Universität Bielefeld Fakultät für Mathematik

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Workprogramme for an extension of the Project INTAS-93-2618
for the time July 1, 1997 – June 30, 1999

1. Title:

Linear Algebraic Groups, Algebraic K-Theory,
and related Structures in Algebra and Topology

2. Scientific Objectives:

The research of this project will concentrate on the following: Investigation of the rationality linear algebraic groups, the splitting behavior of linear algebraic groups and their projective homogeneous varieties, the study of anisotropic groups, further investigation of splitting properties of quadratic forms, study of algebras with involution, investigation of central simple algebras with involution, descent and decomposability questions on such algebras, investigation of Lie algebras and their cohomology groups and their nonsplit and central extensions, in particular the computation of those groups, investigation of central simple algebras with involution Properties of the Brauer group for certain function field extension, K-theory of varieties, K-theory of normed algebras, Mac-Lane cohomology.

The scientific objectives of this project is to perform research in areas related to these fields.

3. Description:
3.1. Background:

The theory of linear algebraic groups governs a major number of algebraic disciplines, such as modules over rings, quadratic forms, Azumaya algebras, Lie algebras, classical groups and related structures. While the historical roots of this theory go back into the last century (at least), more recently, beginning in the sixties, the discipline of K-theory has been developed to study those and more general structures as well. This branch soon turned out to develop very powerful methods for various branches of mathematics, especially in algebra and topology.

In particular there is the field of Lie algebras and their cohomology groups, the theory of Azumaya algebras and Brauer groups, and the theory of semisimple algebras with involutions, the theory of quadratic forms, and algebraic K-theory. All these are or provide “related structures” in the theory of linear algebraic groups in the sense that the properties of these structures are essentially governed by their groups of automorphisms which are algebraic groups.

3.2. Research activities :

The influence of cohomological invariants on the splitting behavior of linear algebraic groups will be investigated, new insights into rationality questions are expected. The properties of anisotropic semisimple groups will be investigated by i) classical methods (over local and global fields by means of number theory and class field theory), ii) cohomological methods, iii) splitting patterns of such groups, iv) anisotropic splitting, and an approach towards a sort of classification of such groups is intended.

Further progress is expected to be made on the research about semisimple algebras with involutions, on questions concerning Brauer groups, the computation of the K-theory of a wide class of varieties, e.g. for the class of homogeneous varieties. A description of K-groups by an exact category in terms of generators and relations with possible applications to regulator maps between K-groups with cohomologies are intended.

3.2.1 Short term visits (roughly 1-2 weeks each) in order to participate on ad hoc workshops at any of the participating institutions, depending on their schedule.

3.2.2 Participation at various scientific conferences, for example the International World Congress of Mathematicians August 18-27, 1998, and its satellite conferences, in particular on “Linear Algebraic Groups and Related Structures”, organized by I. Kersten, A. Merkurjev, U. Rehmann, at the Sonderforschungsbereich “Diskrete Strukturen in der Mathematik”, Bielefeld, Germany, from August 10 till August 14, 1998 (Satellite conference of the ICM98, Berlin).

Research of the individual partners of the contract, in particular on the following fields:

3.2.3 Bielefeld: Anisotropic algebraic groups, their splitting and excellence properties (cooperation with all other labs exists).

3.2.4 Besancon: Research on semisimple algebras with involutions and on their decent and decomposability properties (cooperation with Bielefeld, Minsk, St.Petersburg exists)

3.2.5 St.Petersburg: K-theory of varieties, description of K-groups, study of homogeneous spaces over classical algebraic groups, splitting behavior of quadratic forms, properties of semisimple groups over arbitrary fields (cooperation with Besancon, Bielefeld, Minsk exists)

3.2.5 Minsk: Behavior of the Brauer group under certain special field extension, R-equivalence and rationality questions for algebraic groups, arithmetic groups (cooperation with Besancon, Bielefeld, St. Petersburg exists)

3.2.7 Alma-Ata: Constructive cohomology theory for Lie superalgebras, their first and second cohomology groups, noncommutative extensions of Lie algebras, i.e. Leibniz algebras (cooperation with Bielefeld and with several other Western sides exists and with Tbilisi is intended).

3.2.8 Tbilisi: Hodge algebras, C^*-algebras, K-theory of normed algebras, Hochschild cohomology of algebras, Mac-Lane cohomology, Leibniz representations (cooperation with many Western research groups including Bielefeld exists, with Alma-Ata intended).

3.3 Expected results:

Progress towards a classification of anisotropic groups over arbitary fields and of the structurs of semisimple algebras with involutions, understanding of their decent and decomposability properties, understanding of K-theory of varieties and of homogeneous spaces over classical algebraic groups, results on Brauer groups and on R-equivalence and rationality questions for algebraic groups, arithmetic groups, on cohomology theory for Lie superalgebras, on K-theory of normed algebras, Hochschild cohomology of algebras, Mac-Lane cohomology.

4. Management information

4.1 Division of tasks

The division of tasks between the participants in the project is as follows:

University Bielefeld, Bielefeld, Germany
(Scientific official responsible for the project: Mr. U. Rehmann)
Project coordination.

U.F.R. des Sciences et Techniques, Besancon, France
(Scientific official responsible for the project: Mrs E. Bayer-Fluckiger)
Help with project coordination.

A.Razmadze Mathematical Institute, Georgian Academy of Sciences, Tbilisi, Georgia
(Scientific official responsible for the project: Mr. T. Pirashvili)
6 Scientists to attend conferences and visit one or more of the Western laboratories

Institute of Mathematics, Kazakhstan Academy of Sciences, Alma Ata, Kazakhstan
(Scientific official responsible for the project: Mr. A. Dzhumadel'daev)
3 scientists to visit one or more of the Western laboratories

Steklov Mathematical Institute, St. Petersburg, Russia
(Scientific official responsible for the project: Mr. A. Suslin)
8 scientists to visit one or more of the Western laboratories

Institute of Mathematics, Minsk, Belarus
(Scientific official responsible for the project: Mr. V. Yantchevskii)
3 Scientists to visit one or more of the Western laboratories

4.2 Planning:

The contract will last 24 months, the activities will take place according to the detailed informations described below.

4.3 A breakdown of the costs for travel and subsistence:

It is intended to pay a salary to the same persons as in the former project, however, for a longer time (in the former project they were paif for 6 months, now they are intended to be paid for 15 months.)

Labs:	travel and subsistence	salaries	computer equipm.	overhead	total
1. Bielefeld (U. Rehmann)	2000			500	2500
2. Besançon (E. Bayer)	2000			500	2500
3. Pomi St. Petersburg (A. Suslin)	5890	7950		3460	17300
4. Minsk (V.I. Yantchevskii)	5540			1385	6925
5. Alma-Ata (A. Dzhumadeldaev)	3980	1560		1385	6925
6. Tbilisi (T. Pirashvili)	7965	3115		2770	13850
Total					50000

4.4 Detailed information

The number of people, duration and destination of visits involved is given in the description below.

As the actual visits shall be synchronized to the local scientific schedule for the labs Bielefeld and Besançon, which are made up on a monthly basis, at this time it cannot be fixed who should visit where at what time.

For example, at the Sonderforschungbereich "Diskrete Strukturen in der Mathematik" Bielefeld, there are more than 400 mathematical talks of visitors from all over the world. At any time of the year there are usually more than 30 visitors. In order to gain an optimum of scientific success it is vital that, for a particular visit, a time is chosen which allows the visitor not only to meet the local permanent scientists but also other guests whose research is relevant to their field.

It should be mentioned that, as part of the Sonderforschungsbereich, the field of Linear algebraic groups, directed by H. Abels and U. Rehmann, as well as the field of Algebraic K-theory, directed by R. Vogt and F. Waldhausen, both have permanent seminars with many short and long term visitors each.

1. Lab Bielefeld (Germany, U. Rehmann, 7 participants, Budget 2500). There is an overhead of maximal 20 % = 500 ECU which will be used for administration. It is intended to keep this as low as possible. The rest of this money, at least 2000 ECU will be used for visits of one of the Western scientists at one or some of the NIS labs in order to perform joint research, or alternatively for one of the purposes described below (under 3.-6.), if it turns out that there is some additional demand.

2. Lab Besançon (France, E. Bayer, Budget 2500). There is an overhead of maximal 20 % = 500 ECU which will be used for administration. It is intended to keep this as low as possible. The rest of this money, at least 2000 ECU will be used for visits of one of the Western scientists at one or some of the NIS labs in order to perform joint research, or alternatively for one of the purposes described below (under 3.-6.),

if it turns out that there is some additional demand.

3. Lab St. Petersburg (Russia) 8 participants:

Local coordinator: A. A. Suslin (POMI; algebraic K-theory) I. Fesenko (K-theory and class field theory), N. Gordeev (Chair of algebra and number theory of St. Petersburg University of Education; invariant theory, linear algebraic groups), O. Izhboldin (K-theory of fields), A. Merkurjev (Chair of general mathematics; algebraic K-theory, quadratic forms, Brauer groups), A. Nenashev (POMI; algebraic K-theory) I. Panin (POMI; algebraic K-theory, linear algebraic groups) S. Yagunov (K-theory, postgraduate student of A. Suslin).

(Budget: 17300 ECU) A salary shall be paid to 4 persons. Total of salaries: 7950 ECU. 8 persons should have the possibility of a 2-3 weeks visit at one of the western labs or one of the conferences mentioned above (each visit 415 ECU). Alternatively, fewer persons could be visiting for some longer time. A. Merkurjev shall travel from St. Petersburg to Bielefeld for 10 days in August 1998 in order to organize the joint conference with I. Kersten and U. Rehmann on "Linear algebraic groups and related structures",

4. Lab Minsk (Belorussia), 3 participants:

Local coordinator: V. I. Yanchevskii (Head of Dept. and member of Acad. Sci. of Belarus, linear algebraic groups and reduced K-theory), V. I. Chernousov (arithmetic of algebraic groups), A. S. Rapinchuk (arithmetic of algebraic groups).

(Budget: 6925 ECU) V. Iantchevski will visit Bielefeld in for 1 month in order to perform joint research with U. Rehmann. V. I. Chernousov will visit several conferences.

5. Lab Alma-Ata (Kazakhstan), 3 participants:

Local coordinator: A. Dzhumadil'daev (Lie algebras and algebraic groups), K. Abdukhalikov (group theory, linear groups and Lie algebras), Rashid Kerimbaev (defining relations and cohomologies of Lie algebras)

(Budget 6925): A salary shall be paid to 2 persons. Total of salaries: 1560 ECU. For travel and subsistence 3980 ECU is required (possibly one 10-14 days stay of A. Dzhumadeldaev at Strassbourg).

6. Lab Tbilisi (Georgia), 6 participants.

Local coordinator: T. Pirashvili (K-theory, Hochschild homology), J. Gubeladze, H. Inassaridze (K-theory of normed rings), M. Jibladze (topological Hochschild homology), T. Kandelaki (K-theory of C^*-algebras, and Banach algebras), D. Pataraia (simplicial descriptions of plus-construction),

(Budget: 7965): A salary shall be paid to 6 persons. Total of salaries: 3115 ECU. For travel and subsistence 7965 ECU is required.

Activities:

i) Various short time visits of NIS participants at Western labs, depending on their schedule.

ii) August 10-14, 1998: Conference on "Linear algebraic groups and related structures", organized by A. Merkurjev, St. Petersburg, I. Kersten and U. Rehmann at Bielefeld, Germany on this occasion: visit of A. Merkurjev at Bielefeld (10 days) in order to prepare and organize the conference.

ii) August 18-27, 1998: Visit of various NIS scientists at the International World Congress of Mathematicians ICM98, combined with visits on other Western labs.

iv) Short term visits, in particular of the eastern participants, at any of the Western institutes, depending on their schedule.

Final remark

The following Western participants are involved in the whole project. It is understood that if any of the NIS participants wants to use the travel and subsistence money of this project in order to perform joint research in the area of this project with one of these scientists that he should do so, provided the project coordinator and the respective local coordinator both agree on this.

Germany (7 participants) U. Rehmann (Bielefeld; linear algebraic groups, algebraic K-theory), H. Abels (Bielefeld; linear algebraic groups, transformation groups), I. Kersten (Bielefeld; linear algebraic groups, Brauer groups, Azumaya algebras), M. Rost (Bonn; quadratic forms); R. Schwänzl (Osnabrück; topology, K-theory), R. Vogt (topology, K-theory), F. Waldhausen (Bielefeld; topology, K-theory).

Belgium (1 participant) J.-P. Tignol (Louvain-la-Neuve; linear algebraic groups, Azumaya algebras, quadratic forms).

Denmark (4 participants) H. H. Andersen (linear algebraic groups), M. Bökstedt (algebraic K-theory), J. Dupont. I. Madsen (Aarhus),

France (6 participants) E. Bayer (Besançon; linear algebraic groups, quadratic forms). J.-L. Colliot-Thélène (University of Paris Sud, Orsay; algebraic geometry, algebraic K-theory), B. Kahn (Paris VII; algebraic K-theory, quadratic forms), M. Karoubi (Paris VII; K-theory, topology). C. Kassel (Strasbourg; Lie algebras, K-theory). J.-L. Loday (Strasbourg; cyclic homology, Lie algebras, K-theory),

Italy (4 participants) L. Barbieri-Viale (Genova; Applications of algebraic K-theory to algebraic geometry, motivic cohomology), C. Pedrini (Genoa; Applications of algebraic K-theory to intersection theory on singular varieties), L. Robbiano (Genova; Commutative algebra and applications to computer algebra), A. Vistoli (Bologna; Algebraic groups, intersection theory in algebraic geometry).

The Netherlands (1 participant) W. v. d. Kallen (Utrecht; K-theory).

United Kingdom (2 participants) John Jones (Warwick; K-theory). Andrew Ranicki (Edinburgh; L-theory, applications of K-theory to topology)