INTAS 93-2618: Periodic Progress Report
for the period of June 1, 1995, till June 30, 1996

Initial remark: This report covers the first 13 months of the whole project. As June 1996 was a very active month for the project, with 2 essential international conferences (cf. [C2], [C3] below), where several of the results of the project have been reported, it seems to make sense to cover this month also in this report.

1. Title

Linear Algebraic Groups, Algebraic K-Theory, and related Structures in Algebra and Topology

2. Objectives

The research of this project is concentrated on the investigation of the splitting behavior of linear algebraic groups, the investigation of splitting properties of quadratic forms, the investigation of Lie algebras and their cohomology groups and their non-split and central extensions, in particular the computation of those groups, investigation of Azumaya algebras and Brauer groups of local hyper-elliptic curves, investigation of algebraic K-theories for various objects including homogeneous spaces (of algebraic groups) and special normed algebras over normed commutative rings.

3. Research Activities and Results Obtained

3.1 Description of the research activities

The main activities of the project were done by individual research in the above mentioned fields. Several interim results have been reported on the occasion of three international conferences mentioned below, also some individual visits of the NIS participants at various western universities. All fields mentioned in section 2. (Objectives) have been investigated intensively.
On the occasion of 3 international conferences, which were partially financed by this project, several participating individuals reported their results. These conferences were:

[C1] Euro-Conference on "Linear Algebraic Groups and Related Structures", organized by Ina Kersten (Bielefeld), Alexander Merkurjev (St. Petersburg) and Ulf Rehmann (Bielefeld), held at the Sonderforschungsbereich "Diskrete Strukturen in der Mathematik" at the University of Bielefeld (Germany) on July 3 - July 7, 1995.
International conference with 57 participants, sponsored mainly by the DFG (Deutsche Forschungsgemeinschaft), by the European community as a Euro-Conference, and by INTAS 93-2618 (Participation of 5 scientists from the former Soviet Union states was financed totally or in parts by INTAS.)

[C2] Conference on "Algebraic K-Theory", organized by Dan Grayson (Urbana,Illinois) and Ulf Rehmann at the "Mathematisches Forschungsinstitut" Oberwolfach (Germany), held on June 10 - 14, 1996. International conference with 44 participants.

[C3] Euro-Conference on "Linear Algebraic Groups and Related Structures", organized by Jean-Louis Colliot-Thélène (Paris), Eva Bayer-Fluckiger Besançon), Jean-Pierre Tignol (Louvain-la Neuve), Paul Van Praag, held on June 17 - 21, 1996, at Louvain-la-Neuve (Belgium). (Participation of 3 scientists from the former Soviet Union states and of one scientist from Western Europe was financed totally or in parts by INTAS.)

Besides these conferences there are various research activities of the single labs partially in interaction with each other , which are described separately for each participant of the project below.

3.2 Results

The results of the project are or will be documented by at least 38 publications described in detail below in section 4.1.1. Out of these publications, 29 have already been finished and most of them already have appeared in or have been submitted to international scientific journals. 9 publications are mentioned as being in preparation, their topics will be investigated during the last months of the project, and this will be the expected further achievement of the project.

4. Management and Financial Information

4.1 Project management

4.1.1 Co-operation with reference to each contractor

Participant 1 – Bielefeld:

Activities: Organization of conferences [C1], [C2]; Research in linear algebraic groups and related structures, in collaboration with A. Merkurjev (St. Petersburg), who visits Bielefeld as a Humboldt Prize Winner from June 1995 till August 1996.
I. Kersten and U. Rehmann participated in all or some of the conferences C1, C2, C3.
Publications:
[HR-I] J. Hurrelbrink and U. Rehmann, Splitting patterns of quadratic forms, Math. Nachr. (1995), 111-127.
Abstract: General constraints for splitting patterns of quadratic forms are discussed. It is investigated how the splitting of a quadratic form is influenced by the splitting pattern of its Clifford algebra. Criteria for a quadratic form being a "stable Pfister neighbor" are given, an alternative proof of a recent theorem of Hoffmann on the first higher Witt index is given. Also, properties of forms of height 2 are studied in terms of certain cohomological invariants.
[HR-II] J. Hurrelbrink and U. Rehmann, Splitting patterns of trace forms (preprint 1996).

Abstract: The splitting pattern of a quadratic form q over a field k consists of all distinct Witt indices that occur for q over extension fields of k. In small dimensions, the complete list of splitting patterns of quadratic forms is known. It is shown that *all* splitting patterns of quadratic forms of dimension at most nine can be realized by trace forms.

[KR] I. Kersten and U. Rehmann, Excellence properties of semi-simple algebraic groups (preprint 1996).

Abstract: A semi-simple algebraic group over a field k is called excellent, when for every field extension K of k, the anisotropic kernel of G_K is defined over k. This parallels the definition of excellent quadratic forms as given by Knebusch, however, it is not true that a quadratic form is excellent if and only if its orthogonal group is excellent. The excellence of a form implies the excellence of its orthogonal group, but the converse is false. The excellent groups of inner type A_n are determined, in parallel an excellence condition for Azumaya algebras is formulated, and it is proved that exactly those Azumaya algebras are excellent which have square-free index.

[MRY] G. Margolin, U. Rehmann, V. Yanchevskii, Quaternion generation of the 2-torsion part of the Brauer group of a local quintic, in preparation (see also Participant 4).

Abstract: The splitting behavior of the Brauer group for the field extension given by the function field K of a quintic over some local field k is determined by giving the elements of the kernel of the natural map of Brauer groups $\mathrm{Br}(k) \to \mathrm{Br}(K)$.

Participant 2 – Besançon:

Eva Bayer-Fluckiger, Laura Fainsilber, Detlev Hoffmann, Anne Queguiner from Besançon participated at the conferences C1 and/or C3 and contributed by giving talks. The travel of Bayer-Fluckiger from Besançon to Louvain-la-Neuve on occasion of C3 will be paid from funds of this project.

[1] E. Bayer-Fluckiger, J. Morales "Multiples of trace forms in number fields", *Proc. Symp. Pure Math.* **58.2** (1995), 73–81.

Soient K un corps de nombres, G un groupe fini et L une G-algèbre galoisienne sur K. Notons q_L la forme trace de L, et q_0 la forme trace de la G-algèbre déployée. Le but de cet article est de montrer que les G-formes $q_L \oplus q_L$ et $q_0 \oplus q_0$ sont isomorphes si et seulement si le discriminant de toute sous-algèbre quadratique de L est une somme de deux carrés.

[2] E. Bayer-Fluckiger, "Formes quadratiques devenant isotropes sur une extension", *Enseignement Math.*, **41** (1995), 111–122.

Soit k un corps de caractéristique $\neq 2$. Soit q une forme quadratique anisotrope sur k. Si E est une extension de k, on note q_E la forme quadratique obtenue par extension des scalaires à E. On dit que q *devient isotrope* sur E si la forme q_E est isotrope. Soit $k[X_1, ..., X_m]$ l'anneau des polynômes à m variables sur k. Soit $f \in k[X_1, ..., X_m]$ un polynôme irréductible, et notons $k(f)$ le corps des fractions de $k[X_1, ..., X_m]/(f)$. Le but de cette note est de présenter un critère nécessaire et suffisant pour que q devienne isotrope sur $k(f)$, et d'en donner quelques applications. Sous une forme légèrement différente, ce critère avait été obtenu par Witt.

[3] E. Bayer-Fluckiger, R. Parimala, "Galois cohomology of the classical groups over fields of cohomological dimension 2", *Invent. Math.*, **122** (1995), 195–229.

Soient K un corps parfait de caractéristique $\neq 2$, K_s une clôture séparable de K, et $G_K = \mathrm{Gal}(K_s/K)$. Pour tout G_K–module discret C et pour tout m entier ≥ 0, on note $H^m(K,C) = H^m(G_K,C)$. On dit que la *dimension cohomologique* de K est $\leq n$ si pour tout $m > n$ et pour tout G_K–module de torsion C, on a $H^m(K,C) = 0$. On écrit alors $\mathrm{cd}(K) \leq n$.

Soit G un groupe algébrique linéaire défini sur K. On définit le premier ensemble de cohomologie $H^1(K,G) = H^1(G_K, G(K_s))$.

En 1962, Serre a conjecturé que si K est un corps de dimension cohomologique ≤ 2, et si G est un groupe algébrique linéaire semi-simple et simplement connexe défini sur K, alors $H^1(K,G) = 0$. Cette conjecture est connue pour les corps p-adiques (Kneser), et pour les corps de nombres totalement imaginaires (Kneser, Harder, Chernousov).

Dans cet article, nous montrons que la conjecture est vraie pour les groupes classiques, ainsi que pour les groupes de type G_2 et F_4. La démonstration utilise le théorème de Merkurjev–Suslin, des résultats de Yanchevskii, de Merkurjev, de Parimala–Sridharan–Suresh, ainsi qu'un résultat commun avec Lenstra.

[4] E. Bayer-Fluckiger, "Systèmes de formes quadratiques d'algèbre donnée", *C.R. Acad. Sci. Paris* **321** (1995), 1291–1293.

Soit k un corps de caractéristique différente de 2. A tout système de formes quadratiques sur k est associée une algèbre à involution dont le groupe unitaire est le groupe des automorphismes du système. Le but de cette note est de montrer que toute algèbre à involution se réalise comme l'algèbre d'un système de trois formes quadratiques.

[5] E. Bayer-Fluckiger, L. Fainsilber "Non–unimodular hermitian forms", *Invent. Math.* (1996), à paraître.

Soit A un anneau, et soit M un A–module réflexif de type fini. Soit M^* le dual de M. On dit qu'une forme hermitienne $h : M \times M \to A$ est *unimodulaire* si h induit un isomorphisme $M \to M^*$. La théorie de Quebbemann, Scharlau et Schulte apporte de nombreux renseignements sur les formes unimodulaires. Par contre, les formes non unimodulaires ont été très peu étudiées, sauf dans des cas très particuliers. Cet article ramène (essentiellement) la classification des formes non unimodulaires sur A à celle de formes unimodulaires sur un autre anneau.

Une *algèbre p-adique* est une algèbre A de rang fini sur $\mathbf{Z}_p$. On dit que deux formes hermitiennes sont *stablement isomorphes* si elles deviennent isomorphes après addition (somme orthogonale) d'une même forme hermitienne. Un des problèmes de la théorie des formes hermitiennes est de déterminer si deux formes stablement isomorphes sont isomorphes. Si c'est le cas, on dit que le théorème de simplification de Witt vaut pour les formes hermitiennes sur A. Ce théorème n'était connu que dans peu de cas (certaines algèbres commutatives, ainsi que certaines formes hermitiennes sur les ordres héréditaires). Cet article établit le théorème de simplification de Witt en toute généralité. On obtient aussi une généralisation du théorème de Springer aux algèbres p-adiques, ainsi que des applications à la forme trace.

[6] E. Bayer-Fluckiger, R. Parimala, "Hasse principle for the classical groups over fields of virtual cohomological dimension 2", preprint 1995.

On dit qu'un corps k est de *dimension cohomologique virtuelle n* si $\mathrm{cd}(\mathrm{k}(\sqrt{-1}) = \mathrm{n}$. Notons Ω l'ensemble des ordres de k. Pour $v \in \Omega$, on note k_v la clôture réelle de k par rapport à v. Soit G un groupe semi–simple, simplement connexe de type classique, ou de type G_2 ou F_4. Dans cet article, on montre que l'application canonique

$$H^1(k, G) \to \prod_{v \in \Omega} H^1(k_v, G)$$

est injective.

Ceci généralise la conjecture II de Serre ainsi que le "principe de Hasse" classique pour les corps de nombres (du moins lorsque G est de l'un des types ci–dessus).

[7] E. Bayer-Fluckiger, "Lattices with automorphisms of given characteristic polynomial", en préparation.

Rappelons qu'un réseau est appelé f–réseau s'il a une isométrie de polynôme caractéristique f. Comme le groupe des isométries d'un réseau est fini, l'existence d'un f–réseau entraîne que f est un produit de polynômes cyclotomiques. Le but de cette recherche est de donner la liste complète des polynômes f pour lesquels il existe un f–réseau unimodulaire. Plus généralement, les méthodes de l'article permettent de caractériser les formes discriminant des f–réseaux.

Participant 3 – St. Petersburg:

O. Izhboldin, A. Merkurjev, I. Panin, N. Nenashev from St. Petersburg participated in all or some of the conferences C1, C2, C3 and contributed by giving talks. The stay of Panin, Nenashev and Izhboldin at C1 and the stay of Panin and Izhboldin at C3 was supported by this project. They all including S. Yagonov contributed by writing several research papers (see below). A. Merkurjev from St. Petersburg visited Bielefeld for one year (ending in August 1996) as a Humboldt prize winner (upon suggestion by the project co-ordinator of this project) to do research in the area of linear algebraic groups and related structures. He co-organized the conference C1 at Bielefeld, and contributed to conferences C2, C3 by giving talks there.
i) Ivan Panin.
a) The K-theory of algebraic tori and toric varieties were computed. In particular it was shown that $K_0(T)$ for a tori T is a finitely generated Abelian group.
b) Index reduction formulas for inner forms of flag varieties were calculated. More precisely, inner forms of the flag varieties for classical groups were considered.
c) A variant of Gersten's Conjecture was proved for the K-theory of Azumaya algebras over a smooth local ring of geometric type. The corollary of this is the positive solution of Grothedieck's conjecture on principal homogeneous spaces for the special linear group $SL_{1,D}$.

Publications:
[A] A. S. Merkurjev and I. A. Panin, *K-Theory of algebraic tori and toric varieties*, to appear in K-Theory, 1997.
[B] A. S. Merkurjev, I. A. Panin, and A. R. Wadsworth, *Index reduction formulas for inner forms of flag varieties, I.* to appear in K-Theory, 1996.
[C] I. A. Panin and A. A. Suslin, *On a Grothendieck's Conjecture for Azumaya algebras.* Preprint POMI, 1996, submitted to St.-Petersburg Math. J.

ii) Oleg Izhboldin.
a) The cohomology groups $H^n_p(F((t)))$ and $H^n_p(F(t))$ were computed for any field F of characteristic $p > 0$, in terms of invariants of the field F. In particular the following theorem is proved: The homomorphism $H^n(F(t)) \to \coprod_v H^n(\hat{F}(t)_v)$ is injective, where v runs over all discrete valuations of $F(t)$, and $\hat{F}(t)_v$ denotes the corresponding completion of the field $F(t)$.
b) For any $n \geq 3$, a field F and an n-fold Pfister form ϕ were constructed such that the field extension $F(\phi)/F$ is not excellent. Moreover for any field and quadratic form ϕ over F the following theorem is proved: The field extension $F(\phi)/F$ is universally excellent if and only if ϕ is a Pfister neighbor of dimension ≤ 4.
c) Quadratic forms ϕ with maximal splitting were investigated under some condition on the dimension of ϕ. In particular we find a simple proof of the following fact: any quadratic form with maximal splitting of dimension $2^n - 5$ is necessarily a Pfister neighbor.

Publications:
[A] *On the cohomology groups for the field of rational function*, Amer. Math. Soc. Transl. (2) Vol. **174**, 1996, 21–44.
[B] *On the non-excellence of field extension $F(\pi)/F$*, Doc. Math. J. DMV, **1**, 1996, pp. 127–136.
[C] *On the quadratic forms with maximal splitting*, to appear in St.-Petersburg Math. J., 1996.

iii) Alexander Nenashev.
a) We prove that for an arbitrary exact category A, every element of $K_1(A)$ can be represented by algebraic data called a double short exact sequence.
b) We establish relations between the images of double short exact sequences in $K_1(A)$.
c) We prove that the relations established in (b) provide a complete system of defining relations for $K_1(A)$. Thus we obtain a presentation of K_1 by generators and relations in terms of the underlying exact category.

Publications:
[A] *Double short exact sequences produce all elements of Quillen's K_1*, Preprint MPI 95-113; to appear in Proc. of Poznan K-theory Conf., in Contemp. Math. Proc. Series, AMS.

[B] *Double short exact sequences and K_1 of an exact category*, Preprint MPI 95-118; submitted to K-Theory.
[C] *Double short exact sequences* III, in preparation.

iv) Serge Yagunov.
We completely calculate the homology of bi-Grassmannian complexes with rational coefficients. As an immediate application we obtain a rational case of van der Kallen's stabilization theorem.

Publications:
[A] *Homology of bi-Grassmannian complexes*, Preprint 1996 (K-theory Preprint archive http://math.uiuc.edu/K-theory/0101), submitted to K-theory.

Participant 4 – Minsk:

A. Chernousov and V. Yanchevskii from Minsk participate in this project. Chernousov is visiting Bielefeld right now for several months, supported by Alexander v. Humboldt foundation, and working in the area of this project. Chernousov was supported by this project in order to attend a conference in Barcelona. Yanchevskii has visited Bielefeld several times, supported by the Bielefeld Sonderforschungsbereich 343. He attended the conferences C1 and C3 under support by this project.

Results:

1. By means of the Tate algorithm the complete list of minimal Weierstrass equations for all non-dyadic local elliptic curves is found.
2. The complete description of the torsion part of the group of rational points (as an abstract Abelian group) for any non-dyadic local elliptic curve is obtained.
3. For the 2-torsion part of the Brauer group of any non-dyadic local elliptic curve a representation of all its elements by unramified quaternion division algebras is found.
4. For a non-dyadic local hyperelliptic curves the problem of the description of the m-torsion part of its Brauer group (as an abstract Abelian group) was reduced to the problem of the computation of m-torsion of the group of rational points of its Jacobian.
6. The abstract Abelian group structure of the 2-torsion part of the Brauer groups of non-dyadic local hyperelliptic curves is described.
7. The representation of the elements of the above 2-torsion part by means of unramified quaternion algebras for non-dyadic local hyperelliptic curves with good reduction was found.
Publications:
1. V. Yantchevskii, G. Margolin, Brauer groups of local elliptic and hyperelliptic curves and central division algebras over their function fields
Preprint 95 - 42 Universität Bielefeld
2. V. Yanchevskii, G. Margolin. Brauer groups of local hyperelliptic curves with good reduction. Algebra and analysis v. 7.(1995)N6p 227-249 (in Russian).

3. G. Margolin, U. Rehmann, V. Yanchevskii, Quaternion generation of the 2-torsion part of the Brauer group of a local quintic, in preparation (see also Participant 1).

Participant 5 – Alma Ata:

Participants are A. Dzhumadil'daev, K. Abdukhalikov, R. Kerimbaev. They contributed to the project by the below mentioned publications. Dzhumadil'daev has attended the conference C1 (supported by this project) and has visited Bielefeld in June/July 1996. For this last visit, his travel will be supported by this project, while his subsistence was supported by the Sonderforschungsbreich "Diskrete Strukturen in der Mathematik" at Bielefeld.

Results:

According to our research plan we should calculate the first and second cohomologies of Cartan Type Lie algebras with coefficients in irreducible modules. All other things like nonsplit extensions, deformations and defining relations will follow from this result. Physical applications need some final numerical results. It is useful to prove not only the finite-dimensionality of the cohomologies, but also to find their dimensions. It is interesting not only to show the existence of nonsplit extensions, but also to find exact formulas for cocycles and decompositions of cohomology groups over classical subalgebras.
We proved the (almost) equivalence of the following three things:
i)nonsplit extensions of Cartan Type Lie algebras (Virasoro Type Lie algebras);
ii)defining relations of maximal subalgebras of Cartan Type Lie algebras;
iii)invariant operations on $\wedge^k(\mathcal{L}_1) \to \mathbf{C}$
We prove that Cartan Type Lie algebras have a finite number of irreducible tensor modules with nonzero second cohomologies. For example, the Witt algebra $L = W_n = Der\mathbf{C}[[x_1, \ldots, n]]$ has exactly $5(n = 1), 8(n = 2, 3), 9(n > 3)$ tensor modules with nonsplit extensions. In particularly, nontrivial central extensions appear only in the case $n = 1$. The corresponding central extension is well known as a Virasoro algebra. We find new local deformations in addition to the Moyal bracket for the two-sided Hamiltonian Algebra. We also find new local deformations for Lie algebra without divergence in three variables. Our results show that $H^2(\mathcal{L}_1, \mathbf{C})$ is very large, for example $dim H^2(\mathcal{L}_1, \mathbf{C})$ is equivalent to $n^6/8$, for $L = W_n, S_n$ and $8n^6/9$, for $L = H_n, K_{n+1}$.
These results allows to introduce a new cohomology theory for Lie algebras. Usually, the cohomologies are considered in the exterior powers of Lie algebras (Chevalley-Eilenberg). J.-P.Loday defines cohomologies in tensor powers (Leibniz cohomologies). We define cohomologies in mixed powers (two arguments are symmetric and others are skew-symmetric) and find for them an exact cohomological sequence connecting Chevalley-Eilenberg cohomologies, Leibniz cohomologies and our cohomologies. It will be very interesting to study category-functorial properties of new cohomologies and make applications in structural theory of non classical Lie algebras. But they are subject of future research.
The main difficulty of the calculation of $H^2(L, U \otimes M_0)$ concerns the calculation of the second cohomology of a nilpotent subalgebra $\mathcal{L}_1$ with coefficients in the trivial module $H^2(\mathcal{L}_1, \mathbf{C})$. Among all of the four series of Cartan Type Lie algebras $L = W_n, S_n, H_n, K_{n+1}$

the cohomology algebra $H^*(\mathcal{L}_1, \mathbf{C})$ is completely calculated only in the case $L = W_n, n = 1$ (L.Goncharova, I.M.Gelfand, D.B.Fuks, D.Feigin, etc). Until now there are known at least eight proofs of this result, but it still remains as one of the difficult results of cohomological theory of Lie algebras. We calculate the cohomology of nilpotent subalgebras for any L and any n, but make a restriction for the degree of cohomology: $k = 1, 2$. Our approach is based on studying the L_0-module structure of $H^*(\mathcal{L}_1, \mathbf{C})$. More exactly we study the decomposition of tensor products and exterior products of L_0-modules $L_1 \wedge L_s$. Theoretically this decomposition should follow from Weyl, Freudental and Kostant theorems, but practically it is not so easy to find this decomposition. Luckily it turned out that in the case of $L = H_n$ all L_k are irreducible L_0-modules. This allows to simplify the structure of $L_1 \otimes L_s$ and with the help of computer computations we find the number of irreducible components, highest weights and conditions for highest weights that gives nontrivial cocycles. The last question whether a given highest weight is a trivial cocycle or not needs complicated calculations for the other three series $L = W_n, S_n, K_n$. For example, L_0-module L_k for $L = W_n$ is decomposed into the sum of two irreducible submodules and the question about $H^2(\mathcal{L}_1(W_n), \mathbf{C})$ can be reduced to the question about $H^2(\mathcal{L}_1((S_n), \mathbf{C})$. In the case $L = S_n$ $L_1 \otimes L_s$ decomposes into the sum of 11 irreducible components. At now we finished considering 8 components. Unfortunately, in the case of $L = K_n$ the homogeneous components L_s are not irreducible over L_0 and the number of irreducible components grows with the growing of k. That is why the structure of $L_1 \otimes L_k$ as an L_0-module is more complicated. For finishing our research plan in case of characteristic 0 we must:
1) find the decomposition of $L_1 \otimes L_k$;
2) find height weights of irreducible components;
3) among height weights find such weights that correspond to nontrivial cocycles.
Unfortunately in third step there is no canonical method of finding trivial and nontrivial cocycles. One should guess the exact formulae for the cocycle ψ with given highest weight λ, then one should guess ω such that $\psi = d\omega$, if ψ is trivial cocycle or one should prove nonexistence of such ω . We find these cochains ψ and ω with the help of computer calculations.
These results were obtained by A. Dzhumadil'daev and his students R. Kerimbaev, S. Adbukasimova, with assistance of K. Abduchalikov. Partially these results are published in [1-4].
For completely considering the cases of $L = W_n, S_n, K_n$ we need at least a good computer and two more years.

Main seminar talks

1. Joint seminar of prof. B.Pareigis-J.Wess (Munich University), "Cohomologies of Lie algebras"
2. Seminar in Algebraic Geometry of prof. Yu.I.Manin (Max-Plank Institute,Bonn), "Virasoro Type Lie algebras: cohomologies and defining relations"
3. Conference "Algebraic groups" (U.Rehmann, Bielefeld), "Virasoro Type Lie algebras"
4. Algebraic Seminar of K. Roggenkamp (Stuttgart University), " Cohomologies and representations of modular Lie algebras"

Publications

1. A.S. Dzhumadil'daev. Virasoro Type Lie algebras and deformations, to appear in: Zeitschrift für Physik, Ser. C, September 1996
2. A.S. Dzhumadil'daev, R.K. Kerimbaev. Defining relations of nilpotent subalgebras of contact Lie algebra, Inst. Applied and Theoretical Math., 1994, Dep. KazGosINTI 03.05.94, No.4851-Ka94
3. R.K. Kerimbaev, Second cohomology of nilpotent subalgebras of Hamiltonian algebra, Alma-Ata, 1994, Dep. KaGosINTI, 16.06.94, No.5089-Ka94
4. A.S. Dzhumadil'daev, R.K. Kerimbaev. Irreducible components of the group $H^2(\mathcal{L}_1)$ of general and special Lie algebras, 1994, KazGosINTI, 29.07.94, No.5194-Ka94.

Participant 6 – Tiblisi:

Participating researchers: H. Inassaridze, M. Jibladze, T. Kandelaki, D. Pataria, T. Pirashvili, M. Uridia. They contributed to the project as described below. T. Pirashvili visited the Sonderforschungsbereich Bielefeld for several months, his travel expenses were covered by this project. D. Jibladze was invited by Max-Planck-Institute Bonn from Oct. 1, 1995 till Dec. 31, 1995. His travel expenses were also covered by this project. From there he attended a conference and several Institutes in Western Europe: The Peripatetic Seminar on Sheaves and Logic at the University of Edinburgh, Oct 7-8, 1995, were he gave a talk. He also was invited to visit the Mathematics departments at Utrecht and at Genoa during that time, some of his expenses were covered by this project.

Results:

In [1], the final positive answer to the isomorphism problem for commutative monoid rings has been obtained, namely, it has been shown that a finitely generated submonoid M$\subset\mathbf{Z}^r$ is determined up to isomorphism by the corresponding monoid ring R[M], for any commutative ring R.
In [2], Quillen's algebraic K-theory of discrete rings is extended to the category of normed algebras over a commutative Banach ring with unit and its relationship with topological K-theory is established.
In [3] it is proved that Kasparov's bivariant K-functors form a right universal sequence of functors.
In [4], a cohomology theory is defined for monoids in a monoidal category, which in relevant particular cases covers cohomology of small categories, Hochschild cohomology of algebras, etc. A particularly interesting new case is the cohomology of operads. In a sequel to this paper, a trace map from the non-linear K-theory to this operad homology will be constructed.
In [5], those 2-categories whose nerves admit a cyclic structure are characterized.
In [6] certain $\mathbf{Z}_2$-graded analogs of Cuntz C*-algebras are constructed, which give homotopical interpretation of KK-groups. This algebra is proved to have the same KK-groups as the algebra of 2×2-matrices.
In [7], the notion of the internal category in a left exact category is extended to left exact co-simplicial categories. Assigning to a topos a certain left exact co-simplicial category, the category of internal categories in it is used to define the fundamental group by means of

"discrete" categories over the terminal object. When the topos is the category of sheaves on the locally compact and locally simply connected space, then it's fundamental group is the same as the fundamental group of the underlying space.
In [8], the ring structure of the Mac Lane cohomology of the ring of integers is calculated. The additive structure was already known by Bökstedt. However calculations in [8] do not use Bökstedt's results, so it is somewhat an algebraic counterpart of his sophisticated topological methods.
In [9] it has been shown that the category of Leibniz representations of a classical semisimple Lie algebra has global dimension 2. In case of sl_2 the classification of all indecomposable Leibniz representations is given.
In [10], among other results, it will be proved that algebraic K-functors have the following property: if R is a simplicial ring of finite length (this means that the corresponding Moore complex has a finite number of nontrivial components), then the simplicial group $K_n(R)$ obtained by degree-wise evaluation has also finite length, for any $n \in \mathbf{Z}$.
Publications:
1. J. Gubeladze, The isomorphism problem for commutative monoid rings, submitted to J. Pure & Appl. Algebra
2. H. Inassaridze, Algebraic K-theory of normed algebras, submitted to K-theory, 1996.
3. H. Inassaridze, Universal property of Kasparov bivariant K-theory, to appear, 1996.
4. M. Jibladze (with H.-J. Baues and A. Tonks), Cohomology of monoids in monoidal categories, preprint.
5. M. Jibladze, Cyclic structures on nerves of 2-categories, in preparation.
6. T. Kandelaki, On the universal C*-algebra generated by partial isometry, to appear.
7. D. Pataraia, Internal categories in the left exact cosimplicial category, to appear in Georgian Mathematical Journal.
8. T. Pirashvili (with V. Franjou), On Mac Lane cohomology for the ring of integers, submitted in Topology.
9. T. Pirashvili (with J.-L. Loday), Leibniz representation of Lie algebras, J. Algebra, 1996.
10. T. Pirashvili, Simplicial degree of functors, in preparation.

4.1.2 Planning

The time planning was essentially in accordance with which was indicated in the INTAS cooperation agreement.

4.1.3 Problems encountered and changes required to the co-operation agreement

No serious problems were encountered. The only changes to the cooperation agreement are that no participant so far wanted to use money for computer equipment, as it was more desirable to use the money in order to attend conferences or visit colleagues.

4.1.4 Use of equipment

– None –

4.2 Financial Information

4.2.1 Cost breakdown for the period covered by this report

Labs:	travel, subsist.	salaries	comput. equipm.	overhead	total
1. Bielefeld (U. Rehmann)	—			—	—
2. Besançon (E. Bayer)	—			—	—
3. Pomi St. Petersburg (A. Suslin)	3235	2862			6097
4. Minsk (V.I. Yantchevskii)	1787				1787
5. Alma-Ata (A. Dzhumadeldaev)	733				733
6. Tiblisi (T. Pirashvili)	2244	748		332	3324
Total					11941

Hence approximately 60 % of the total amount of 20000 ECU has been employed.

The funds were transferred to the NIS participants in cash on occasion of several visits of them at Bielefeld or on occasion of one of the above mentioned conferences C1, C2, C3, or by transactions to western bank accounts which some of them have. Funds for the western participants were transferred by ordinary bank transfer. No problems were encountered here.

Project finances other than the INTAS grant: Some visits of NIS participants at Bielefeld were supported by the Sonderforschungsbereich (SFB 343) "Diskrete Strukturen in der Mathematik", sponsored by the Deutsche Forschungsgemeinschaft (Subproject "Linear algebraic groups") and also by some Euro-Conference Project. For example, the funding for the conference C1 was done as follows: ca. 20000 DM was paid by SFB 343, ca. 15650 ECU was contributed by the Euro-Conference Project, and INTAS contributed as described above. The conference C2 was sponsored by the Oberwolfach Institute, and the conference C3 was sponsored, among others, by resources of the Belgium state, by the Euro-Conference Project mentioned above, and by INTAS. Also the German Alexander v. Humboldt foundation financed a one year research stay of A. Merkurjev (St. Petersburg) at Bielefeld (ending in August 1996) on the basis of a Humboldt prize for him (upon suggestion of the project coordinator of this INTAS project, U. Rehmann), and also a one years stay of A. Chernousov (Minsk) at Bielefeld as a Humboldt fellow, starting in June 1996.

4.2.2 Cost breakdown for the period covered by this report

Labs:	travel, subsist.	salaries	comput. equipm.	overhead	total
1. Bielefeld (U. Rehmann)	800			200	1000
2. Besançon (E. Bayer-Fluckiger)	800			200	1000
3. Pomi St. Petersburg (A. Suslin)	505	318			823
4. Minsk (V.I. Yantchevskii)	-47		1030		983
5. Alma-Ata (A. Dzhumadeldaev)	513	624	900		2037
6. Tiblisi (T. Pirashvili)	1496	498		222	2216
Total					8059

Hence approximately 40 % of the total remains to be employed.

5. Summary

Linear Algebraic Groups, Algebraic K-Theory, and related Structures in Algebra and Topology

Periodic Project Report on INTAS-93-2618, June 1, 1995 – June 30, 1996

Project Co-ordinator: Ulf Rehmann, University of Bielefeld

This project deals with the investigation of the splitting behavior of linear algebraic groups over arbitrary fields, the investigation of splitting properties of quadratic forms, the investigation of Lie algebras and their cohomology groups and their nonsplit and central extensions, in particular the computation of those groups, investigation of Azumaya algebras and Brauer groups of local hyperelliptic curves, investigation of algebraic K-theories for various objects including homogeneous spaces (of algebraic groups) and special normed algebras over normed commutative rings.

The main activities of the project were done by individual research in the above mentioned fields. Several interim results have been reported on the occasion of three international conferences mentioned below, also some individual visits of the NIS participants at various western universities. All fields mentioned in section 2. (Objectives) have been investigated intensively.

On the occasion of 3 international conferences, which were partially financed by this project, several participating individuals reported their results. These conferences were:

[C1] Euro-Conference on "Linear Algebraic Groups and Related Structures", organized by Ina Kersten (Bielefeld), Alexander Merkurjev (St. Petersburg) and Ulf Rehmann (Bielefeld), held at the Sonderforschungsbereich "Diskrete Strukturen in der Mathematik" at the University of Bielefeld (Germany) on July 3 - July 7, 1995.

[C2] Conference on "Algebraic K-Theory", organized by Dan Grayson (Urbana,Illinois) and Ulf Rehmann, held at the "Mathematisches Forschungsinstitut" Oberwolfach (Germany) on June 10 - 14, 1996.

[C3] Euro-Conference on "Linear Algebraic Groups and Related Structures", organized by Jean-Louis Colliot-Thélène (Paris), Eva Bayer-Fluckiger Besançon), Jean-Pierre Tignol (Louvain-la Neuve), Paul Van Praag, held at Louvain-la-Neuve (Belgium) on June 17 - 21, 1966.

Besides these conferences there are various research activities of the single labs partially in interaction with each other , which are described separately for each participant of the project below. The results of the project are or will be documented by 38 publications.