulsory schooling: grades 1–5 (elementary) and 6–9 (junior high). The State Education Commission, in charge of all Chinese education, has demanded that the concentration of attention on the "rate of passes" on entrance examinations alone be stopped. A change of emphasis from "education for the elite in order to pass the examination, to education for all citizens in order to enhance their knowledge" is now widely recommended. Mathematics education reform is under way in China.

Current Situation

In China, just as in other countries, women tend to avoid mathematics, science, and technology. Even though our country places emphasis on equality between females and males, and mathematics courses in general are compulsory until senior high school, there is still a concern over women's low representation in mathematics and related fields.

For example, mathematics competitions are small "hot spots" of social life in China. There are various competitions organized at the school, county, city, and provincial levels. Since 1989, the Chinese team has won first place in the International Mathematics Olympics (IMO) several times, but the holders of the gold medal in the IMO are all boys. There are 20 boys in the preparation team for participation in the IMO next year, and no girls. Of course, many girls have won mathematics competitions at a variety of other levels.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

We have some special mathematics schools that provide particular training for those who are interested in mathematics; they take mathematics courses after school (generally on Sunday). In the mathematics school of Shanghai, there are several hundred students; however, the girls make up only about 10%.

In 1990–91, the second International Assessment of Educational Progress (IAPE) was conducted. The subjects were divided into two groups: one age 9, the other age 13. For the 9-year-olds the test items were mathematics and science, and for the 13-year-olds mathematics, science, performance and geography. Twenty countries participated in the project.

China participated only in mathematics and science assessments of 13-year-olds (about 1500 students). The mean percentage of correct responses for mathematics was 80, placing China first among the 21 populations, but the mean percentage of correct responses for science was only 67, putting China in 15th place. The detailed achievement of boys and girls is as follows:

Means and Standard Deviations

	Mathematics	Science
boys	81.7 (1.0)	69.4 (1.2)
girls	78.5 (1.1)	64.8 (1.1)

Further statistical analyses show significant differences for the mathematics and science achievement for boys and girls.

A nationwide survey of mathematics in Chinese junior high schools was carried out in 1987 in 15 provinces and municipalities throughout China. Approximately 50,000 grade 9 students from 1,209 classes of 614 middle schools took the examination. The examination was divided into two parts: part I emphasized the mastery of basic knowledge and skills, part II emphasized the ability to analyze and solve problems. The means for boys and girls are shown below.

	Means for boys		**Means for girls**	
	Part I	Part II	Part I	Part II
whole country	74.87	58.52	72.15	55.27
factories and mines	65.78	46.67	63.75	54.20
cities	72.83	55.47	71.82	54.20
towns	74.95	58.87	72.78	55.60
countryside	75.80	59.78	73.06	56.71
upper schools	82.34	69.00	80.82	66.56
middle schools	74.23	57.38	71.19	53.76
lower schools	68.81	50.29	65.80	47.40

Boys outperformed girls. We used the U-test to examine the statistical significance at the 0.05 level. The results were as follows:

Values of U test

	Part I	Part II
whole country	12.49	11.85
factories and mines	1.96	1.56
cities	2.02	2.00
towns	3.35	4.01
countryside	10.11	8.97
upper schools	4.53	5.31
middle schools	9.40	8.91
lower schools	6.04	4.86

There were significant differences in all but the schools attached to a factory or mine. However, the differences varied between provinces and kinds of schools.

In another project, the purpose was to explore the development of mathematical thinking ability of students in junior high school. Although the thinking ability of boys was slightly greater than that of girls, there was no significant difference. We found that:

1. In different features of thinking activity, boys and girls have their own strong points. Generally speaking, the girls are independent and stable, whereas the boys are flexible and reducible.
2. The development of thinking ability of girls is earlier than that of boys. As observed, girls' ability grows quickly at junior high grades 1 to 2, whereas boys' ability grows quickly at junior high grades 2 to 3.

Why have women been seriously underrepresented in mathematics and related fields? Why is the gender difference in mathematics achievement found sometimes and why does it vary in different places and times? What are the determinant factors?

In my personal view, I agree with the simple premise that there are no physical or intellectual barriers to the participation of women in mathematics, science, and technology. I agree that one cannot explain boys' superiority in mathematics on the basis of biological differences. I believe the determinants are social and cultural barriers, that is, the traditional ideas of people, force of habit, public opinion, the attitudes of teachers and parents, and all consequences arising therefrom, in particular, the impact on students' psychological factors (e.g., belief, confidence, the attitude towards difficulty, and so on).

It is well known that China was a feudalistic country for a long period of time. Therefore, discrimination on the basis of gender has a long history. Although times have changed and our country has carried out a series of policies to ensure equality between the sexes, feudal thought has not been thoroughly eradicated; quite a few people, including teachers, parents, and women themselves, think that women are inferior to men in many respects.

We have done a study of 60 teachers and 510 students in junior high school for the purpose of exploring the factors that influence the gender difference. Some findings are:

1. Teachers give different assessments to boys and girls. They frequently attribute boys' poor achievement to inadequate effort and girls' to poor ability.
   They analyze the causes of success and failure for boys and girls as follows:

For boys:

the causes of success		the causes of failure	
a. intelligent	83.3	a. not diligent	71.7
b. has interest	36.3	b. naughty	41.7
c. concentrates in class	26.3	c. has no interest	20.0
d. diligent	15.0		

For girls:

the causes of success		the causes of failure	
a. diligent	80.0	a. not intelligent	68.3
b. concentrates in class	38.3	b. not diligent	16.7
c. has interest	18.3	c. has no interest	15.0

   24.3% of the students think the cause of mathematics anxiety is that "the teacher thinks I do not have the aptitude for mathematics"; 62% of them are girls, showing that girls are more apt to be influenced by the teachers' assessment than are boys.
2. Students' attitudes towards mathematics learning: Most of them hold the view that "girls are inferior to boys at learning mathematics," and the reason is that "girls are not as intelligent as boys."

To the question, "Who can learn mathematics better, boys or girls?", the students responded as follows:

Who can learn mathematics better?

	Boys Better	Girls Better	Both better
Boys	202	5	47
Girls	199	4	55
Total	401	9	102

Some girls stated that "teachers always say boys are more intelligent than girls," and that "people often say boys generally learn more quickly than girls when they enter junior high school, particularly in geometry." The attitude people unwittingly adopt may have an unjustifiably negative effect on girls, causing them to lose confidence in their ability to learn mathematics.

These attitudes are primarily manifested by the degree to which boys are more encouraged in the study of mathematics than are girls. As a result, girls' perceptions of the difficulty of mathematics are different; more boys hold positive attitudes than girls when faced with difficulties.

I believe the root cause of the gender differences in mathematics lies in tradition. There has not been much attention devoted to the issue of gender discrimination. In order to change the current situation, much has to be done; it is an important and difficult task. Below are some thoughts about ways in which this task may be accomplished.

1. *Changing the negative attitude of traditional ideas about gender and mathematics education.* It is not possible to change long-standing ideology in a short time. It requires every effort from all aspects, including the social context, cultural pattern, educational system, and public opinion. For instance, governments should formulate appropriate policies to protect women's rights, parents should hold more positive attitudes.

Since 1949, our country has formulated policies and taken various measures in order to ensure equality between men and women, to get rid of the remnant of the influence of feudalism. For instance, implementing compulsory schooling and compulsory mathematics courses, and setting up girls' schools. Progress is being made rapidly, but the problem still remains.

2. *Devoting more attention to the role of teachers.* Teachers are among the most important educational factors affecting students' learning. Teachers' positive assessment of students would alter their attitudes, interest, self-confidence, etc. To some extent the words of teachers have greater influence on students than those of parents.

We do need to improve teacher training concerning the attitude toward gender. Changing traditional ideas should begin with changing teachers' ideas. Teachers should be aware of gender bias, which exists in teaching practice. As teachers, we must have fair views and scientific attitudes. In this respect we can motivate students' interest, encourage enthusiasm, and attract their attention to mathematics. In a word, teachers should always adopt a positive attitude to students.

3. *Cultivating students' strong psychological development.* The traditional ideas, force of habit, and public opinions are all external factors that influence mathematics performance through students' internal factors – their psychological development.

We must encourage students consistently, foster in them the ability to adopt a correct assessment of themselves, especially girls as they underestimate their level of performance more frequently than do boys.

WOMEN AND MATHEMATICS IN NEW ZEALAND UNIVERSITIES

Gillian Thornley
Massey University

Women as Students

The University of New Zealand was established in 1874 with a charter that permitted women to be students equally with men. This was only achieved through intensive lobbying (Grimshaw, 1972). That same year K. M. Edger applied to sit the entrance scholarship in mathematics. So it was that in 1877 we had our first woman graduate, Kate Edger, with a BA in mathematics and Latin. She was the first woman to graduate BA in the British Commonwealth, and Queen Victoria sent a letter of congratulation. (Grace A Lockhart graduated BSc from Mount Allison University, New Brunswick, in 1875.) Kate then took up a teaching position at the newly established Christchurch Girls' High School and studied part-time for an MA in Classics. At the age of 26 she was the foundation Principal of Nelson College for Girls. Seven years later she offered her resignation on account of her marriage, but the College Board of Governors persuaded her to stay another year until the birth of her first child. She raised a family and continued in teaching and other work (sometimes part-time) for a further 20 years.

Many other women followed Kate Edger to university, and by 1893 more than half the students in the University of New Zealand were women. (Needless to say, the proportion of women students was much lower through most of the 20th century, and we are only just reaching equal numbers again in the 1990s.) Some of these women chose to study mathematics and to continue on to graduate level.

Early calendars of Canterbury College record that 7 women were among the 31 students who gained master's degrees with honours in mathematics or mathematical physics in the period 1889–1919, and a further 4 women from Victoria College gained master's degrees with honours in mathematics in the period 1901–1911.

A study of the academic records of science students in the University of New Zealand 1933–1950 (Dick, Williams & Straker, 1955) shows that during this period women gained 16% of the BSc degrees in pure mathematics and 9% of the master's degrees in mathematics. The latter represents eight MAs and seven MScs. It is almost certain that women would have gained a higher proportion of the BA degrees in mathematics.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

More than a generation later, we have the following data from Education Statistics of New Zealand (1977-1991) for numbers of graduates in mathematics, statistics, and operations research, over three recent five-year periods.

Bachelor Degrees	**Women**	**Men**	**Women as % of total**
1976 - 80	297	754	28%
1981 - 85	318	754	29%
1986 - 90	395	761	34%
B(Hons) and Masters Degrees			
1976 - 80	60	287	17%
1981 - 85	61	229	21%
1986 - 90	63	190	25%
Doctoral Degrees			
1976 - 80	2	27	7%
1981 - 85	2	10	17%
1986 - 90	3	14	18%

The increase in numbers of women studying mathematics at the bachelor level is not really flowing on to post-graduate degrees, despite the steady increase in percentage of women in honours and master's degrees! In 1993, 16 women are enrolled in PhDs (31% of the total), so the next three or four years should bring increased numbers of women graduates at that level.

Women as Staff

What happens to women graduates in mathematics? Do they claim a reasonable share of the creative jobs in the field or do they end up in the kitchen and nursery?

Many of the early women graduates went into school teaching, providing staff for the new girls' schools, which in turn provided students for the university. Dick, Williams, and Straker (1955) give details of the employment of some of the science graduates from the period 1933-1950. Half the 300 women graduates in mathematics were evenly distributed between teaching, government departments, and "overseas," while the next significant block was in scientific and industrial research. One disturbing aspect is that six of the seven women with MScs in mathematics fell into the category "not employed – married." Needless to say, no men were in this category.

An analysis of surveys taken in the year following graduation from 1973 to 1986 (Purser & Wily, 1987) shows women with mathematics degrees participating in all employment categories, although they are grossly under-represented in university teaching, and to a lesser extent in science and engineering and in insurance, while being over-represented in the statistics and manager/advisory officer categories.

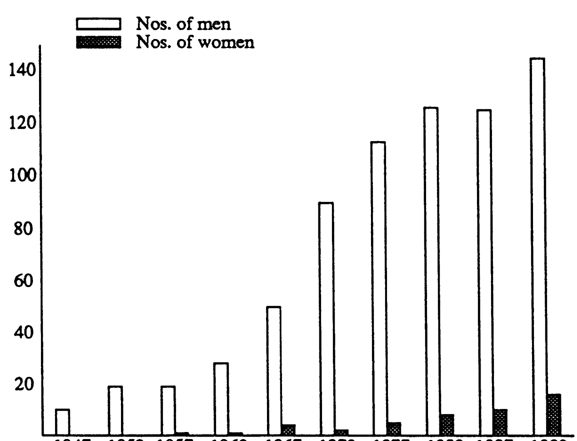
In the past, the universities and the Applied Mathematics Division of the Department of Scientific and Industrial Research (DSIR) have been the major employers of doctoral graduates in mathematics. Have women been gaining their share of these jobs?

According to the records in university calendars, the first woman to join the university mathematics staff was Christina Bell, who was "Assistant to the Professor of Mathematics" at Otago University from 1924 to 1928 (there were no other mathematics lecturers at Otago). Christina graduated MA with first class honours in 1925, and the lecturing position was apparently a temporary one. She took up secondary school teaching and became Principal of Whangarei Girls' College.

During the second world war women appeared among the university staff again, but in temporary positions. Theodora Marsh MSc (NZ) was assistant lecturer at Otago from 1939 to 1942 and assistant lecturer (temporary) at Auckland from 1942 to 1946. (Was she the one MSc in mathematics from the 1933–1950 period who escaped marriage and unemployment?) Una Powell MA (NZ) MA (Cambridge) was assistant lecturer at Canterbury from 1942 to 1945.

Dr. Mary Harding MA (NZ) PhD (Edinburgh) was the first woman appointed to a lectureship. In 1953 she was appointed Warden of Helen Connon Hall and part-time lecturer in mthematics at the University of Canterbury. She relinquished the Warden's position in 1958 to lecture full time. At that stage she was the only person with a doctorate among the mathematics staff of six at Canterbury. She had completed her doctorate in the 1930s and this was her first full-time appointment to a university mathematics department. She had previously taught in girls' schools and been principal of a ladies' college.

The graph shows the subsequent growth of university mathematics staff, with men and women shown separately. These are numbers of staff (lecturer and above)



New Zealand Universities - Permanent Staff in Mathematics, Statistics and Engineering Science at five-year intervals, according to records in university calendars.

In 1993 women make up 10% of the mathematics staff taken over the country as a whole. Nevertheless, one large mathematics department has no women academic staff, and has been in this situation for many years (possibly about 20).

The Applied Mathematics Division of DSIR had a slightly better record over the period of its life from 1942 to 1992. On average, women were almost 22% of the scientific staff in the mathematical physics, statistics, and operations research sections. It should be noted that the Applied Mathematics Division employed people with bachelor and master's degrees, as well as those with doctorates. Of the women employed, 23% had doctorates compared with 29% of the men. Last year the Department of Scientific and Industrial Research was restructured into several Crown Research Institutes. A smaller Applied Mathematics Group is part of Industrial Research Ltd., but some mathematics/statistics positions have been scattered among other Institutes. This is not good news either for mathematics or for women.

Conclusion

It is clear that women are still seriously under-represented both among the students and among the staff of university mathematics departments. The two problems are not unrelated. The small number of women staff are scattered over six universities and 10 departments, giving only a token woman or two in each one.

Over the past four years women post-graduate students with Graduate Assistant duties have raised the "woman-presence" considerably in our department. Two of them completed their PhDs this year and three more have begun. However this does not necessarily lead to jobs on the permanent staff. At present there is intense international competition for university positions, and overseas experience tends to be rated above the local version. We look forward to the time when women mathematicians in New Zealand attain that critical mass that will enable real change to occur.

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Panel 1:

GENDER AND MATHEMATICS EDUCATION

Moderator:

Mogens Niss

Panelists:

Gila Hanna, Calle Jacobsson, Christine Keitel, Anna Kristjànsdóttir and Gilah Leder

The overall task of the opening panel was to set the stage of the study conference. This is reflected in the fact that the title of the panel is identical with the title of the entire conference. More specifically, the panel was asked to address the following issue:

Where are we **today**, what are the most important current **problems**, and in what **directions** should **we** move – everything considered in terms of **research** and **development** perspectives on **gender**?

The crucial or controversial elements are emphasized in bold-face letters: *Where* indicates that the panel was invited to describe and analyze the present situation without taking any specific, established understanding for granted; *today* indicates that yesterday's situation was to be considered primarily to the extent that it sheds light on the present state of affairs; the attention paid to *we* is meant to remind us that it is not clear at all who "we" are, neither from a local nor from a global perspective; as far as *problems* are concerned it was an aim for the panel to strive to identify and deal with genuine and deep difficulties, obstacles, controversies, or dilemmas, not superficial, accidental, or fashionable ones; *directions* were emphasized as it is not a trivial matter to choose the ones in which we could/should move, perhaps because it is not trivial to decide and determine the end points of the motion. Finally, it should be kept in mind that an ICMI study is always devoted to *research* and *development*; this does not imply that attitudes, values, issues of policy, or politics should be left out of consideration – in addition to being an impossible endeavour this would also be an absurd one – only that these factors should be subject to reasoning, investigation, knowledge, analysis, and arguments; the term *gender* is underlined only to remind us that there are two of them and that from the very beginning the study was designed to deal with both genders.

Finally, the panelists of Panel 1 were not asked to try to reach any sort of consensus, or the opposite. Rather they were expected to articulate their possible

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

disagreement so as to make the ensuing discussion illuminating, meaningful, and challenging.

Condensed versions of the opening statements of the panelists, given in alphabetical (surname) order, follow below.

Opening remarks

Gila Hanna:

First, let me say that it is not the aim of the present conference to race towards universal solutions to the problems of gender and mathematics education. Instead, I see this conference as an opportunity to explore differing and often opposing views, and in the end to deepen our understanding of the important issues through this exchange of ideas. I presume we hold many different views of mathematics education in general. As far as I know, we also have different ideas on gender and mathematics education in particular: On how to look at the issues, on which issues are more important, and on an agenda for action. During this conference we should be prepared to argue for the positions we hold. But we should also be prepared to listen to others, and indeed to abandon some of our present beliefs if in the course of the discussions we find them to have been unfounded. Since I see no virtue in premature theoretical closure, I look forward to a few days of fruitful exchanges and lively discussions.

Learning and cognition

I would like to start by stating the obvious: People change constantly; they do new things; they behave in new ways. So that when we classify people, the classifications that were judged adequate at a certain time may no longer be appropriate at a later time. In addition, the classifications themselves may interact with the classified, so that it becomes necessary to constantly revise our definitions. In particular, the classification into gender, though it seems straightforward at first blush, is by no means a static one. This is because the object of classification is no longer the same. By this I mean that the set of characteristics which we had in mind 20 years ago when we spoke of the "typical" male or female student, whether in elementary school, in secondary school or in university, no longer corresponds to reality. In fact, our image of gender has changed for the better, as far as mathematics education is concerned.

Our expectations of what males and females can achieve in various domains have also changed. I myself start with the premise that there is no physical or intellectual barrier to the participation of women in mathematics, science and technology. This very seemingly innocuous statement brings me to an important current problem, namely the issue of gender differences in cognition.

On this issue, it so happens that otherwise antagonistic groups, those who hold the opinion that women are naturally inferior in mathematics and science, and those who do not hold this opinion, have found themselves on the same side of the fence. They both claim that men and women have "different" ways of knowing.

Much current thinking on modes of thought defines rationality not as monistic but as pluralistic. Cognitive scientists paint a complex picture of what it means to think and to learn. They talk about a plurality of modes of thought, which may depend on content (science, mathematics, literature), on culture (oral, western, oriental), on modality (narrative, logic), and, as some would like to add, on gender as well.

But if we accept the pluralistic view in general, does it make any sense to talk of "women's way of knowing?" Can women claim a "different voice" when it comes to learning and cognition? Will it ever be possible to design a research program to support or refute these claims? These are among the open issues that might well be discussed at this conference.

Factors generating inequities

In this conference we would also like to identify and examine factors that might generate inequities. As listed in the discussion document, there are a number that seem to warrant close scrutiny: Attitudes of students, teachers, and parents; social perceptions of gender roles; cultural patterns; expectations; mathematics as a discipline, the question of identifiable gender biases in the theory and discourse of mathematics; curriculum and school setting.

Though women have made enormous gains on the professional front, aided by active organizations and, of course, by appropriate legislation, they still have a long way to go. The doors to education are now wide open, at least in North America, and, as far as I know, in several European countries. Although formal barriers have been removed, however, women have not rushed to avail themselves of the new opportunities. We have to ask ourselves why this is so. What are the informal barriers to women reaching for equity in mathematics, science and technology? What can and should be done to remove them?

Calle (Carl) Jacobsson:

The dominant problem in Sweden and in most other countries is the very small percentage of female students, and the extremely small percentage of female professors, in mathematics at university level. It seems that women choose not to study mathematics, and especially not to pursue a career in our subject.

In the Department of Mathematics at Stockholm University we are running a project for gender inclusive mathematical education called *Women in Mathematics* (Kvinnor i Matematiken – KIM). During the first year we tried to identify factors that discourage women from engaging in studies of higher mathematics. Ylva Elvin-Nowak, of the project team, conducted a survey that consisted of a questionnaire, answered by some 200 students and 30 university teachers, and 17 in-depth interviews. This survey made us see three big problems with current university mathematics education:

(1) A poor study climate, with little contact between students and teachers and amongst students themselves.
(2) Mathematics is a very difficult subject to most students. The students complain that teachers often have a very good command of mathematics but little understanding of students' difficulties.
(3) Female students, more than male, feel it is important to see applications of the subject, but women tend not to find mathematics as applicable as do men.

During the second year of the project, 1992-1993, we tried several experiments to remedy some of these problems. In my opinion, the following experiments were the more interesting and promising:

(1) All female beginners were put into gender-balanced groups (half the number of the groups); the rest of the groups were purely male.
(2) A simple model for working with mathematics in small groups was introduced. First came a lecture hour; next the students worked in small groups; finally, in the last hour, the lecturer gave a sum-up. This model turned out to be very popular with the students. It also seemed to be fairly efficient as far as learning was concerned.
(3) A seminar series for the teaching staff, on gender and mathematics teaching, with six half-day seminars, was arranged in 1992–1993. Discussions were inspired and informed both by the results of the KIM-survey and by invited speakers.

Of course, these can only be minor steps towards the creation of a gender-inclusive climate and education at our department. I want to suggest further reform steps that may prove useful:

(1) A significant part of the education takes the form of applied projects and investigations performed by groups of students, supervised by the teacher. This has been done, for instance, in Roskilde, Denmark, and Southampton, UK, for many years.
(2) Older female students, graduate students, and teachers serve as role models and contact persons ("mentors") for the younger female students.

However, there are two basic requirements for getting started, and I think if these requirements are not met things become very difficult:

(1) A significant number of the teaching staff must be willing to contribute to a gender-inclusive education.
(2) The people in power in the department and in the university must not be passively against reform in this area. In fact, it is very much preferable that they are indeed actively *for* them.

Of course, nobody is overtly against more women entering education, but lack of interest can often be quite repressive as well. In my experience, it sometimes happens that very disinterested people can become rather upset by these issues and become "aggressively indifferent."

Christine Keitel:
When thinking about the problem of gender and mathematics education, I see myself as a victim as well as a contributor. I am a victim because, like most of us, I had to go through the experiences of a woman as an outsider in mathematical studies during my entire education. On the other hand, we might well see ourselves as contributors to the problem, because nevertheless we were successful. As teachers of mathematics or as university professors we became integrated into the system, with all its opportunities but also with all its deficiencies and failures, and maybe we now contribute to its functioning or malfunctioning. I sometimes fear that the women who completed their mathematics education successfully and became professors have already taken over, mostly unconsciously, some of the behaviour we are struggling against.

Concerning the work which has been done in recent years on our main theme, the most important development, as I see it, is a shift of perceptions, a shift from "the problems of girls and women in mathematics education" to "the problem of gender and mathematics education," as reflected in the title of our conference. This is an important step forward, and leads to other changes in approach. The first is a shift from biological aspects of differences to more general, social aspects, or in other words a shift from searching for deficiencies attributed to one or the other side (though mostly to the women and girls) to studying relationships between gender and mathematics education, a view which includes both sides.

I do hope that this shift to "gender and mathematics" makes for more of a balance. This morning Elizabeth Fennema emphasized this new perspective and proposed to consider the problems of men and boys as well. We all know that only *some* males excel in mathematics and that most boys have the same problems with mathematics as do girls. If we consider the issues from a sociological perspective, we also have to address problems of social class. And we can also view even research on "women and mathematics education" in a new way: we could use our results as indications of fundamental deficiencies in the whole enterprise of mathematics education as such.

I am afraid that the one-sided methods and restricted paradigms that we usually apply in research in mathematics education have biased our investigations of the problems of gender and mathematics education. Maybe we should look for a new start in research, with a more interdisciplinary, more philosophical, and more hermeneutic perspective. Gender investigations have been dominated not only by approaches belonging to positivist, empiricist psychology, but also by a heavy focus on "numbers" and "facts." I do not want to abolish this type of research, but it does transmit prejudices about research questions and paradigms which then tend to shape the results. Fortunately, we now have some studies, such as those that follow the case-study or narrative approach, which constitute an important corrective to data gathering and comparison because they require interpretation and stimulate new research questions.

Another bias in our research may be that we mainly center on cognitive and psychological matters. When using the term "cognitive" I am not referring to the distinction between "cognitive" and "affective," but to "cognitive" as a psychological construct which tends to see the girl, the woman, the pupil, the individual, as an actor mainly determined by psychological characteristics. An alternative view sees the individual as a social, interacting subject, who cannot be adequately studied by means of cognitive psychology alone, but only by reference to sociological or political frameworks.

Finally, turning to educational policy as one outcome of our research, let me recall a type of argument which is often encountered in the context of gender and mathematics. The argument runs that women and girls should pursue mathematical studies because mathematics is useful for everybody. The search for usefulness in debates on mathematics education has, in my opinion, to be fundamentally questioned. We do not have any evidence that mathematics education really *is* useful in the literal sense. Mathematics education is useful for passing exams, but I see a sharp contradiction between the mathematical knowledge needed for exams, on the one hand, and that needed for jobs and professions on the other. A lot of political advertising is going on here, and one has to question how mathematics is actually used in our society. I am not talking about the superficial kind of usefulness which students often ask about, nor about the usefulness of mathematics in the mechanics of everyday life. What I would like to see is a meaningfulness and usefulness in people's social and professional lives that would enable them to exercise judgement in the world in autonomous, competent, and self-determined ways. It seems to me that because of our limited perception of mathematics and mathematics education we have been led to disregard this wider value of mathematics. I believe this also contributes to the severe deficiencies of mathematics education in general as well as to the gender problem in particular.

Let me add a final remark. Perhaps the greatest and most unconscious use of mathematics lies in our obsession with numbers as arguments. Everything in our society is either connected to or justified by numbers, mostly in relation to money. Money provides the super-rational argument. Its rationality stems from the fact that money itself is materialized mathematics and borrows its rationality from mathematics. This identification of numbers with money is inculcated very early in primary school (we need only recall the overwhelming number of text problems that deal with shopping), yet it is seldom balanced later by other kinds of insight. It thus presents a basic example of an unconscious, questionable, but unquestioned use of mathematics.

Anna Kristjánsdóttir:
Gender and mathematics education is a complicated issue with many perspectives. To a great extent, it is culturally bound and has deep roots in the, often unspoken, beliefs and attitudes of both women and men. Personally, I feel in no position to express myself in general terms, so I have chosen to confine myself to talking about the part of the world and the kind of society I am familiar with.

There is a gender imbalance in mathematics education, evident in upper secondary education and even more so at university level. This fact needs little discussion, but the causes and reasons do. Although gender imbalance at lower stages is not evident in work or test results, we cannot immediately refute the possibility that the roots of the imbalance at higher age levels lie in the lower stages and are interwoven with the choice of content, ways of working, homework activities, and communications that take place in the classroom.

The message in school concerning how to succeed in mathematics is often misleading. "Do your multiplication tables, your long division, your fractions, your algebra manipulations!", "Do this accurately and in the way you're taught to, restrict your questions to 'how to proceed'!" Children who accept such requirements without any urge for deeper meaning, for connections, or for greater freedom in dealing with the subject matter, are likely to find themselves in a rootless world called mathematics from which they may hope to escape soon.

Reports on children's behaviour in class, their acceptance of implicit rules, efforts to attract attention, and so forth, often show an imbalance between the two genders into which we should look more closely in relation to the learning of mathematics.

Mathematics plays a vital role in many areas of school and adult life. My own survey studies show that children in the age range 11–14 do not see the connections that mathematics has with other subjects. So they are left to the public opinions of what mathematics is linked to – to physics more than to needle work, to geography more than to art, etc. Then, too, their attitude towards mathematics is coloured by the strong gender-subject grouping that exists in the environment but is not discussed, just accepted.

Mathematics is becoming more visible in the media, but often in a misleading way that is neither subject to criticism by people working with the media nor by the public. A survey of mine indicates that, at the same time, mathematics is becoming less visible in important aspects of the daily life in children's homes. Highly visible paper-and-pencil calculations are, for instance, gradually disappearing. This implies that education in mathematical awareness is becoming more important, and so is the teaching of a sensible use of technology in mathematics. Also in this respect, there are dangers of an imbalance due to the skewedness encountered in the two genders' interest in technology.

The discussion of mathematical problems follows the logical rules of mathematics. These rules are very different from the ones used in many other areas. You do not arrive at a logical conclusion in mathematics by establishing an agreeable compromise. The way discussions are conducted in mathematics seems strange, even hostile, to some, while natural to others. However, logical discussions need not be sharp, fast, devoid of humour. Making mathematical discussions more open to explorations would, we may hope, stimulate and encourage a greater variety of individuals to engage in a mathematical discourse – and in mathematics.

Gilah Leder:

Introduction

Almost three centuries ago the well-known author Daniel Defoe wrote:

> I have often thought of it as one of the most barbarous customs in the world that we deny the advantage of learning to women...If knowledge and understanding had been useless additions to sex, God Almighty would never have given them capacity; for he made nothing needless. (Defoe, 1697/1969, pp. 283-284)

What progress has been made since then? Do females and males now participate equally in mathematics courses and related pursuits?

The present

Contemporary data indicate that there is much overlap in the patterns of participation in mathematics courses of females and males. Nevertheless, some differences in favour of male enrolments continue to be reported for higher level and more intensive mathematics courses, related applied fields, and occupations that require substantial proficiency in mathematics.

There is also much overlap between females and males in achievement in mathematics. However, subtle performance differences, generally in favour of males, continue to be reported on selected mathematical tasks assessed through standardized and large-scale testing. What might be a possible explanation for the small, subtle, but nevertheless pervasive differences described? Examining the broad environment in which schooling in general, and the learning of mathematics in particular, take place is a useful starting point. Two specific examples allow us to focus on important background elements.

Example 1: Staffing at a university

Recent (March 1991) statistics suggest that there is a "good" gender balance at the university at which I am currently working: 53.1% of its employees are males, 46.9% females. When only academic staff appointments are considered, the figures are less balanced, however: 68,8% males and 31,4% females. The discrepancies in male:female rates of employment increase further with level of seniority.

On the academic teaching and research staff, there are very few women at the senior level. Only 15.4% of Senior Lecturer positions are held by women. The number of women employed above this level is extremely small (Monash University Equal Opportunity Committee, 1992, p.2).

The data presented here are consistent with those that would be gathered at many other Australian universities. They are likely to convey, reinforce, and perpetuate beliefs that certain occupations are more suitable for males than females.

Example 2: Data on other occupations

Several months ago a popular and widely read metropolitan newspaper (*The Age*, published in Melbourne, Australia) highlighted a report that examined current participation rates of males and females in a wide range of occupations. The following formed part of the list:

Hairdressing apprentice in 1990: 763 males and 14,459 females
Apprentice in 1991: 122,200 males and 17,080 females
Teaching student: 20,629 males and 54,143 females
Business student: 62,182 males and 42,643 females
Technical studies: 11,924 males and 941 females
(Adapted from Horan, Wall & Walker, 1992)

Is it unreasonable to suggest that statistics such as these could influence students', parents', and teachers' perceptions that likely career directions are based on gender divisions as well as on ability and inclination?

It is, unfortunately, all too easy to identify many more examples of cultural gender stereotyping and gender bias – some subtle, others direct. We cannot ignore society's beliefs, conventions, and expectations when we consider gender differences in mathematics learning.

Future directions

There is much that *can* be learnt from previous research. To maximize the chances of doing so we must engage in constructive dialogue, particularly with those in other countries and with researchers working in paradigms and disciplines different from our own. It is essential that as part of this dialogue due recognition be given to the context in which data were gathered and findings were obtained. We should tackle the difficult task of exploring how research questions might sensibly be refined further so that the many subtle, indirect, and less readily quantifiable factors are not ignored. We should welcome and foster diversity in the formulation and exploration of relevant issues. We also need to consider seriously the challenge of identifying and discussing gender differences in mathematics learning without at the same time further perpetuating these differences.

Reporter:
Mogens Niss

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Panel 2:

FEMINIST PERSPECTIVES ON GENDER AND MATHEMATICS

Moderator:

Elizabeth Fennema

Panelists:

Leone Burton, Suzanne Damarin, Ann Koblitz, Beth Ruskai and Jeremy Kilpatrick

Welcome to the panel on feminist perspectives on gender and mathematics. In order to set the stage for the comments, think about the subject of the panel. A feminist perspective involves viewing the world from a female point of view and while it can be understood and even presented by both males and females, it must include female voices. There is a wide diversity of female voices which are explicating feminist perspectives, and if after hearing what is said here, you don't believe that there is a wide diversity of feminist perspective, we certainly will have failed. By participation as panelist or audience, we should develop understanding of alternative perspectives. And as we develop understanding of others viewpoints, we will enrich our own views.

There are four women on the panel (I'm excluding myself) each of whom has a very strong voice and a strong and well articulated feminist perspective about gender and mathematics that they will present. They need no introductions as their words speak for them. Each will present her views. After that, you will have the opportunity to have your voices heard. Following that Jeremy Kilpatrick will have an opportunity to have his voice heard.

Leone Burton:

Thank you, Elizabeth. I would like to remind you all of something said by the colleague from the national agency for education in Sweden when she opened the conference. She drew our attention to the way that Latin, and Greek, operated as gatekeepers to education in some countries in the nineteenth century. In this century we all know the gatekeeper effects of mathematics and physics. I would like to offer you a conjecture that, in the next century, Subject X will operate as the gatekeeper and, if it does, somewhere in the back of university campuses, there will be esoteric groups of almost forgotten women academics, who will be busy-

B. Grevholm, G. Hanna (Eds).
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ing themselves with mathematics. And the powerful men will have moved to Subject X. They will be competing for positions of power and the females in Subject X will be raising questions about gender equity. However, we are here to talk about gender and subject. Therefore, before asking pedagogical questions, for me there are prior questions which relate to the philosophy and epistemology of mathematics. The kinds of questions to which I am referring are: What is mathematics? What does it mean to know mathematics? Various sources have pointed out that mathematicians do not waste time asking these kinds of questions. They engage in the doing of mathematics. But that does not make the questions less valid, nor less appropriate, nor, let me emphasise, less difficult. What contribution, therefore, might answers to these questions make to our social justice debate?

I am starting with two axioms.
Axiom one: there is differential engagement with mathematics at both the public and the private level across different societies at this time. I do not think many people who are here would disagree with that axiom. Otherwise, I do not suppose you would be here. So let us look more seriously at starting axiom two: mathematics is a socio-cultural artifact. Different times, different cultures, different social conditions have produced different mathematics. In that process, it is legitimate, it seems to me, to ask the question: Who has accorded the value which has designated some mathematics as important, some as trivial? How have these decisions been made and how have they been implemented? Those questions are questions that relate to the sociology of mathematics knowledge. They are not questions that I am attempting to answer today, but I have mentioned them because I want to indicate where race and gender can be seen to be important in generating interesting questions. And interesting questions that are difficult questions.

So let us go back to starting axiom two.
Mathematics, I am conjecturing, is a socio-cultural artifact. Therefore, it is not objective, not absolute, not value free. Since Gödel, we all know it is not certain although it is very clear that sociologically speaking that certainty has not been abandoned. So there is a discontinuity between the position that I am espousing and the social description of mathematics and its public positions. But if you will stick with me for a moment on my starting axiom two, I am not asking you to agree with me, but to join with me in a thought experiment.

What do we lose by accepting the axiom? We lose objectivity, we lose certainty, we lose mathematical 'truths'. What do we gain? I would suggest that, potentially, we can re-focus on internal consistency from the stated axioms of a mathematical problem or situation through to the articulation of a generalization. So, in strictly mathematical fashion, I have stated my starting axiom (two) which I will attempt to substantiate, to generalise, in order to ask questions about implications in an internally consistent manner. However, one word of warning. Internal consistency does not imply eternal existence.

What do we gain, therefore, from an epistemological position which attempts to establish inclusivity on a social justice line? Well, first of all, it seems to me we gain honesty. We throw away some myths that surrounds mathematics and people's experience of mathematics. But we also gain intellectually challenging questions. If mathematics is a product of different places and times, we must be able to imagine and derive different mathematics. Let me give you an example of what I mean. When my son was three to four years old, he started, spontaneously, to count the buttons down the front of his cardigan in twos. Now, I think you will indulge me and agree that from the socio-cultural perspective of western European mathematics, that was a bright child. I am Australian. I might have been an Australian Aboriginal mother. What would my child have done equivalently to demonstrate mathematics of that degree of sophistication? Well, it certainly would not have been to count the buttons down the front of the cardigan in twos. Because Aboriginal society does not accord number or comparison the value that our society does. But it does accord a very great value to spatial orientation. And, indeed, if I had taken my son out into the bush and left him there, as of course I, as a European could not have done, but if I had been an Aboriginal mother presumably I could, he would have found his way home. There is a little story that if you say to Aboriginals "What happens if you get lost?" They say, "I go home." The Aboriginal child, therefore, would have been a product of a society which places high value on spatial orientation in exactly the same way that my child was a product of a society obsessed with numerical and comparative orientation. What I am saying is that the values are equally valid as an entry by a child into the corpus of mathematical knowing but that corpus looks and feels very different. Now, forget arguments about sophistication and primitiveness. Because those kinds of judgements stem from our own culture-centricness. Certainly the Aboriginal child was not doing calculus but, in their own culture, there are sophisticated knowings which deserve our respect rather than competition to establish superiority. If we are honestly convinced that the mathematics which we have constructed is superior to a mathematics which they have constructed and which we do not understand, as educators we have a responsibility to make connections and respect the learner's perspectives rather than impose a singular view-point.

To begin to get this sense of multiplicity, accord value differently and think about what mathematics might look like from different perspectives, I think we have to identify contradictions. So, I am saying that mathematical statements are not necessarily truths. But they are contingent statements of internal consistency. And, therefore, they are not necessarily unique.

What does it mean to know some mathematics under this kind of analysis? I would like to suggest five areas that for me come out of the literature written both by mathematicians and by feminist philosophers of science. First of all, to know some mathematics means to have personal, cultural and social knowledge interlinked; to be able to cite the knowing in both a personal and a socio- cultural context. It means that the knowing and the knower are intimately connected. Second, the knowing is non-judgmental; it recognizes and celebrates different, and flexible, approaches. Just as a throw-away example I am sometimes horrified when sitting near a mathematician who is in a session where somebody presents a geometric proof which is entirely visible and says out of the side of their mouth, "That's not a rigorous proof." That kind of judgmental statement historically is not born out by the way in which proofs have been offered in the mathematical community. So I think we have to be non-judgmental and flexible. Third, the knowing involves the aesthetics of mathematical thinking. And a lot of mathematicians write about the power of aesthetics. Fourth, the knowing nurtures insight and intuition, again, born out very strongly by what mathematicians say about their experience of doing mathematics. And fifth, the knowing has to be global. There has to be a sense of connectedness across knowing.

I am offering that outline as an epistemological perspective on knowing mathematics. If you know a piece of mathematics and you look at my list of five areas, it is for you to judge whether or not they reflect your understanding of your knowing. I have tried it and intend to go on trying it. The responses that I have had from mathematicians so far are positive. They say something along the lines of: Yes, this does seem to me to be about how I know mathematics or how I understand my knowing of mathematics. I am not ready yet to say that I can articulate something substantial but that the signs are encouraging. So this is still all conjectural at the moment and you do not have to go with my conjectures. But, if my five categories carry any weight, they seem to me to indicate strong reasons why, if they are not integral to the discipline, we have a noninclusive mathematics experience. The categories that I am identifying are categories that are both race and gender inclusive and address questions about the nature of knowing mathematics which have not, in the past, attracted very much attention. Thank you.

E: I would agree that that is a strong voice. The next strong voice is Suzanne Damarin.

Suzanne Damarin:

In my work, I try to take into account a substantial subset of the wealth of feminist writing available: feminist pedagogy, feminist theory, feminist epistemology, and feminist philosophy of science, most of which is not directed specifically toward mathematics education or mathematics. In this literature, I seek something that might be helpful to our concerns. It's a pretty simply stated problem, but a bit harder to do, and in order to do so, I give that literature a certain reading. One aspect of that reading is I have to have some sense of what I mean (or what the author might mean) by the word "women". So I'd like you to think for a minute with me about what you mean when you say, or what you think I mean when I say "women", and what you read when you read the word "women". Do you read "all women"? Do you read "at least one woman"? "Real woman"? "Typical women"? "Some women"? "Significantly many women"? I could go on and on ... we all give that word some particular reading. Among many feminists, there's a concern that we avoid essentialism, that is, that we resist trying to characterize "all women." When applied to gender categories, essentialism is the tendency to ascribe some psychological, emotional, spiritual, or other characteristics to all women (or, alternatively to all men) as necessary characteristics. For instance, historically, the idea of maternal instinct has been ascribed to all women. And I think that idea has been discarded. It's no longer the case that people go around and say all women have a "maternal instinct." In the history of mathematics, there's been a lot of essentializing of women, and in the history of education likewise. A part of Evelyn Fox Keller's project, I think, has been to reveal some of the ways characteristics thought to be essential to women were tied into certain aspects of the practice of science. And if we think also of Rousseau, and Emile's education as opposed to Sophie's education, there's an essentializing of women that takes place there. As I read feminist literature and as I think about women, I want not to essentialize. I believe this is typical of feminism these days; there is a realization that we can't talk about necessary characteristics of all women.

Yesterday, Gila Hanna mentioned the existence of a controversy about the work of Carol Gilligan and the authors of Women's Ways of Knowing. The controversy which surrounds those folks is over whether they are essentializing, that is whether they are trying to say that all women have certain characteristics: cognitive characteristics in the case of Women's Ways of Knowing and moral ones in the case of Gilligan. I read those authors in different ways. I don't really care in my reading whether they are trying to ascribe attributes to all women. I care, instead, whether they are saying interesting things about some women, and whether the ideas that they convey are interesting in relation to thinking about mathematics education.

Turning to some of these interesting things, I'll start with Women's Ways of Knowing. What's interesting in this book to me as I think about women in mathematics, is first of all, the discussion of abstraction. The authors say that women are not opposed to abstraction but resist it when it precedes experiences from which they can abstract. I (the students) can describe the full cube. She leads them in reflecting upon their knowledge of what it is they see, how they construct the full cube from the part they do see, and how they learn. The women interviewed who spoke of this class reported what we might call an "aha" experience and great respect for the teacher (who happened to be a woman). It is a specific kind of meta-cognition, perhaps, we're talking about here, but one that I've seen described in this feminist literature and that I haven't seen in other kinds of literatures. I find it valuable. Some other valuable ideas from Belenky et al. have to do with their vocabulary and also with the idea that vision is a male metaphor for knowing; they would find hearing or feeling more feminine metaphors.

Another feminist who was influenced by Gilligan's work, and whom I take seriously is Nel Noddings, a philosopher of education who was for 20 years a mathematics teacher. She has taken Gilligan's work as a base and developed a moral theory, an ethics of caring. When those of us who know mathematics read her, we see immediately that Noddings sets up the caring relation as a set of ordered pairs. If you read through her book (some of which I disagree with very strenuously), you find that she periodically come to examples drawn from the mathematics classroom. One of things she says which is very provocative is roughly, "If I'm a mathematics teacher and I have a student who comes to me and says, 'I hate mathematics,' I might be inclined to say, 'Come and do these exciting activities with me and you will come to love mathematics as I love mathematics.'" – But, Noddings goes on to say, "If I really care about the student, I must respect that this student hates mathematics." For me, this was a new idea: that I must respect the fact that the student hates mathematics, and that this requires that I must help that student understand the implications, both the roots and the implications of hating mathematics in today's society. I think that's a provocative idea.

Sandra Harding and other feminist standpoint theorists ask that we consider science from the perspectives of women. Since I'm extending this work to mathematics, I work to consider mathematics from women's experience. And in my own particular interests, I'm looking at the experience of mathematics by people who would prefer to have no experience of mathematics at all. And I ask what contributes to the construction of this view of mathematics? (I talked a little bit about this yesterday.) There's mythology and discourses to be considered. The idea I'm working with the most seriously is that for people who don't have a good relationship to mathematics, mathematics is an area of competing discourses, beginning with being told, "It's important to learn this. On the other hand, it's not important for women to learn this." Despite the efforts of mathematics organizations, many women are still caught in this contradictory discourse.

There are a few other feminists I'd like to mention and I don't have very much time. Maybe I'll just close by noting that the French feminist philosopher Luce Irigaray talks about the notion that, if there's a problem around gender, that problem is rooted in difference. And, therefore, she says, the solutions must be found in difference. In relation to women and mathematics, that's also, I think, a provocative idea because, while we've identified differences, so much of what we've done in gender and math has been to seek to eradicate difference, not to use it. And Irigaray is suggesting that we should capitalize on difference. That seems to me to be consistent with another feminist theorist, Donna Haraway. Among the many wonderful things she writes, Haraway talks about networks of power, how we tend to feel trapped by networks of power and how these very same networks can be the key to our liberation if we learn to traverse them in our own ways and toward our own contrary ends.

Ann Hibner Koblitz:

Let me start by giving some background. I am a historian of science, all of whose work has been concerned with the interactions of gender and science in various contexts (Koblitz, 1987; Koblitz, 1991; Koblitz, 1993). Much of my writing has been on the mathematician, socialist, and feminist Sofia Kovalevskaia. But also, my husband Neal and I have a small non-profit foundation dedicated to the encouragement of women in science, technology, and medicine in so-called developing countries. We sponsor prizes for women scientists in Peru, El Salvador, and Vietnam, and help fund small activities such as regional conferences or science fairs or other occasions in which young women can get to know practicing women scientists in their own country.

In addition, I am a feminist who is actively involved in my institution's women's studies program. I should perhaps also mention that at the request of UNIFEM and the Statistical Division of the United Nations I am putting together a section on women in science and technology for volume two of *World's Women.*

One of the things I find most attractive about late 20th century feminism is that, at least in theory, we are working toward inclusivity, and acknowledgment of complexity, diversity, nuance, and contradiction. In the feminist parlance, we are opening the way to a number of perspectives and voices, and we are highlighting the intricate and ever-changing interactions of gender, race, class, and culture.

As a feminist historian, I think there are fundamental problems with anything that smacks of essentialism – that is, with anything that attributes gender differences to biological, genetic, psychosocial, or other immutable factors. One problem with such theories is that women's position in the sciences is constant neither across cultures nor across time periods. What in one country or time might be considered unfeminine might in another country or time not be so. Specifically with regard to the mathematical sciences, there are several countries, including the Philippines, Turkey, Mozambique, and Mexico, for example, in which women constitute a rather high percentage of mathematics-related professions, especially when compared with the numbers in northern Europe, Canada, or the United States (Faruqui, Hassan & Sandri, 1991; Lovegrove & Segal, 1991; Stolte-Heiskanen, 1991; UNESCO, 1993; United Nations Statistical Division, 1992). Yet most feminist theorizing about gender and mathematics assumes that women's participation in these fields is uniformly low.

The theorists also assume that Victorian era bourgeois stereotypes concerning femininity and gender polarities are uniform across all cultures, classes and historical periods. This is far from being the case, even within western Europe. In Italy, for instance, the stereotype is different from that in the U.S. or Sweden or the U.K. Women are purported to be "natural" theoreticians (hence the relatively large number of Italian women mathematicians and computer scientists), while men are supposed to be more practical by nature (and thus become engineers rather than theoretical scientists).

Moreover, the position of women in mathematics and the sciences can change quite rapidly for the better (or worse). The changes are far too rapid to be explainable by biological theories of difference, or by psychosocial theories such as Nancy Chodorow's (1978).[1] Gender and science theorists have the unfortunate tendency to make generalizations despite clear historical and cross-cultural counterexamples. For example, it simply is not true that women's status in the sciences has remained unchanged since the Scientific Revolution, though that is a claim often made by theorists like Evelyn Fox Keller (1985), Sandra Harding (1986, 1991), and their imitators.

In like manner, I am disturbed by certain aspects of work by Belenky and her collaborators (1986), Gilligan (1982), and others on women's purportedly different ways of knowing. Belenky and her collaborators, for example, assert that the gender differences they describe are independent of culture, ethnicity, and class. This is highly questionable. A colleague of mine at Hartwick College, Katherine O'Donnell, has conducted years of field research with poor migrant farm women in New York state. At the beginning she had had the expectation of finding support for the writings of Belenky and Gilligan (O'Donnell, 1994). Now, however, she is firmly convinced that, on the contrary, the interactions of gender, culture, race, and class are far too complex to be encompassed by a simplistic gender polarity theory (O'Donnell, to appear).

To bring this discussion closer to mathematics, let us take the work of Sherry Turkle (1984) on children and computers. Under the influence of object relations theory and gender and science theory, Turkle has created the concepts of "soft mastery" and "hard mastery" to categorize her analysis of the use of computers by school-age boys and girls. For Turkle, "hard mastery is the imposition of will over the machine through the implementation of a plan... the hard masters tend to see the world as something to be brought under control." Soft mastery, on the other hand, is more interactive – "the soft masters are more likely to see the world as something they need to accommodate to, something beyond their direct control." Though Turkle gives examples of soft masters of both sexes, she says that girls tend to be soft masters, while hard masters are "overwhelmingly male."

Large parts of the theory sound quite plausible at first. Certainly it is true that in most societies males and females are socialized differently, with female socialization tending more toward valuing qualities like accommodation and interaction, and male socialization tending more toward control. Upon reflection, however, some people have raised questions about the validity of the theory and voiced concerns about its social implications.

It has been pointed out (by Beth Ruskai (1990), for example) that the theory contains certain assumptions about the nature of computer science that are rather far off the mark. What Turkle dichotomizes as "hard" and "soft" mastery might be better characterized as two inextricably interwoven parts of the creative scientific process. The attempt to label people as being *either* hard *or* soft therefore misses a crucial point of what it is to do science.

Turkle's theory makes no allowances for historical and cross-cultural variation. It ignores the evidence that women's participation in the sciences – including computer science – has varied widely from one decade to the next and even between neighboring countries (Lovegrove & Segal, 1991; Koblitz, 1991). Moreover, stereotypes regarding women's innate capacities and their relation to mathematics and computer science vary from culture to culture.

There is also reason to be uncomfortable with the way Turkle's theory dovetails with current western European and North American stereotypes about women's intrinsic nature. In practice, these stereotypes can contribute to discrimination against women in the workplace, and to the segregation of women in so-called "pink collar" ghettoes like data processing.

Sherry Turkle is fairly skillful in inserting caveats in her generalizations, and in reminding us that the picture is complex. (Harding, Belenky, and Gilligan are not nearly so careful.) Unfortunately, however, the caveats and reservations rarely make it into popular accounts of the work. What gets picked up in the media is the rather simplistic idea that girls cannot be attracted into computer-related fields unless the machine can be portrayed as artistic, relational, and "soft." We do not work in a vacuum. If the media *can* distort a theory, they *will.*[2]

In the U.S., one of the most common manifestations of sexism and racism in the classroom is a refusal to intellectually challenge girls and members of minority groups. They are condescended to and patronized, and do not receive adequate exposure to the more rigorous, thought-provoking, and elegant aspects of mathematics. Their understanding thus rarely attains the level of the systematic and the structural; they seldom arrive at the stage where they can see much point in doing mathematics. Unfortunately, this phenomenon can be exacerbated by overzealous followers of the ideas of Turkle, Gilligan, and Belenky.

In conclusion, I would like to emphasize that the experiences of women in mathematics, both historically and across cultures, have been diverse, and in many cases, contradictory. And the interactions of gender and mathematics are complex and dynamic, as the research of many speakers at this conference demonstrates. I would argue that this diversity and complexity is far too rich to be encompassed by any single explanatory scheme, especially by one, such as gender and science theory, which is so tied to polarities and to absolutes.

Mary Beth Ruskai:
As Elizabeth emphasized, there are many different feminist viewpoints. But the terms "feminist critiques" or "feminist theories" are often applied to a rather narrow set of views in a way that colors the discussion. Thus, anyone who disagrees is labelled as non-feminist, even if that person has worked hard for what feminism traditionally meant, namely for equity and advancement of women. Elsewhere, I have made specific criticisms of the works of Sandra Harding, Sherry Turkle, and others. Since time is limited, I will concentrate here on some general issues closely related to this question of essentialism.

Not only do I frequently find errors and inconsistencies in the writings of a group that I refer to as "gender difference theorists", but I feel that they are basically asking "the wrong questions." Since that's a charged phrase, let me explain. If one wants to instigate constructive social change by increasing the contributions of women to science and mathematics, the gender difference theorists are focussing on issues which may very well exacerbate the problem rather than alleviate it. Let me illustrate this with a personal anecdote. After graduate school, I took a postdoc in theoretical physics in Switzerland in 1969, the year the Swiss finally gave women the right to vote in federal elections. However, it was not until 1979, a full ten years later, that women were allowed to join the Swiss Alpine Club. So I had to find other groups to ski, hike and climb with, most of which were dual gender groups. But I also joined the Club Suisse de Femmes Alpinistes, and went with them on a week-long ski mountaineering trip. At that time, I was by far the best trained in terms of stamina and endurance although I didn't have the same mountaineering and ski skills as the Swiss women who had been doing this all their lives. Our two male guides set a brisk pace up the mountain as we followed laden with skis and gear for a week. After about an hour, I was the only one still keeping up with the guides, who stopped to let the others catch up. After about 15 minutes the last two, who were somewhat older, came huffing and puffing up the mountain and practically collapsed as they dropped their packs. When they had finally recovered, both of them looked over at me and said with great concern, "Are you all right?" (Laughs.) As the week progressed, the male guides and the other women accepted me as part of the group, realized my strengths in stamina, my weaknesses in mountaineering skills, and gave me help and encouragement when I needed it so that I could really enjoy the trip. However, despite blatant evidence to the contrary, these two elderly women remained convinced of – and regularly commented upon – my immanent collapse. Now this is an absolutely classic. I picked it deliberately because bias by elder Swiss women against visiting Americans is not a serious societal problem, likely to elicit emotionally charged responses. Nevertheless, it exhibits all the elements inherent in racial and sexual discrimination. I was being judged on the basis of their ideas of what American tourists are like and not on my own merits.

And this, I believe, is the fundamental problem, to judge people by group categories instead of by their individual attributes. Gender difference theory, aggravates this problem because it focuses on specific gender characteristics within a group and gender differences between the groups. But the focus should be on individual differences. It is entirely irrelevant whether or not, in particular circumstances, the attributes that are associated with a group are statistically valid, or whether the attributes are thought to be of biological or cultural origin. It's far more important to judge people as individuals. Despite our best efforts most of us have hidden biases, not only ethnic, but against short people, single people, certain hair styles, etc. Focussing on group attributes is not the way to overcome these biases.

I realize this may sound suspiciously like something that most of us at one time in our lives believed in and have been disillusioned by. Namely, that we should be completely gender neutral and do everything on the basis of pure merit. But our biases are simply too ingrained for that. Instead, we've had to accept things like affirmative action. For example, I've learned that you'll rarely have any women speakers at math conferences unless you make an active effort. But when you do make that effort, you easily find half a dozen who are at least as good as the men speakers, so that you can invite them on their merits after all.

But ultimately we need to focus on treating people as individuals. Let me give another example. In the last few years, I've changed my own teaching style so that I lecture less and the students participate more. This was partly a response to reports that girls were much more successful in "cooperative learning" environments. I merely insist that the students work in class, although I do encourage them to do this in groups. What I discovered should not have surprised me, but it did. Many of the girls dislike this intensely because they feel, no matter how low-key I try to make it, that they're being put on the spot and tested immediately. So it has not been as successful with some girls as I had hoped. However, I have become so convinced that the students learn more, that I will not abandon this method simply because some girls don't like it; instead, I'm determined to try to find ways to make it more palatable for those girls. On the other hand, it has been far more successful with black students, although my sample is very small (for unfortunate reasons that I won't go into here). In particular, two men who might very well have failed if taught in the traditional way earned strong B's. These two black men were quite different, the reasons why they might have failed were different, and the reasons why they responded positively were different. The only common thread is that because I interacted with students working in class, I was able to view each of them – not as yet another poorly prepared black male sitting in the back of the classroom failing exams – but as unique individuals with strengths and weaknesses. This led to interactions that did seem to work in the sense that they responded very positively and succeeded.

My attitude is that you should treat people as individuals. It may well be true that more girls need encouragement than boys. But our attitude should be to identify and encourage those students whose confidence level is such that they need it. If it turns out to be 95% girls, fine; if it's 60% girls, fine; if it's 20%, fine. We must respond to the needs of the student, keeping aware that what works for one group doesn't necessarily work for everybody.

One of the things that gender theorists have emphasized with which I agree is that the background and viewpoint from which one comes to a subject is important. For example Karen Johnson made a very convincing case (in Physics Today Sept. 1986, pp. 44-49) that Maria Mayer's nobel-prize work on the nuclear shell model was a direct consequence of her previous work in atomic and molecular calculations, which gave her a different viewpoint from most of the (male) nuclear physicists. In my own work, I had an interesting multi-national collaboration in which I was able to "smell" the solution to one unsolved problem in a paper that I found rather technical and hard to understand. So I asked the probabilists in our group to look at it. One of them translated it into operator theory language so that our entropy equivalence question was reduced to the equivalence of eigenvalues of certain matrices. From this he felt it was intuitively obvious that they were not equivalent and we should search for a numerical example. As an operator theorist, I took one look at his notes and in two more lines used max-min to prove that they actually were equivalent. Our backgrounds were very, very important to our different insights. But in both of these examples, it was scientific background, not gender, that made the difference. I have not yet seen a convincing case for gender-based perspectives providing important scientific insight except in cases, such as hormone research, were gender is itself intrinsic to the subject being studied. There's another point to be made here. As soon as I communicated my observation, my colleague changed his position and agreed. Just because there are situations where you can have definite answers does not make mathematics unfeminine or uncreative. In this example it was a tremendous advantage. We could agree that one problem was settled, and go on other things.

I'd like to return to the point that characterization by gender reinforces conformity, while what we need is an appreciation of diversity. In some of the gender difference literature, the arguments seems to me particularly flawed because they define feminine in terms of certain well-known, or common characteristics. They also define science in terms of other characteristics (sometimes of questionable validity, such as non-creative and non-intuitive) which they consider masculine. If one points out that there are examples of many women who don't fit this stereotype, the response is that the paradigm is cultural and not biological. So that the women who don't fit are masculine and the men who do fit are feminine. To a mathematician this sounds suspiciously like "assuming what you're trying to prove".

One of the often-told stories about the great mathematician Emmy Noether is that because she was so successful at mathematics, which in her culture and generation was considered unfeminine, she was often referred to as "Der Noether", using the German masculine pronoun. Sometimes when I read this so-called "feminist theory" or "gender difference" literature, particularly the stereotypes and the remarks about women scientists who do science in the traditional way, I feel that they are simply calling us all "Der."

Jeremy Kilpatrick:

I don't know whether or not I'm being asked whether I'm a feminist. After this discussion, I'd say that I guess I am. Even before the discussion, I was trying to work my way into this topic to understand it better, and the panel has helped me over some rough spots.

I was asked to give a summary of the panel's remarks, but I'm not going to attempt that. I didn't think it made sense before, and I certainly don't think it makes sense now. Either you were paying attention and have your own summary in your head, or you weren't paying attention and cannot possibly digest any summary I might give you now. In a few minutes, I cannot hope to pull together the many points these speakers have made and summarize them fairly. Instead, I'm just going to offer some observations about what I think we've been doing here over the last two hours.

This panel has offered us what I hesitate to call a crash course in feminism but what might at least be considered a condensed view of feminism as it relates to mathematics and mathematics education. For me, at least, the panel has been enormously helpful because of the seriousness, intellect, and talent the four panelists have brought to the discussion, as well as the nature of the questions that have come from the audience.

All of us seem to be trying to figure out what this thing called feminism is and what it might have to offer our subject, which is the teaching and learning of mathematics. I think we're all struggling with these questions. Some of us are more familiar with the relevant literature than others. And as we know, reading the literature is only part of coming to understand a topic. I think the discussion today has helped all of us,whether we've read the literature or not, to interpret some feminist perspectives. We see a field that is struggling to achieve an identity and a coherence. We recognize the great complexity of the ideas relating to feminism.

I was struck by the agreement among the panelists about the issue of *essentialism*. It has been productive to have that term at least partially explicated this afternoon. Some of the feminist literature I've read has bothered me in ways I couldn't figure out. Thinking about essentialism has helped me see better the problem I've been having. Perhaps it has helped you, too.

In a field that is in the process of being born, it is not surprising that there are struggles over terminology and questions such as, "What do you mean by that?" "Do you agree with me on that?" I particularly welcome Suzanne Damarin's suggestion that we consider the literature and what we have heard in terms of the reading we give to it. We can read something for purposes we have at the moment and use it positively, regardless of the intentions or background of the person who wrote the passage or made the assertion.

I am impressed by the courage manifested by people who adopt a feminist perspective. It represents an extraordinary accomplishment in the face of many obstacles and many forces arrayed against it. We have heard a lot about the power sources in our societies and, as well, the distortions that the media, the public, the politicians, and others have about mathematics. We recognize that as a community of mathematics educators, we need to try to create a better view of what our subject is and can be. Perhaps feminist perspectives will give us some ways to achieve that, to get beyond the stereotypes people have of our subject.

Leone Burton struck a chord when she talked about the personal view. I would like to share with you a personal view of my own. Many years ago I worked on a research study in which we compared the achievement of boys and girls in classes that had been taught from different mathematics textbooks. The study compared the effects of books used during the new math era. We found various sex differences that made little sense to us and that seemed to have no practical implications. One of us – not me – made the facetious statement that we leave the interpretation of these findings to "those in the women's lib movement." That statement came back to haunt us.

I struggled for a long time with the findings. It seemed at first that the only implication might be that publishers issue one set of textbooks for boys and another set for girls. That was the simplistic view – this is confession time – that I took of the differences we had obtained. Because I didn't think that publishing different textbooks was either feasible or justifiable, I couldn't see any implications for practice.

Of course, the rest is history. Various researchers took those findings, and others like them, and developed not only a better understanding of the sources of such differences but also some powerful intervention techniques to reduce them. The understanding of gender and mathematics that has been built over the past two decades has powerfully influenced the mathematics education community.

Even though the field of feminist perspectives toward mathematics and mathematics education is young, we can already see it changing many people's thinking. One difficulty, however, is that it happens to coincide with many other changes in mathematics education. It's very hard to disentangle the sources of change, but I don't think it's necessary to do so. We should just proceed with what we have learned and with the perspectives we can share.

I would like to end with a couple of remarks that Bent Christiansen made last night and try to tie them to what we have heard today. Bent quoted Hans Freudenthal's observation about mathematics starting and staying in reality. For me, one of the strengths of feminism is how it helps us see that people do have different realities. When we talk about mathematics starting and staying in reality, we have to ask, whose reality? That can be a powerful message for us.

Another powerful message from Bent, and I'm sure you caught it, comes from the glory he takes in the human experience and in the fact that doing mathematics is part of the human experience. Well, doing research is also part of the human experience. Feminist perspectives can help us see the human side of research. I'm intrigued by the way in which people who have a feminist commitment can, on the one hand, bring passion and eloquence to their subject and, on the other hand, demonstrate a high level of academic scholarship. We badly need both of these qualities in research in mathematics education. That is why I'm particularly appreciative of this panel's contribution today.

Notes

[1] In general terms, object relations theory says that a girl infant never has to disidentify herself from her mother. Therefore, she never sees the world as alienated from herself in the same way that a boy infant does. A boy baby, on the other hand, realizes very early that he is not the same gender as his mother. He has to distance himself from her, and thus begins to objectify the world. Because object relations theory attributes gender differences in intellectual outlook to an immutable mechanism of early childhood, it is almost indistinguishable from a genetic or biological theory. Moreover, there is no way the theory can account for differences between individuals, or for change over time or across cultures.

[2] We had a distressing illustration of exactly this point in Swedish newspaper coverage of our conference. The headline of the 9 October 1993 *Dagens Nyheter* story was "Mathematicians disagree whether biology makes a difference," and Miguel de Guzman was misquoted as saying that gender differences manifest themselves from day one in the classroom!

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Reporter:
Elizabeth Fennema

Panel 3:

THE ROLE OF ORGANISATIONS IN GENDER AND MATHEMATICS EDUCATION

Moderator:

Christine Keitel

Panelists:

Josette Adda, Gerd Brandell, Kari Hag and Cora Sadosky

I would like to welcome you all to Panel 3. I am moderating this panel as the current Convenor of the "International Organisation of Women in Mathematics Education" (IOWME); this panel is about organisations. After many challenging and exciting discussions and working group meetings, we still have important questions to discuss; our major concern here should be the role of organisations in gender and mathematics education. To start the discussion, I would like to go back to some of the last points of the discussion of Panel 2: This afternoon, we heard about empowering and the question of retaining, distributing, or sharing power. This, of course, immediately refers to political problems and possibly to political actions. Do we believe in being able to better connect ourselves, to better and more successfully work and act together if we establish organisations? And how best to do it? I invite you to discuss this aspect with our representatives of already distinguished organisations closely concerned with gender and mathematics, and draw some conclusions from their experiences and views for the factual and the possible role of organisations in other countries or on the international level, in order to pursue the purposes of this conference. We will proceed as follows: Each panelist will introduce herself with a statement of about 10 minutes. We begin from a more national and European perspective and proceed to the more international one; in doing so, we hope to develop a more substantiated view of how to work in and cope with organisations. May I introduce to you my dear colleagues : Gerd Brandell from Sweden and Kari Hag from Norway who start, speaking on the national, "Nordic," and European level of organisations on "Women and Mathematics"; then there is Josette Adda from France who opens up our discussion, referring also to agencies and organisations that are not particularly concerned with the question of women and mathematics education, but have started to address them; finally, Cora Sadosky from the United States will inform us about the

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Gender and Mathematics Education, an ICMI Study

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Association of Women and Mathematics in the USA, which has become a world-wide and influential organisation.

Gerd Brandell:
I teach mathematics to engineering students at the Universitiy of Luleå. Luleå is in the north of Sweden. This university has its roots in a relatively young technical university and an old teacher training college. The research is mainly in the areas of science and technology, but we also have research going on in economics and in pedagogy. We have problems defining ourselves, sometimes calling the university a technical universitiy, sometimes calling it just a university. You might have the impression from the address-list of this conference that my dear collegue from Luleå and I belong to different universities, but we do not. As you already might have heard, there are only 21 positions for full professors in mathematics in Sweden – none of them is held by a woman. Of approximately 200 positions as senior lecturers, only 8 are held by women. One interesting fact is that out of these eight women three have been or are head of the department. Right now Gudrun Brattström is head of the mathematical department of Stockholm University and I am head of my department.

My main concern with the issue of this conference is to understand and to try to influence the extreme imbalance of the mathematical community of Sweden. I first got involved with this question when I went to the ICME-III conference in Karlsruhe in 1976. Since then there have been contacts between IOWME (The International Organisation of Women in Mathematics Education) and Swedish mathematicians. But the starting point for the active work in Sweden was the ICME-VI conference in Budapest in 1988. A group of women returned from the conference very determined, very inspired, and soon things began to happen. A conference on "women and mathematics" was held in April 1990 in Malmö, organized by Barbro Grevholm, Ingegerd Palmer, and Kerstin Ekstig. Some 120 persons participated in that conference, a report was published, and we have all the material in the book exhibition for you in the main building. The report from this conference is in Swedish but you may get an impression if you take a look into it what the conference was about.

From this conference organised by an informal group of involved women, an association now comprising about 400 members was established. A newsletter is regularly distributed about twice a year; the current issue is the fifth. The second conference of "women and mathematics" was held in Luleå in June 1993, with Barbro Grevholm, Britt Marie Stocke, and myself as organizers; about 185 persons participated in that conference, mainly colleagues from Sweden and other Northern countries. We also invited a few lecturers from other countries. The next conference is planned for 1996 in Gothenburg. Of course, members of our network are participating in the organising of this conference. One of our members is active on the organising committee of the biannual Swedish conference on mathematics teaching, which is a really big event with several thousand participants. Members of our network also are working through other organisations. As an example, members of the network are on the board of the Swedish Mathematical Society. We also have members working on a national committee set up by the Swedish government to prepare the new national mathematics curriculum. The network cooperates with other organisations, such as the European Women in Mathematics (EWM). Through all these channels we try to influence issues concerning women and mathematics.

Right now all is going well and we feel that the network is in a period of rapid development. This conference obviously will mean much for the future development of the Swedish network.

I would like to raise two questions about the organisation and its future and I would be very interested in hearing from you about your experiences and your views of these questions. The first concerns the structure of the organisation. The network is quite informal and it depends heavily on the unpaid work and the very strong devotion of a few persons. My experience from 10 years of working to establish a centre for women's studies at my own university tells me that such an organisation is vulnerable. Right now, the issue of more women in science and technology is popular among politicians and among people who have influence and power, but who knows how long that interest will last. What we do know is that sooner or later it will become harder to get economic support, and we also know that if we proceed we will meet more obstacles and more determined opposition. That is something of which I am very convinced. My first question is: In what ways should we cooperate with or become part of greater organisations such as the Swedish Mathematical Society or the Organisation of Swedish Mathematics Teachers in order to secure our future work and long-term survival without losing our independence? My second question has to do with the different levels on which we are working. We discussed at the Malmö Conference and agreed that it is a good thing that we have among our members teachers from all levels of our schools as well as teacher-trainers and university mathematicians. This helps us in understanding the whole question of gender and mathematics education and gender and mathematics. It is also related to the fact that this country is a small country and we are not so many.

But how will we manage to handle the needs of various subgroups who need to work with specific questions related to the level that they are working in? We observed at the Luleå Conference that some teachers working at the primary level or university mathematicians wanted to concentrate the program and discussion more on their own level. So there obviously is a need for working through subgroups. But how can we manage to do this without splitting up and weakening the whole network? This is my second question – the first is about power and influence, the second about internal organising.

Kari Hag:
I am associate professor of mathematics at the Technical University of Norway, located in the middle of Norway. I would like to show you some statistics about female mathematicians at the university level in the Nordic countries.

Women (%) in Mathematics

	Ph.D	Tenured	Full Prof.
Denmark	20	3	0
Finland	8	2	2
Iceland	0	0	0
Norway	5	7	6
Sweden	10	5	0

(Source: European Math. Society 1992)

As to Norway, the only positive feature is that the percentage does not decrease when moving up the university ladder. This might be due to the "equal opportunity policy" that we have had in the eighties in Norway. There has been extreme pressure on the Norwegian research and development system to work for equal rights and equal conditions for men and women. All universities and colleges have an equal opportunities committee; I have been the chair of one for some time at my university. I have also been on a national committee for recruiting girls into mathematics, sciences, and technology. Finally, I was the chairperson of the Norwegian Mathematics Council from 1990 to 1992, and I guess that is the main reason why I am here on the panel today.

Now I would like to present to you the structure of the Norwegian Mathematics Council. Firstly, it is different from the Norwegian Mathematical Society.

The Norwegian Mathematics Council

Broad representation:
- Elementary school
- High school
- Teacher's colleges
- Technical colleges
- Universities
- Industry
- Research councils

Concerned with:

- Mathematics education including preschool math and research courses
- Mathematics and society

Arrangements:

- SÅNN,JA!

Network Newsletter

As you can see, there are representatives from all kinds of schools and all universities, from industry and research councils as well, altogether some 30 members. The main concern is with mathematics education. Here, to give you some idea about the range again, it includes issues from pre-school mathematics to research courses. However, we have been mostly involved with the teacher training, which has been the main theme, and in addition topics like mathematics and society, and so on. The Council has published booklets and also organised seminars and some conferences; the last conference was "SÅNN,JA", last May in Kristiansand in lovely Southern Norway. Our conference was on mathematics didactics and women in the mathematical fields. We called it "SÅNN,JA": the title is a pun. It is pronounced the same as Sonja, and Sonja Kovalevska was the first female mathematics professor in modern times (and I should remind you that at the time Sweden and Norway were united, so we could say that she was *our* first math professor, too). "SÅNN,JA" in Norwegian also means "such, yes" or "that is the way to do it." We hoped to find some ways to improve the situation of women in mathematics, science, and technology. This, of course, we did not really accomplish, I think, but we made friends and had interesting lectures and discussions. Gila Hanna, our chair at this conference, was one of the main speakers in Kristiansand. The "SÅNN,JA" report contains some of the articles in English, some in Norwegian and one, I think, in Swedish.

The dotted lines on my transparency mean future development, and what we hope, at least the three Norwegians who are here, is to start a network with a newsletter, inspired by the Swedes. We would like to have this network and newsletter under the direction of the Mathematics Council. I think it is a good idea to have an informal broad network with a newsletter, tied in with a formal organisation with some power on the national bases. So, for a small country like ours, only half the population of Sweden, I myself think that it is a good solution. I would like to hear your opinion, and, of course, we want to cooperate with the Swedes and with the Danes and the Finns and so on, as the other Nordic countries come along. We have a National Coordinator in the IOWME and we also have a National coordinator in the "European Women in Mathematics." That is Ragni Piene who is a professor of mathematics at the university of Oslo. The organisation "European Women in Mathematics" was formally established in Poland last summer, informally it already existed in 1986. And if you want to hear more of this story, I think Bodil Branner is the right person here to tell you about that.

I have some of the statutes here for that organisation and I will read what I think is the most important.:

Article 1: "European Women in Mathematics" (informally EWM) is an organisation established in accordance with the laws of Finland. Its seat is in Helsinki, Finland.

Article 2: The purposes of EWM are to encourage women to take up and continue their studies in mathematics, to support women with or desiring careers in research in mathematics or mathematics-related fields, to provide a meeting place for these women, to foster international scientific communications among women within and across the fields of mathematics, to promote equal opportunity and equal treatment of women and men in the mathematical community, to cooperate with groups and organisations with similar goals.

Article 3: A member of EWM may be any woman related to Europe who supports the purposes of the organisation.

Josette Adda:

I come from France. I am specially interested in didactics of mathematics. I am a professor of the science of education ("Sciences de l´Education") at the University of Lyon 2 and a research director of didactics of mathematics at the University Paris 7. I want to consider two meanings of the word "organisation": first, I shall speak about associations that are free and private and, after that, I shall speak about official organisations. But, before that, I think that it was a good idea to speak about the situation in the universities as my colleagues from Sweden and Norway did. I did not bring with me the latest numbers but I remember that, two or three years ago when there was a paper about it, there were about 25% of women at the level of assistant professors and "maitres de conférences" in mathematics and about 10% for full professors. I hope it is a little bit better now (considering the recent recruitment for professors), but I am not sure of it.

The work of associations is linked with this problem. The most important French association for our theme is *"Femmes et mathématiques"* (women and mathematics); the members of this association are essentially researchers in mathematics, but a few are teachers and or engineers. It is linked with the European association "Women and Mathematics" (EWM) and it is a very active group, with meetings once a trimester and a newsletter. In these meetings, we discuss the evolution of women in mathematics in France. We also have each time a lecture on mathematics by a French woman mathematician or sometimes by a foreigner coming to Paris. Although it is a national meeting, it is most often held in Paris. If you have questions I could provide you with more precise information, but we shall come across the same issues again when describing the other associations. For example, I must say some words about the French section of IOWME. I am the National Coordinator of IOWME in France, but I have very little to do, other than linking with "Femmes et Mathématiques." There is also an important association of mathematics teachers of our country (*Association des Professeurs de Mathematiques* – APM), with regular meetings, which I often attend when there are activities about women. We try to establish links with international associations, but it is a big problem to us that most newsletters from international associations (for example of IOWME) are written in English and when I offer these newsletters I find only very few interested people because of the language. So I have at least some people who want to have the news but it is not a big group. Most action is done by "Femmes et Mathématiques".

I think it is important to speak about the *SMF ("Société Mathématique de France")*, which is the French Mathematical Society. Of course, its issues are not just gender issues, but more and more it supports "Femmes et Mathématiques." In nearly each "Gazette" (the newsletter of the SMF), you can read about gender issues. However, not everything is perfect: for example, in one newsletter, one could find on one page a call for a position for "*un* président" and, on the next page, the call for "*une* secrétaire." (It is quite impossible to translate into English!). In fact, there was a time when a woman was president of SMF, and the APM, the association of teachers of mathematics (these are teachers primarily of secondary schools, with some of primary and universities, too), has very often had female presidents, and in secondary schools about 50% of the math teachers are women. Perhaps we can come back to this later if there is time. In each meeting of the APM there is a workshop about the problem of gender and mathematics education. So, although there is not a specific association of women teachers, we have a very active association for non-sexist education (*"Association pour une Education Non Sexiste"*), which is not especially related to mathematics education, but which does good work and is linked with us, for example, about questions of sexism in textbooks. We also have an association of women with diplomas (*"Association des Femmes Diplomées"*).

I will now speak of official organisations. There was a ministry of second level called a *"Secretariat d'Etat" for the Women Rights* – created when the left was in government. It has changed now – this year France changed government and we no longer have this ministry for women's rights. There exists a minister of Human Rights (in French "Droits de l'homme"!) but the problems of women's rights are not included there. Most women (of any political side) are not unhappy about this change because what was done in the "Secrétariat d'Etat" for women's rights is now directed by the Ministry of Social Affairs, which is in the hands of Simone Veil, who has done very much for women in France (especially the law for abortion). She has declared her intentions to cooperate with associations ("Les associations ont toujours été à l'avant-garde du combat pour l'égalité. Porteuses des aspirations des femmes dans la diversité, elles sont le moyen d'expression indispensable de ces aspirations auprès du gouvernement et elles complètent son action. J'attends donc beaucoup de la reflexion des associations et de leurs propositions.").

From the reports of the projects of the former and current ministries you can see that the new one takes in charge all that was done before. What was done? There were some groups called *"Délégation Regionale aux Droits des Femmes,"* from departments in each region with charge of women rights. Of course, employment, contraception, abortion, etc., were of major concern, but so too was the orientation of students. With the former ministry, there was first a *scholarship* given to female students who wanted to do university studies in science. It was not only for mathematics, it was science and technology. However, this scholarship, after some years, cost more than foreseen by the government, and it was changed into a prize called *"Prix de la Vocation"* and this is given to 40 girls in each region. Also, in each academy, there exists an office called *"Chargé de Mission à la Diversifaction de l'Orientation des Filles"* who has to inform and to campaign to change the vocational choices of girls. Then, too, we have more than the prize: information, for example, is provided to teachers and to public schools. All these official organisations try to link information with the free associations, and women coming from "Femmes et Mathématiques" and from the associations of teachers go to give lectures about the problems of gender for teachers, especially for groups in the institutes called *Instituts Universitaires de Formation des Maîtres (IUFM)* (for teacher training) during the initial training or to inservice teachers. I consider this quite a step forward.

Cora Sadosky:
I am a Professor of Mathematics at Howard University in Washington DC, which

is the main historically black university in the United States. This university was created during the American Civil War in order to train black people who were excluded from education. However, the reason I am speaking here today is because I am the President of the *Association for Women in Mathematics* (AWM).

This panel is about the role of organisations. In the United States we have a positive experience on how organisation has helped us, by pulling together our strengths and resources, to achieve significant changes in our struggle for equality and the right of women to mathematics at all levels.

I am going to talk mainly about AWM, established in 1971 to encourage women to study and have active careers in the mathematical sciences. Equal opportunity and equal treatment of women in the mathematical sciences are the objectives of AWM. The Association now has more than 3,500 members – both women and men – from the United States and from around the world, and represents a broad spectrum of the American mathematical community. Many of the AWM members are college students and teachers of mathematics at different levels, that is, not all of them at universities.

AWM was created 20 years ago to promote the equal rights of women in mathematics at every level. We believe very firmly in the need of equal opportunity for women both in mathematics research and teaching at all levels: The world cannot afford to lose half its potential population of teachers and researchers in mathematics. Nowadays it is well known that women are able to do mathematics, and that they have done and are doing top mathematics. Yet we believe that the opportunity of women to have access to mathematics is still a problem, and that this must change.

Women must have the opportunity to do mathematics at every level they want. We believe it is very important that women mathematicians be professors at the top universities. In the United States we are now celebrating the fact that the first woman has been nominated as a professor in the Department of Mathematics of Princeton University, the number one department of mathematics in the nation. Emmy Nöther could not be a professor at Princeton when she arrived in the United States before the Second World War. Everybody knew she was excellent, one of the best mathematicians in the world. But she was a woman and, as such, could not become a university professor. Sixty years had to elapse before Ingrid Daubechies, a young woman from Belgium, could become a full professor at Princeton university.

Why is it important to us that women mathematicians be recognized for their mathematics, that they attain professorships at leading universities, become members of the National Academies of Science, and be invited lecturers at important international conferences? It is important because that implies recognition by the community at large, and enables young people to sustain their quest of excellence. Women in prominent places signal the possibilities ahead, their presence dispels the vulgar belief that "women do not belong" there.

Twenty years ago AWM was seen as a very controversial organisation. Since 1971 the initial group has grown and became more established – which does not mean more Establishment. We still confront the mathematical community, but we have learned the power that a large membership gives to our positions. In fact, AWM's tenets have permeated the culture of the other American mathematical associations, and are permeating those of American colleges and universities.

I have been asked to describe briefly some of the activities of our association. AWM's main organisational tool is our *Newsletter,* which is the link with our associates and with the mathematical community at large. People can catch up with the news, discuss topics of science policy, read book reviews, and debate issues related to women in mathematics and to mathematics in general. The pages of the *Newsletter* are truly open to its membership and are used by its membership.

AWM has several programs. We are interested in education at various levels, and one of our most successful programs is the *"Sonja Kowalewskaya High School Days."* This program is for both teachers and students in high schools. Its aim is to introduce women high school students and high school teachers of both genders to exciting applications of mathematics, and to encourage both students and teachers to think of mathematics as an appropriate field for women to enter. Although each program must be adapted to local needs and interests, they all consist of a full day of interaction among a high school group and mathematicians from the area.

Other programs are geared to different levels. The *Travel Grants* are for enabling professional mathematicians to attend national and international meetings. The *Workshops for Female Graduate Students and Postdoctoral Mathematicians* highlight their achievements in research.

Annually AWM awards two prizes. One is named after our founding member and former president, the *Alice T. Schafer Prize.* It is a prize given to undergraduates for excellence in undergraduate work. The other prize is the *Louise Hay Award* for mathematical educators. It honours one of our best colleagues, a now deceased American mathematician and mathematical educator, widely recognized for her strong leadership in nurturing the talent of young people, a truly outstanding educator herself. Its importance is to establish recognition of the best women in mathematical education in North America. Starting in 1994, both prizes granted by the AWM will be presented at the Joint Prize Sessions of the American Mathematical Society and the Mathematical Association of America, which means that our awardees will achieve national recognition as well.

AWM has different programs aimed at different groups of its constituency, but all of them are quite punctual, because, although we have a large membership, we are not a "big" organisation, certainly not a rich organisation, and we are mostly a *volunteer* organisation. Thus, when people ask me questions such as

"Why don't you do more for mathematics in high school?" I have to explain that AWM does what its members do. Each of the projects I described has been born from the efforts of one individual or of a small group of individuals. This is one of the lessons we have learned: people have the power to act at local levels, and together we can coordinate those actions for the good of the community.

Take the *Sonja Kowalewskaya High School Days*, for instance. It started as a local project. After years of sponsoring them, we have finally succeeded in getting financial support that will enable people to conduct them nationwide. This is the kind of project that can be copied internationally, adapting it to local resources and needs.

This brings me to the last point I want to make about our activities. We are very interested in international contacts, and we consider both the *European Women in Mathematics* and the *Joint Committee on Women of the Canadian Mathematical Society* as our close associates. We have been very close to both since their inception. We have already initiated joint ventures with both of them. We hope to continue doing so at the International Congress of Mathematicians in Zurich in August 1994 and in the following ICMI Congress. We are equally interested in women in mathematics worldwide and are establishing good contacts with colleagues in Africa and Latin America. We are very open to talk about our experiences, which we think are very good, without any will to impose the fruits of our experiences onto others, with their own backgrounds and national situations. We do not believe we are the guardians of any truth, but we have a long experience in the struggle for women equality in mathematics and we are eager to share it.

I feel comfortable talking about how good our experiences are because I have personally done very little of the important work of AWM, and some of the people here from the United States – in particular Mary Gray, AWM's founder and first president – have done much more. I know that most people here who are mathematicians from the US or Canada, as well as many of the European people, already belong to our association, so they know about it and about all I have been talking about. They know that AWM is a very especial group and that it gives its members a lot. Working through AWM, American women in mathematics have been able to make an impact on the mathematical culture of the United States. I want to insist on the fact that I think what makes AWM exceptional is that it has put a lot of weight on the power of organisation. We have been able to link the work of many women, different women, women with diverse backgrounds and interests, working at different types of institutions – in schools, colleges, universities, industry, government – women with different political and social views. We have been able to set aside our many differences to speak with one voice for the right of half the population to the joys and labours of mathematics.

Our experience tells us that instead of continuing to speculate about how it is going to be when women do mathematics, it is good to realize that it is already happening everywhere, and it is high time to help it to happen more and more and more.

Reporter:
Christine Keitel

Panel 4:

INTERNATIONAL PERSPECTIVES

Moderator:

Jean-Pierre Kahane

Panelists:

Setsuko Hazama, Elfrida Ramos de Matos Ralha, Teresa Smart and Maria Trigueros

In a recent report, Unesco 1993, claimed that "Gender parity is virtually attained [so far as female enrolment in education is concerned] in the developed countries and Latin America and the Caribbean. In this [latter] region tertiary education is the only level of education where parity has not been reached. In other developing countries, girls and women are clearly worse off and the higher the level of education the greater the disparity. ... The Arab states represent the region with the lowest proportion of girls in primary education. In higher education the gap is most marked in Africa ... ." Such data present an overall view but it is clear that gender biases can exist within mathematics even though parity may well exist within the educational system as a whole. In this session, speakers from very different parts of the world were able to give some indication of what was happening in their countries.

Setsuko Hazama (and Hanako Senuma, who co-authored the Japanese contribution) saw the main "gender" problem in Japan to be that society and educational bodies do not recognize any problem and, as a result, there are no concerted moves to effect changes. There has been a gradual improvement in the removal of gender biases, but this appeared to be unplanned. In Japan, unlike many other countries, female mathematics teachers were still in a minority. In elementary schools they comprised 56% of the teaching force, but this dropped to 20% in the junior high schools (12-15), and to only 9% in the senior high schools (15- 18). Senior high school is not compulsory but is attended by about 94% of the age cohort. Within it mathematics is compulsory for the first year and the number of female students opting to study mathematics at this level and subsequently at university appears to be increasing rapidly. Nevertheless only 20% of mathematics undergraduates are females and at graduate school this percentage decreases rapidly: to roughly 10%.

Although the situation might be improving, it is still the case that there are very few female senior high school and university teachers of mathematics. Thus the female voice is rarely to be heard on influential committees. In over 50 higher-educational institutions offering courses on mathematics education there is only

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

one female professor. This imbalance can also be seen in, for example, textbook authorship. Japanese textbooks tend to be written by groups of authors, but inspection of six texts written to meet the demands of the new (1992) curriculum for the elementary school revealed that three had no female authors and that, of the others, the best had 1 in 12 of the authors female. Six junior high school texts revealed roughly the same pattern: 3 with no female contribution, the others with a best representation of 6%.

Research indicates no differences in attainment and attitudes at junior high school level, but relatively big differences in the senior high school. Some of these problems may arise from stereotyping in texts and an emphasis in them on males, but perhaps the most important arise from outside the mathematics classroom: from career expectations combined with those of early marriage; from the extreme pressures currently exerted by the entrance examinations held by senior high schools and higher educational institutions; and from a failure to set mathematics teaching within a social context (which steps are now being taken to correct).

The position in Portugal is very different from that in Japan. It is only recently that the school-leaving age was raised from 12 to 15 and it is still the case that the majority of young people do not attend senior high school. Portugal's educational system, then, may be seen to be lagging some decades behind those of many of its European neighbours. That is not all for the bad, however, for it does mean that teachers still retain a higher standing within the community than they have preserved elsewhere in Europe and North America. Unlike Japan, women occupy a greater position in the teaching force and are in the majority amongst undergraduates studying mathematics - two facts which may not be unrelated. Yet allegiances can be fickle as one example showed. A "Chemical Engineering" degree course on which 70% of the students were women was renamed "Industrial Chemistry": there were no changes in course content, but the gender balance was completely reversed - now 80% of the students were men. The recent expansion of the educational system has meant that many "non-mathematicians", eg engineers and economists have been recruited into teaching. Fewer women seem to be attracted to teaching and the status of the profession appears to be declining. Several forces have combined to bring about a marked increase in the provision of private tuition. This gives teachers a second income, but also "justifies" the government in keeping teachers' salaries low. Private tutoring on a large scale, as has been found elsewhere, has significant effects on students' attitudes in the classroom.

The Mexican educational system has also expended enormously in recent years: in particular, at the university level. Great numbers of extra university staff were recruited: the result being ill-prepared teachers and unattractive salaries. About two thirds of the mathematics staff were recruited before they had obtained a bachelor's degree. Now the position is more stable and most of the faculty have at least a master's degree. Women currently comprise about 34% of the faculty staff in science and mathematics compared with 44% in education and the humanities. There are more women in private than in public universities and since research tends to be concentrated within the latter, this means that many women are teachers rather than researchers.

Expansion of higher education has led to some neglect of other sectors. The status of teachers has dropped along with the number of well-prepared entrants to the profession. Women form a major part of the teaching force: 80% at the elementary level.

Female participation in higher education rose from 17.3% in 1969 to 34.5% in 1985 and continues to grow. However, within mathematics the percentage of female students has remained fairly constant since the mid-1970s at around 37%. There appears to be a strong link between social class and the holding of sex stereotypic views on women in mathematics – both women students and women teachers tend to come from higher social classes than their male counterparts.

As in many other countries very little research is being undertaken on gender issues. There is a shared feeling concerning the importance of peer group and societal influences but this awareness is not being reflected in actions.

The report from England took a slightly different form. There, gender problems had been very much on the agenda in the 1980s, but were now in danger of being forgotten. So far as data were concerned, it was noticeable that gender imbalances up to the age of 16 had now largely disappeared. However, as elsewhere, once mathematics became optional, girls were more likely to drop it and so the percentage of females entering for the 18+ specialist mathematics examination or for degrees in the subject showed little change.

However, political changes appeared to have had serious consequences for active work on gender. Whereas, for example, the influential "Cockcroft" report of 1982 had devoted a large appendix to gender issues, the new National Curriculum had nothing to say on the matter. Changes in the financing and responsibilities of the local education authorities which in the past had supported gender initiatives, meant decreased resources for in-service courses and work in the classroom. A change in the role of (the much depleted force of) Her Majesty's Inspectors from advisers and commentators to 19th century inspectors of standards and attainment, had removed help from that source. Whereas in the 1980s there was an atmosphere of research generated by Valerie Walkerdine and others, this had now disappeared. Financial stringency and political pressures had diverted money and interest towards assessment. Even at national meetings such as the British Congress on Mathematical Education, there now appeared to be little specific interest in gender issues. Within education certain initiatives which appeared advantageous to girls, such as the inclusion of more coursework in the 16+ "external" examinations, were being suppressed or restricted as more "pencil and paper" tests are being imposed on schools.

Nevertheless, gender-related work continued, if on a reduced scale and without so much institutional recognition and help. Most importantly, however, it was not now being perceived quite so much as a single, independent problem, but as one which cannot be divorced from other social problems such as race and culture.

Reporter:
Geoffrey Howson

Panel 5:
RESEARCH PERSPECTIVES

Moderator:

Maria Trigueros

Panelists:

Karin Beyer, Helga Jungwirth, Meredith Kimball and Else Marie Staberg

Research on issues related to gender and mathematics education started about 20 years ago. The first studies were centred on quantitative techniques and focused mainly on assessment, specifically on standardized tests.

During the past few years qualitative research perspectives have provided an opportunity to look forward and examine the alternative paths that can lead to a better understanding of the role of gender in mathematics education and the particular actions to be taken to ensure better mathematics for all.

The panelists concurred on the importance of research, because it deepens our understanding of the factors that can lead to an integration of gender and culture in mathematics education.

The study of gender problems is a complex social issue. There are plenty of intertwined factors that contribute to the present situation in the mathematics classroom. It is important to analyze social interactions and the context in which they arise. The main question is, what kind of research is more appropriate and reliable?

There was also a general opinion that research in gender and mathematics education is a very sensitive field. From previous research we have some descriptive information, but that information is not always perceived as neutral and it is very easily misunderstood.

Both quantitative and qualitative research are fundamental in the understanding of complex situations such as this one and, as the possibility of funded research may be limited, it is extremely important to focus on research that will eventually give results that can be implemented in actions for change.

The lack of a theory or of coherent models to guide such research can lead to method-driven investigations, where the answer may be correct but nothing can be said about the question. It is important to reflect on paradigms.

Karin Beyer stressed the need of eclectic research, where quantitative and qualitative methods are used together and always accompanied by clear statements about values and goals and by different interpretations of the results so they

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

are not misunderstood. New research should take the concern for change as a useful criterion of validation and focus on investigations where researchers, teachers, and students work together looking for changes in the practice of education. New research in which context, cultural and social aspects of the teaching and learning situations are taken into account is needed.

Helga Jungwirth continued with the discussion about qualitative methods; she said that most of the research that has been done so far is deterministic in the sense that it looks for laws to predict behaviour, taking as a basic law the gender-role socialization. She thinks this makes sense if the social reality is considered homogeneous, but that this is not the case.

Qualitative methods give results that cannot be generalized. This is often perceived as a problem but from a constructivist position, where models are considered as viable constructs, they are valid as long as results obtained from them are helpful.

One possibility for implementation of these changes in methodology is to rely on symbolic interactionism and ethno-methodology as a theoretical framework. The basic assumption is that people construct subjective meanings for things but nevertheless arrive by social interaction at a common reality and to shared meanings. Handling and modifying meanings is accomplished through an interpretative process of the individual in dealing with things encountered.

This methodology is helpful for analyzing interactions in the classroom with respect to gender differences. The actions of one participant are then analyzed in terms of the interactions of the whole community; that is different from the sum of the particular interactions – it is an emergent event. This methodology is useful because it prevents the interference of stereotypes and gives one new insights on the problem, tracing it back to its sources and pointing out possible paths to solve it.

Meredith Kimball talked about past research on achievement in mathematics. Depending on the definition chosen to measure achievement research results, there is little difference between genders: standardized examinations favour males, school grades favour females. In spite of the small difference there is a persistent belief in the superior performance of males in mathematics.

One important factor that contributes to this misinformation and to its persistence is the belief that mathematics belongs to the realm of the masculine (that is, of symbolic gender). Exploration at different depth levels show that it is possible for mathematics to be a gender symbolic system even though women and men do equally well. Studies show also that in much of the research on gender issues in North America, and in the media presentations of those results, there is the underlying assumption that men should do better in mathematics than women. Only more research directed to the analysis of the persistence of peoples' belief systems can guide the attempts to alter them.

Else Staberg based her classroom research on a feminist perspective and in a slightly more general setting, the science classes. In her viewpoint, research should be emancipatory, and the issues of gender, class, and ethnicity cannot be considered as isolated. There is a considerable interplay between different factors, and gender is more and more looked upon as a construct structured by society.

She agrees that science and technology are conceived as masculine domains that can be criticized from a feminist perspective, taking individual differences into account. This approach allows one not only to describe but to explain and to understand girls' choices. One difficulty that showed up was that teachers became objects of the research; this posed a difficult ethical question.

The panel conclusions indicate the need of a theory-based and more focused research for the years to come. Theory is necessary for the proper formulation of the research problem and in the planning of the research program. The methods should be chosen according to the questions that are perceived as important under the light of the theory. When theory is not complete, inspiration from the practice can be helpful.

Some research themes were suggested explicitly by Karin Beyer, Meredith Kimball, and the public as examples of interesting topics.

- Comparisons in gender and mathematics in two different countries. Comparisons in large-scale testing can be meaningless because of the differences in culture but comparing the way gender roles are developed in two different cultures can be enlightening.
- Classroom processes in terms of teaching strategies that include cultural and gender factors.
- Teaching strategies to promote independent autonomous and responsible learning styles for all.
- Strategies for effective mentoring of female students and staff members at the universities.
- Implications of socialization changes of the genders in mathematics education.
- Analysis of people's beliefs systems in relation to gender differences and mathematics achievement.
- More work on boys' behavior in classroom.

During the discussion, the problems of publication of research results, because of the bias against quantitative research, was addressed and there was a widespread emphasis on the importance of editing books on gender topics.

A reasonable amount of research is being done that will aid in understanding the problem; this will open doors to actions and participation of the mathematics community. But there remains much to be done to arrive at an integration of females and minorities in mathematics.

Reporters:

Carlos Bosch and Maria Trigueros.

Working group A1: STUDENTS – PERSONAL AND PSYCHOLOGICAL FACTORS

Leader:

Gilah Leder

Our working group focused on the personal and psychological factors relating to students in mathematics education. We had a wide-ranging discussion, reflecting the differing backgrounds and experiences of the group members and the number of interacting factors which complicate discussion of the student-related issues.

Initially we brainstormed ideas and discussed input from other sessions as it related to our topic. Student attitudes, effects of curriculum change, social and cultural factors such as parental encouragement, cooperative and competitive learning environments and alternative assessment practices exemplify the breadth of discussion.

The use of groups as a strategy for increasing girls' participation and improving outcomes was a recurrent theme. People shared their experiences of initiating or participating in group work and topics such as organizational strategies, differential participation and monitoring were discussed.

In the second session each participant contributed a relevant piece of research or personal anecdote. The variety of settings from which our group members came was reflected in the range of material described. Personal experiences gave insights into work with students at all levels – using group work in teacher training, socio-cultural factors and their influence on females, tertiary learners choosing institutions for their style of teaching, single sex and co-educational settings, communication between students and teachers. These contributions, and others, cast light on the position and achievement of girls and women across a broad spectrum of educational settings.

Several group members shared research results and research questions, highlighting the importance of systematic observation and analysis in determining the experiences of learners in the classroom. For example, an Australian study showed that despite the appearance of cooperation in a mixed group engaged in non-traditional mathematics problems, gender stereotyped tasks were undertaken. Canadian research highlighted the tendency for students to attribute the perceived qualities of the subject to the teacher. Whether differential access to home resources influenced students' performance on homework-based courses was a

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

question nominated for further research. Children in one multi-grade school setting showed a tendency towards choosing children of similar ability to work with when offered a choice. This factor seemed to be a stronger influence than gender or age.

We went on to consider how students might be disadvantaged by factors in the learning context, resources, the content of mathematics and its assessment. We also considered the nature of the interaction between gender and other factors such as socio-economic status, ethnicity, and language.

As part of the final session we each provided a summary statement and we discussed these in groups. The comments were grouped into three broad areas:

the diversity of viewpoints at the conference, the idea of a feminist's perspectives and their implications for classroom practice and research, and concern with learners - the relationship between cognition and affect and the importance of affective responses.

While expressing a wide range of opinions and ideas we were able to deepen our understanding of the issues relating to students as mathematics learners.

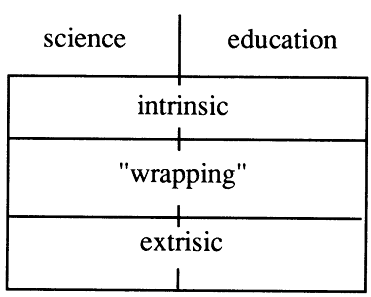
Participants:
K. Beyer, T. Bibelnieks, B. Christiansen, A. Cordeau, Y. Elvin-Nowak, L. Finne, E. Fennema, H. Forgasz, J. Gaffney, F. Grant, K. Hag, M. Haynes, K. Saif, C. Jacobsson, B. Reilly and N. Sukthankar.

Reporter:
Fiona Grant

Working group A2, subgroup 1:
MATHEMATICS AS A DISCIPLINE

Leader:
Gila Hanna

The first session began with the group trying to clarify its thoughts about what was meant by "Mathematics as a Discipline." We explored a number of key phrases: the "teaching of mathematics," the "understanding of mathematics," and the "nature of mathematics." Some time was spent discussing interaction and negotiation in the classroom, the building of concepts, the use of language, and ways of presentation. The focus of the discussions frequently shifted, from "mathematics" to "the teaching and construction of mathematics." We considered a diagram illustrating relationships between the science of mathematics and education of mathematics and the intrinsic, extrinsic, and "wrapping" aspects to this relationship. Some participants assumed that the greatest weight was being given to the intrinsic elements but proponents of the diagram argued that the "wrapping" and external aspects were just as important.



In session 2 there were several strands to our discussion: firstly, a recurring theme of the distinctions between the "finished product," the process of creation, and the solving of mathematics problems. There was a concern that mathematics not be seen as "God-given" and yet, in contrast, that there are unchangeable particular results.

In this matter, several strategies were suggested: placing an emphasis on teaching within a historical context, spending more class time on the processes of creation and problem solving (both doing it and discussing it). The question was raised whether one should discuss philosophy of mathematics in the classroom and the general view was: not directly but through the above strategies.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

The long-term nature of the mathematics curriculum implies a need to make connections – if one doesn't teach mathematics in a way that prepares for these connections, difficulties will arise. But where was gender in this? We resolved to concentrate on this aspect in the next session.

Although , in session three, agreeing that mathematics as a discipline is a deep and philosophical question that we could instantly answer, nevertheless several aspects of the question seemed fairly clear to us.

1. Mathematical meaning is constructed in the classroom through interaction. Typically, the structure of these *interactions* is closer to practises that boys have developed in their culture than girls in theirs.
2. The image of mathematics ("wrapping") is important to the learner. Bringing in history, social context, and local culture are important ways of clarifying to the learner that mathematics is a human activity and not a finished body of knowledge.

After some discussion on whether the nature of mathematics would be similar or different if women had developed it, we concluded that different people make different mathematics but that the results stand. Mathematics *has* developed differently in different cultures. Chinese mathematics (in methods and emphases) is very different from Greek mathematics – very different but not incompatible.

We suggest that the same would be true for "male" and "female" mathematics.

The workshop finished with a brief discussion of a GASAT paper that seemed to be suggesting, among other things, that women were less capable of coping with abstract ideas than men!

Participants:
Josette Adda, Awadh Aram, Thora Blithe, Gerd Brandell, Megan Clark, Gila Hanna, Helga Jungwirth, Meredith Kimball, Gordon Knight, Hanne Koch, Anna Kristjànsdóttir, Lisbeth Lindberg, Dagmar Neuman, Mogens Niss, Elfrida Ralha, Kristine Revak, Else-Marie Staberg and Maria Trigueros.

Reporters:
Megan Clark and Maria Trigueros

Working group A2, subgroup 2:

MATHEMATICS AS A DISCIPLINE

Leader:
Christine Keitel-Kreidt

The main object of our very lively discussion was clarification of our different views of mathematics: the different philosophies of mathematics that underlie research and teaching; how mathematics is seen by other disciplines that apply mathematics; the public views of mathematics mediated by schools or media. We discussed the idea of progress in mathematics: are there final ends and certain teleological goals pursued in doing research? Concentrating on the idea of modelling and mathematics as a means for description and prediction, we referred to mathematics mainly as a social enterprise with a certain social history and social conditions of development. As a core of mathematics, we summed up: it is related to creativity and beauty, but aims at simplicity in the sense of the "most basic or fundamental," at generalisation to the far-reaching end.

On the other hand, we agreed that not only the image of mathematics but also mathematics itself is constituted by teaching and communicating, and we asked ourselves: how do we convey our perception of mathematics as a challenging activity and full of creativity, as a fantastic and a rational construction and as a collective development process when teaching? In particular, how do we do convey these ideas when publishing mathematics is our official way of communicating? What will be visible of our main perception of mathematics for others if we just publish or teach purified and polished results? We analysed the question of how we teach, with respect to school mathematics as well as university mathematics, and we agreed that the view of mathematics as unpersonalised, linear ordered, and structured that is offered by the teaching lectures, in textbooks, and in publications completely obscures the way mathematics is created and used. By social acceptance, mathematics has been taken as a world view for all other disciplines, modelling the most appropriate approach to problem solving in nearly all areas. However, the restricted view of mathematics in the public contradicts this use. We found that we as mathematicians and educators are responsible for public opinion about mathematics and that we have to take responsibility for changing the image as well as for misuse of mathematics, that is, a rhetorical use and a cheating by mathematics.

Concerning the gender issue, our discussion tended to take a somewhat speculative turn – not accepted by all members of the group – of whether math-

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

ematics would be different if women had contributed in a more substantial way to the development of the discipline. How could we perceive this? We rejected the speculation about the kind of mathematics as another product, but some of us believed that there might have been a women's focus on different problems, the development and use of another philosophy, another language, more inclusive and less competitive ways of communicating; in particular, there might be a focus on other questions and problems for the application of mathematics, but this can be negotiated as we exert more influence on research and education in mathematics.

Participants:
Liv Berge, Bodil Branner, Susan Chipman, Andrejs Dunkels, Sharleen Forbes, Mary Gray, Liv Sissel Grønmo, Miguel de Guzman, Christine Keitel, Jeremy Kilpatrick, Cora Sadosky, Claudie Solar and Gillian Thornley.

Reporters:
Christine Keitel and Bodil Branner

Working group A3:
SOCIAL, ECONOMIC AND TECHNICAL DEVELOPMENTS

Leader:
Lesley Jones

Questions we decided to address:

1. Could technology provide a way to improve or change mathematics?
2. What are the perceptions of mathematics (of pupils and teachers), and what causes these?
3. Redefining mathematics to relocate its power (hegemony of powerful male mathematicians).

A number of issues were discussed. Some of these are summarised below:

- Technology can be liberating or oppressive. Both of these are relative, not absolute terms.
- Pedagogic issues: Teaching styles can empower or disempower the learner.
- Significance of the context to the introduction of technology. Sometimes it is appropriate to introduce technology because it is particularly useful for a specific piece of mathematics.
- Should we be using technology to *explore* mathematics rather than mathematics to *exploit* technology? What effect does this have on the curriclum?
- Fear of technology: Both teachers and students may have fears about using technology in the classroom.
- Expert novice model: The dangers of establishing a pupil "expert" to advise others about technology.
- Valuing different skills and knowledge: The process can be more important than the product. Technology may cause us to reevaluate what we see as important in mathematics.
- INSET: Dealing with problematic situations. Where do teachers turn for advice?
- Power / control / autonomy: Is the power base in the classroom likely to shift because of technology? Can technology help pupils to be autonomous in their learning? Can the use of devices such as "grading contracts" help students to take more control over their work? If the teacher is giving the grades she is taking responsibility. If the students set the grading criteria and agree or negotiate their grades, they have to take more responsibility.

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

- Assessment has an effect on the curriculum. What is assessed tends to define what is important.
- Technocentrism: Technology can become the focus of classroom activity at the expense of other considerations.

Towards the end of the sessions, the working group decided that they wanted to continue working together. At present, colleagues are in correspondence with each other and considering each of the following "problematics." It is anticipated that the group will publish its findings.

Problematics

1 Technocentrism
2 Assessment defines what is important
3 Boys and mathematics
4 Three-way interaction between technology, mathematics, and gender
5 The metaphoric agenda that lies behind the choice of terms and phrases associated with technology
6 The effect of scarcity of resources.

Participants:
Mary Barnes, Tracy Bibelnieks, Leone Burton, Laura Cavallo, Suzanne Damarin, Jeff Evans, Ann Koblitz, Helen Pilström and Teresa Smart.

Reporters:
Lesley Jones

Working group B1:

ASSESSMENT AND CURRICULUM

Leader:

Mogens Niss

The first session of this workshop was a brainstorming session that produced a variety of questions from the group concerning current assessment. Although curriculum was part of the discussion topic, it was the opinion of the group that the focus would be assessment, with curriculum as a component. The questions generated and discussed in this session addressed broad questions that included:

1 Do current assessment modes favor gender differences?
2 Are new assessments traditionally researchable?
3 Do we know what assessment measures, and do we know what we measure when we measure assessment?
4 What is the purpose and use of assessment?

The second session of the working group was focused around more specific questions that included:

1 Where are the obstacles, problems, gender differences in current assessment procedures or the use of the results of the assessments?
2 Does traditional assessment promote memorization, which in turn has a differential effect?
3 Assuming that assessment leads to gender differences, how do we assess learning styles to understand this effect?
4 What is the gender significance of cues, context, and environment in the mathematics being assessed?
5 How do we assess the assessment?
6 How do we influence the systemic environment to accept alternative modes of assessment?
7 What are the power and control issues in assessment affecting gender issues?
8 If we develop authentic measures of mathematical knowledge, will the gender issues take care of themselves?

The implications of saying "yes" to question 8 were discussed. This resulted in the group feeling that "no" was a more appropriate response. The on-going process of changing assessment must include gender issues. In fact, gender issues could be used as a vehicle for changing traditional modes.

Further issues were discussed in the second session – assessment does not occur in a vacuum and neither does interpretation of results, and therefore they are

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

influenced by many outside factors; also, the way the assessment results are used reflects on the students image and therefore functions as gatekeeper to further study and careers. In light of this, the more basic question of why we have chosen to assess mathematics in the first place was raised. It was pointed out that, like cooking, mathematics is fundamental and essential in everyday life, yet we have chosen mathematics to be academic. However, the group agreed that assessment in mathematics does play a vital role and the discussion returned to the matter of addressing deficiencies in current assessment practices.

The third session focused on the societal resistance to change traditional assessment (society being defined as those who govern the educational system and employers). Assuming that innovative assessment may be better able to accommodate gender issues, society may resist them. Some reasons identified by the group were cost to society, validity and reliability of new assessment results (i.e., the perception of cheating), and a move away from a one-mark indicator that society can use as a filter.

Awareness of the need for change can become a wedge to overcome societal resistance. In this regard, students need to be made aware of the need and purpose of change. Their voice should not be overlooked as one of the important vehicles available to carry this message to society.

The assessment and curriculum workshop concluded with the group voicing concerns that future research of assessment and gender issues must be continued to provide support for change. In the words of Jeremy Kilpatrick, "research is not just an academic exercise – it reshapes our thinking."

Participants:
Saleh Awadh Aram, Liv Berge, Tracy Bibelnieks, Bodil Branner, Megan Clark, Jeff Evans, Sharleen Forbes, Janice Gaffney, Barbro Grevholm, Liv Grønmo, Gila Hanna, Maggie Haynes, Calle Jacobsson, Jeremy Kilpatrick, Gilah Leder, Lisbeth Lindberg, Mogens Niss, Helen Pilström, Mary Beth Ruskai and Khariya Saif.

Reporter:
Tracy A. Bibelnieks

Working group B2:

TEACHERS – PERSONAL AND PSYCHOLOGICAL FACTORS

Leader:

Christine Keitel-Kreidt

The major concern in our discussion was the role of gender issues in the classroom and possibilities for change in and by teacher training for all levels (primary, secondary, tertiary), in all areas (preservice, inservice, continuing edu-cation).

For some time, our discussion was concerned with the question of the "necessary" mathematical knowledge provided by teacher education, including attitudes and beliefs developed by the way it is offered. The question: What is a "sound mathematical knowledge" for teachers of all level or school types, and for secondary and tertiary in particular; what is the "correct" view of maths, the most "substantial" topics and methods, and the necessary "experience" to apply to mathematics and to understand the consequences of this?

The term "sound mathematical knowledge" we confronted with the term "pedagogical content knowledge" in mathematics (mathematics didactics). How do we educate teachers to make them able and aware of the problem to transform mathematical knowledge into an appropriate teaching situation, to reconstruct mathematical knowledge by teaching and learning, to create a rich and "sound" context with differentiated settings recognizing gender issues? How do teachers and teacher trainers convey the psychologically and socially appropriate way for successful teaching and learning, respecting the unplannable aspects of interaction in the concrete classroom? In particular for primary teachers, learning of mathematics cannot be perceived without an integration of context and content; therefore, the question of "sound" mathematical knowledge for primary teachers is specially difficult to answer.

A change in our discussion began by considering mathematics teaching as a mutual interaction. It was emphasized that there is a wrong assumption underlying so much research on gender and classroom interaction as the interpretation of interaction is determined completely by the teacher. Interaction has to be interpreted as active from both sides, teachers and pupils. It is wrong to assume that it should be possible to provide future teachers a full range of appropriate knowledge enabling them to be prepared for all situations in the future classroom and just refer to certain means learned in teacher education.

B. Grevholm, G Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

In contradiction to this assumption, we conceived of the teacher as an active investigator, a problem solver in the practice of the ever-changing classroom situation. In practice, there are constantly new problems to be solved; this has to be taken into account when talking about teacher education, whether in preservice or inservice education. Finally, we discussed different programs that intend to intervene beliefs and ways of conveying mathematics held by teachers and to be seen in daily classroom teaching, the question of gender bias in assessment and its connection to teaching – in particular, we compared teacher-based assessment to external assessment methods that separate teachers from major parts of their responsibility in teaching. We asked for a list of programs for good and successful inservice addressing the gender issue, which might well serve at the preservice education level, also. In order to encourage teachers and ourselves, we plead for not underestimating teachers influence on gender issues in schoolpractice!

Participants:
Thora Blithe, Bent Christiansen, Elizabeth Fennema, Lena Finne, Helen Forgasz, Fiona Grant, Miguel de Guzman, Helga Jungwirth, Christine Keitel, Hanne Kock, Gunilla Ljung, Elfrida Ralha, Claudie Solar and Jan Troják.

Reporter:
Christine Keitel-Kreidt

Working group B3:

SOCIOLOGICAL AND CULTURAL FACTORS

Leader:

Maria Trigueros

We affirm that women can do mathematics and do achieve at the highest levels. This is shown by the growing presence of women professors among the mathematics faculty of universities and the contribution of women as invited speakers at the International Congress of Mathematicians. In the United States of America there have been great changes since 1960 in the numbers of women studying mathematics through high school and on to college/university, so that now equal numbers of men and women are graduating in mathematics at bachelor and master's levels. Most other countries represented in our group have not yet come so far along this road, although one (Kuwait) reported that three quarters of their BA students in mathematics are women. Nevertheless, there are many women worldwide who do not choose to study mathematics beyond the level where it is compulsory and many whose opportunities are restricted in our society because of their lack of achievement in mathematics.

What are the sociological and cultural factors influencing girls and women in relation to the study of mathematics?

Home and Community

In traditional cultures formal education has not been a high priority for girls. They were expected to participate in home and family tasks and to learn the domestic skills from their mothers. This attitude still persists today in many societies. For example, in Papua New Guinea, the traditional bride price takes no account of a young woman's formal education and so acts as a disincentive to education for girls. Even in countries where efforts have been made to remove gender-based barriers to education, parents may retain the prejudice of earlier generations and discourage their daughters from studying mathematics, depicting it as a "hard" subject in which they are unlikely to succeed.

The pressures of the home and family role can reduce the time girls and young women put into their study compared with that available to boys and men. Girls may be expected to take a heavier load of household tasks and to care for younger children, while young women and those returning for second-chance edu-

B. Grevholm, G. Hanna (Eds).
Gender and Mathematics Education, an ICMI Study

Lund University Press, Lund

cation may have to juggle study, work, childcare, and the home. Bright girls often have a broad range of interests and accomplishments in addition to their formal schoolwork. All of this reduces the time and attention they give to their formal study. It influences their choice of subjects and may mean that they do not realise their potential in mathematics.

For teenagers, peer group pressures are very strong. These frequently give the message that "it is not cool to be smart," and those who choose to pursue mathematics and to excel in it risk being ostracized. This is particularly true for young women. In the youth culture, it may be all right for some young men to be bright, but for young women it can appear to restrict their life options. As a result, they may hide their ability or develop a fear of success. A programme in Minnesota that seeks to overcome some of this peer pressure takes bright students out of schools across the city once a week for a special class in mathematics at the university.

Young women are subject to strong cultural pressures associated with motherhood, which influence their choice of subjects and of career. They do not want to risk losing out on things that are important to them as women. They must wonder whether further study in mathematics will lead to a congenial work situation where it is possible for them to combine career and family. Other women in mathematical careers can be reassuring role models.

Mathematics is commonly used as a gate-keeper to a number of well-paid professions such as law and medicine, as well as science and engineering. Young men are expected to achieve in mathematics and have access to these powerful positions in society but there is not always the same encouragement from family and community for young women to keep these doors open.

Some societies do not ascribe the same importance to mathematics. In Kuwait, where all men are guaranteed a highly paid job so there is less incentive for them to study mathematics, three quarters of the graduates are women.

School and University

Within the education system there can be invisible barriers to women and girls studying mathematics. Research has shown that teachers' expectations of pupils are frequently fulfilled, and there are still teachers who expect less from girls than from boys in mathematics. There is a tendency to label students according to their social or gender grouping and ascribe stereotypes rather than treat students as individuals with the capacity to change.

A study in France found that teachers unconsciously favoured boys in subjective marking. Other countries have reported similar cases.

There has often been a worry that children may be introduced to mathematics by teachers who are not entirely comfortable with the subject themselves and who display aspects of "mathematics phobia." At the high school level in the United

States of America, it has been found that some counsellors who advise students on their choice of subjects also suffer from mathematics phobia. Such people can easily discourage girls (who may need reassurance and encouragement) from continuing with mathematics.

Although evidence of the direct value of role models may not be strong, a lack of female role models in the university mathematics departments and in jobs using mathematics must carry a subconscious message to young women contemplating entering the field.

Minority Groups and Mathematics

We found a number of parallels in the difficulties faced by certain racial minority groups and the difficulties faced by women in regard to mathematics. Women from such minority groups can be at a double disadvantage. Nevertheless, our attention was drawn to the results of two studies involving African-American students, where girls/women did better in mathematics than boys/men. Also, among the small group of African-American mathematicians with PhDs there is a higher proportion of women than in the case of white Americans.

Students from minority groups can be disadvantaged by teacher expectations and the labelling of the group as being weak in mathematics.

A lack of role models among teachers or senior students can deter students from a minority group from continuing with mathematics. Sometimes young people from a minority group are keen to position themselves in careers where they can help their own people. They frequently fail to see mathematics as having any relevance to this, although by becoming a teacher they may be able to influence the way mathematics is taught and encourage more of their people to achieve in this area, opening a wider range of career options to them.

The group discussed some intervention programmes to improve the performance in mathematics of minority children. It seems to be a good strategy to involve parents along with children in such programmes, though we noted that this requires child-care facilities so that all parents can attend.

Language plays a very important role, particularly when students are beginning to learn mathematics. Children from minority groups can be at a considerable disadvantage if they are trying to learn mathematics along with a second language. In Sweden this problem is being addressed for groups of Finnish children by providing schooling in their own language up to the intermediate level. In New Zealand some schools provide instruction in Maori, the language of the indigenous people. This is more widespread at the early-childhood level but a few high schools have a Maori bilingual unit, where the aim is to present mathematics (and other subjects) in a Maori context. It has taken patient cooperative work with the Maori people to identify the mathematics within their cultural lifestyle and to find genuine Maori words to describe mathematical concepts. This approach to teaching mathematics is producing significant improvement in achievement for these Maori students.

Comments and Questions

It is commonly emphasised that young women need affirmation of their ability and encouragement to continue in mathematics. The group agreed that this is very important, but we have to ask: How can we change the cultural messages surrounding childhood and elementary mathematics education that apparently leave so many girls/women with little confidence in their abilities in mathematics and an expectation of failure?

We felt that there is insufficient evaluation of the various intervention programmes to improve the participation and performance of girls/women (or minority groups) in mathematics. It seems that some sort of parental involvement can be helpful in these programmes. We were divided on the issue of separate schools/ colleges for women (or minority groups); we agreed that there are advantages and disadvantages but they are good for some students.

We suggest that the approach of the Maori programme (which begins with the cultural context and develops mathematics relevant to it) may be useful with girls/ women also. This is probably already happening within the Equals programmes but is it carrying through to high school and university? Does mathematics itself need to change?

We are concerned that we can contribute to the stereotype of girls not being good at mathematics by always looking for the differences between boys' and girls' performance and providing remedial-type programmes for the girls. We note the success of the Challenge for Excellence programmes at university level, where young women are challenged to work in groups on difficult problems over an extended period. These women produce excellent mathematical work beyond expectation. (Berkley and Mills programmes set up by Association for Women in Mathematics.)

We recommend a positive approach that affirms that girls/women can do mathematics, provides a congenial cooperative atmosphere for them, and expects them to excel.

Participants:
J. Adda, M. Barnes, K. Beyer, G. Brandell, L. Burton, L. Cavallo, S. Chipman, A.M. Cordeau, S. Damarin, A. Dunkels, Y. Elvin-Nowak, M. Gray, K. Hag, S. Hazama, A. Hibner Koblitz, L. Jones, M. Kimball, G. Knight, A. Kristjànsdóttir, D. Neuman, B. Reilly, K. Revak, C. Sadosky, T. Smart, E-M Staberg, N. Sukthankar, G. Thornley and M. Trigueros.

Reporters:
Maria Trigueros and Carlos Bosch.