Universität Bielefeld Fakultät für Mathematik

Prof. Dr. Ulf Rehmann

Declaration of intent for a NIS-EC Cooperation on

**Linear algebraic groups, algebraic K-theory,**

**and related structures in algebra and topology**

## 1. Description of the proposal.

Research in Mathematics, as a supra-cultural, supra-lingual and supra-national science was at every time performed in a collegial spirit by mathematicians of various nations, and even during the days of the cold war there was always a vivid transfer and an exchange of mathematical ideas between Eastern and Western Europe as well as with other parts of the world.

However, as an organizer (and participant) of several conferences in the research areas mentioned in the title, I can definitely say that it was usually very difficult to maintain those connections across the borders which did split Eastern and Western Europe.

Since there was a desire to do joint research between mathematicians working in the mentioned areas, colleagues from the East always have been invited to the Western conferences, but usually the conference chairman was not able to tell whether or not they would be allowed to attend the conference until they really showed up. And usually it were just the senior or well known scientists who were allowed to participate in Western conferences or to visit Western research institutes, while it was usually completely impossible to invite young, unkown mathematicians for such events.

This proposal therefore takes an opportunity to set up a frame of modalities for an intensified exchange of scientists and ideas in an area of mathematics which already has a rich and living history of joint research in Eastern and Western Europe.

I now describe the mathematical intentions of this proposal in some detail.

The theory of linear algebraic groups governs a major number of algebraic disciplines, such as modules over rings, quadratic forms, Azumaya algebras, Lie algebras, classical groups and related structures. While the historical roots of this theory go back into the last century (at least), more recently, beginning in the 60th, the discipline of K-theory has been developed to study those and more general structures as well. This branch soon turned out to develop very powerfull methods for various branches of mathematics, especially in algebra and topology.

Ever since, these two fields have influenced each other in a very fruitful way. I mention the investigations for the congruence subgroup problem which solved difficult questions concerning the subgroup structure of certain classical and more general linear groups by studying questions which are typical for algebraic K-theory like stability questions. Complicated questions about presentations of so called "arithmetic" subgroups of classical and "exceptional" groups over number fields were answered with methods developed in K-theory. The determination of various (co-)homology groups of linear groups were transformed into questions about certain 'K-groups'.

Recent research has uncovered methods which are likely to stimulate further far going investigations of these areas. In general I refer to the volume [7] titled "Algebraic K-theory" and edited by A. A. Suslin, which contains important results of Mathematicians from (nowadays) St. Petersburg going into this direction and concerning most of the structures mentioned above. I especially mention the results of Panin [6] on the K-theory of projective homogeneous varieties which have been used to prove the famous index reduction formulas of Schofield/v.d. Bergh for division algebras, which in turn has recently been brought to a very general form by Merkurjev, Panin and Wadsworth [8]. On the other hand, the importance of these varieties for the splitting behaviour of reductive groups has been investigated by Kersten/Rehmann [3] and it again turns out that those two fields have mutual influence on each other as well as on Azumaya algebras and on quadratic forms (cf. [2, 1, 5]).

Further results including the theory of Brauer groups, algebraic geometry and related areas has been done by Yanchevskii (Minsk), and by Mikhalev (Moscow) and others.

It is vital that algebraic K-theory is not just considered as an isolated theory within algebra. To the contrary, it is quite interdisciplinary within mathematics. Here are some prominent aspects of this feature. First, the construction of higher algebraic K-theory, due to Quillen, uses the elaborate machinery of algebric topology in an essential way; in fact, the very definition of the higher K-groups is in terms of homotopy groups in the sense of algebraic topology—no other definition of the higher K-groups is known to this date. Second, algebraic K-theory has found applications to the topology of manifolds and their automorphisms; keywords are: pseudoisotopy theory, and the algebraic K-theory of topological spaces of Waldhausen. Third, the latter theory necessitated a technical development which has repercussions to algebraic K-theory proper; specifically it has enabeled Thomason to develop higher K-theory for schemes with far reaching applications in algebraic geometry. Fourth, but probably not finally, the theory of the trace map which into higher algebraic K-theory was introduced by Dennis has found a substantial extension to a theory involving the so-called topological Hochschild homology theory. A variant of this, the topological cyclic homology, has recently permitted the solution, by Bökstedt, Hsiang, Madsen, of the K-theory analog of the Novikov conjecture. The topic of topological Hochschild homology has, by the way, already been the subject of a joint project across the former iron curtain; the result of the project is reported on in a joint paper by Pirashvili (Tbilisi) and Waldhausen (Bielefeld).

It is the intention of this proposal to intensify joint research of groups of scientists from Eastern and Western Europe by organizing mutual visits including joint conferences, long term visits (like 1 years stays) of their members at one of the below mentioned laboratories, especially this should be done for young mathematicians like postdocs. Also, this proposal should make it possible to install Computer equipment, especially at the East European labs, insomuch this is required to ease the joint research.

References.


[1] J. Hurrelbrink and U. Rehmann, Splitting patterns of quadratic forms, to appear.

[2] B. Jacob and M. Rost, Degree four cohomological invariants for quadratic forms, *Invent. math.* 96 (1989), 551-570.

[3] I. Kersten and U. Rehmann, Generic splitting of reductive groups, to appear in *Tôhoku Math. J.*

[4] A. S. Merkurjev, Simple algebras and quadratic forms, *Izv. Akad. Nauk SSSR, Ser. Mat.* **55** (1991), 218-224; English trans: *Math. USSR Izvestiya* **38** (1992), 215-221.

[5] A. S. Merkurjev, J.-P. Tignol, The multipliers of similitudes and the Brauer group of homogeneous varieties, preprint 10/1993.

[6] I. Panin, On algebraic K-theory of generalized flag fiber bundles and some of their twisted forms. *Advances in Soviet Math.* **4** (1991), 21-46

[7] A. A. Suslin (Editor), Algebraic K-Theory. *Advances in Soviet Math.* **4** (1991).

[8] Talk of A. Wadsworth at Bielefeld on some joint work with Merkurjev and Panin, Spring 1993.


### 2.1 Existing cooperations with NIS

This year A. Merkurjev (St. Petersburg) is visiting Louvain-la-Neuve. This year A. Vavilov (St. Petersburg), and T. Pirashvili (Tbilisi) are visiting Bielefeld.

Oct. 5 – Nov 5, 1993: V. I. Yanchevskii visits Louvain-la-Neuve.

Oct. 20 – Nov 3, 1993: A. Merkurjev (St. Petersburg) visits Bielefeld.

Dec. 3 – Dec. 16, 1993: B. Kahn (Paris) visits Bielefeld.

Febr. 1 – Febr. 28, 1994: I. Panin (St. Petersburg) visits Bielefeld.

There is an existing research cooperation between Strasbourg (France) and Alma-Ata (Kazakhstan).

The following conferences on the described topics are organized or planned:

June 6–12, 1994: Conference in Lumini (France) on: Formes quadratiques et groupes algébrique linéaires, organized by: Eva Bayer-Fluckiger (Besançon), Jean-Louis Colliot-Thélène (Orsay), Bruno Kahn (Paris), Jean-Pierre Tignol (Louvain-la-Neuve).

July 1995: 1-week Conference in Bielefeld (Germany) on: Linear algebraic groups and related structures, organized by I. Kersten and U. Rehmann.

**3.2 Previous involvement with EC projects**

Waldhausen and I are local coordinators for Germany of the EC-Science project on "K-Théorie et ses applications en géométrie, en arithmétique, en topologie et en analyse".

**4. Scientists and institutes of the cooperation.**

At this stage of the proposal, the list is not considered to be exclusive.

**4.1. NIS States**

**St. Petersburg State University, Russia** (9 participants)

Local coordinator: A. A. Suslin (POMI; algebraic K-theory)

I. Fesenko (K-theory and class field theory), N. Gordeev (Chair of algebra and number theory of St. Petersburg University of Education; invariant theory, linear algebraic groups), O. Izhboldin (K-theory of fields), A. Merkurjev (Chair of general mathematics; algebraic K-theory, quadratic forms, Brauer groups), A. Nenashev (POMI; algebraic K-theory) I. Panin (POMI; algebraic K-theory, linear algebraic groups) A. Vavilov (linear algebraic groups), S. Yagunov (K-theory, postgraduate student of A. Suslin).

**Moscow State University, Russia** (5 participants)

Local coordinator: A. V. Mikhalev (Head of research section of computer algebra; classical groups over rings),

V. A. Artamonov (Serre's conjecture over noncommutative rings), E. S. Golod (commutative algebra) I. Z. Golubchik (automorphisms and subgroups of linear groups), Y. P. Soloviev (topological cyclic homology).

**Institute of Mathematics of Belorussian Acad. Sci., Minsk** (3 participants)

Local coordinator: V. I. Yanchevskii (Head of Dept. and member of Acad. Sci. of Belarus, linear algebraic groups and reduced K-theory),

V. I. Chernousov (arithmetic of algebraic groups), A. S. Rapinchuk (arithmetic of algebraic groups).

**Institute of Mathematics of Acad. Sci. Kazakhstan, Alma-Ata** (1 participant)

A. Dzhumadildaev (Cohomology of modular Lie algebras).

**State University of Tbilisi, Georgia**

Local coordinator: T. Pirashivili (K-theory, Hochschild homology),

H. Inassaridze (K-theory of normed rings), M. Jibladze (topological Hochschild ho-