Universität Bielefeld
Fakultät für Mathematik

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Workplan for INTAS Project INTAS-93-2618

Title:

Linear Algebraic Groups, Algebraic K-Theory, and related Structures in Algebra and Topology

Objectives:

The theory of linear algebraic groups governs a major number of algebraic disciplines, such as modules over rings, quadratic forms, Azumaya algebras, Lie algebras, classical groups and related structures. While the historical roots of this theory go back into the last century (at least), more recently, beginning in the sixties, the discipline of K-theory has been developed to study those and more general structures as well. This branch soon turned out to develop very powerful methods for various branches of mathematics, especially in algebra and topology.

Ever since, these two fields have influenced each other in a very fruitful way. I mention the investigations for the congruence subgroup problem which solved difficult questions concerning the subgroup structure of certain classical and more general linear groups by studying questions which are typical for algebraic K-theory like stability questions. Complicated questions about presentations of so called "arithmetic" subgroups of classical and "exceptional" groups over number fields were answered with methods developed in K-theory. The determination of various (co-)homology groups of linear groups were transformed into questions about certain 'K-groups'.

Recent research has uncovered methods which are likely to stimulate further far ranging investigations of these areas. In general I refer to the volume [7] titled "Algebraic K-theory" and edited by A. A. Suslin, which contains important results of mathematicians from (nowadays) St. Petersburg going in this direction and concerning most of the structures mentioned above. I especially mention the results of Panin [6] on the K-theory of projective homogeneous varieties which have been used to prove the famous index reduction formulas of Schofield/v.d. Bergh for division algebras, which in turn has recently been brought to a very general form by Merkurjev, Panin and Wadsworth [8]. On the other hand, the importance of these varieties for the splitting behaviour of reductive groups has been investigated by Kersten/Rehmann [3] and it again turns out that those two fields have mutual influence on each other as well as on Azumaya algebras and on quadratic forms (cf. [2, 1, 5]).

Further results including the theory of Brauer groups, algebraic geometry and related areas has been done by V. Iantchevskii (Minsk), and others.

It is vital that algebraic K-theory is not just considered as an isolated theory within algebra. To the contrary, it is quite interdisciplinary within mathematics. Here are some prominent aspects of this feature. First, the construction of higher algebraic K-theory, due to Quillen, uses the elaborate machinery of algebric topology in an essential way; in fact, the very definition of the higher K-groups is in terms of homotopy groups in the sense of algebraic topology—no other definition of the higher K-groups is known to this date. Second, algebraic K-theory has found applications to the topology of manifolds and their automorphisms; keywords are: pseudoisotopy theory, and the algebraic K-theory of topological spaces of Waldhausen. Third, the latter theory necessitated a technical development which has repercussions to algebraic K-theory proper; specifically it has enabled Thomason to develop higher K-theory for schemes with far reaching applications in algebraic geometry. Fourth, but probably not finally, the theory of the trace map which into higher algebraic K-theory was introduced by Dennis has found a substantial extension to a theory involving the so-called topological Hochschild homology theory. A variant of this, the topological cyclic homology, has recently permitted the solution, by Boekstedt, Hsiang, Madsen, of the K-theory analog of the Novikov conjecture. The topic of topological Hochschild homology has, by the way, already been the subject of a joint project across the former iron curtain; the result of the project is reported on in a joint paper by Pirashvili (Tbilisi) and Waldhausen (Bielefeld).

It is the intention of this project to intensify joint research of groups of scientists from Eastern and Western Europe by organizing mutual visits including joint conferences, short term visits (for example one months stays) of their members at one of the below mentioned laboratories, especially this should be done for young mathematicians like postdocs. Also, this proposal should make it possible to install Computer equipment, especially at the East European labs, insomuch as this is required to aid the joint research.

Description and time schedule

The project should be run as a networking type project, that is, it should allow to share common scientific goals of the participants and closely co-ordinate their activities.

It seems reasonable to give the scientists involved the opportunity of mutual short term visits in order to perform joint research.

Also, for some sites, there is some need to get computer equipment in order to do research for the project.

There are two conferences planned for 1995 which fit into the context of this project:

1. Conference on “Linear algebraic groups and related structures”, organized by A. Merkurjev, St. Petersburg, I. Kersten and U. Rehmann, at Bielefeld, July 2-8, 1995. This conference is sponsored by different sources:

A) EC Program Human Capital and Mobility Prop. Nr. ERB4050PL934139 (This is part of an approved Euroconference Series under the title “Formes quadratiques, algebre a Division et groupes algebriques lineaires- Relations avec la geometrie algebrique et la K-Theorie” coordinated by Prof. Eva Bayer, Besançon, France)

B) The approved research project “Linear algebraic groups” organized by H. Abels and U. Rehmann at the Sonderforschungsbereich “Diskrete Strukturen in der Mathematik” at the University of Bielefeld.

C) While A) allows to invite young mathematicians from the (Western European) EC states and B) will pay for some invited speakers, this project should be used to pay for the participation of scientists from the NIS labs of this project.

2. Conference on “K-Theory and homotopy theory” at Oberwolfach, Germany, Nov. 5-11, 1995, organized by T. Goodwillie and F. Waldhausen

The budget is distributed as follows:

Western Labs:	
1. Bielefeld (U. Rehmann)	1000
2. Besançon (E. Bayer)	1000
Eastern Labs:	
3. Pomi St. Petersburg (A. Suslin)	6920
4. Minsk (V.I. Yantchevskii)	2770
5. Alma-Ata (A. Dzhumadeldaev)	2770
6. Tiblisi (T. Pirashvili)	5540
Total	20000

More precisely, the project shall run for 18 months and the following activities should be done by support of this project. All the visits and conferences will take place in this time period, all inaugurated salaries of NIS scientist will be paid during this time period as well. It is also expected that it will be possible to buy the necessary computer equipment in this time period.

1. Lab Bielefeld (Germany, U. Rehmann, 7 participants, Budget 1000). There is an overhead of maximal 20 % = 200 ECU which will be used for administration. It is intended to keep this as low as possible. The rest of this money, at least 800 ECU will be used for visits of one of the Western scientists at one or some of the NIS labs in order to perform joint research, or alternatively for one of the purposes descibed below (under 3.-6.), if it turns out that there is some additional demand.

2. Lab Besançon (France, E. Bayer, Budget 1000). There is an overhead of maximal 20 % = 200 ECU which will be used for administration. It is intended to keep this as low as possible. The rest of this money, at least 800 ECU will be used for visits of one of the Western scientists at one or some of the NIS labs in order to perform joitn research, or alternatively for one of the purposes descibed below (under 3.-6.), if it turns out that there is some additional demand.

3. Lab St. Petersburg (Russia) 9 participants:

Local coordinator: A. A. Suslin (POMI; algebraic K-theory) I. Fesenko (K-theory and class field theory), N. Gordeev (Chair of algebra and number theory of St. Petersburg University of Education; invariant theory, linear algebraic groups), O. Izhboldin (K-theory of fields), A. Merkurjev (Chair of general mathematics; algebraic K-theory, quadratic forms, Brauer groups), A. Nenashev (POMI; algebraic K-theory) I. Panin (POMI; algebraic K-theory, linear algebraic groups) A. Vavilov (linear algebraic groups), S. Yagunov (K-theory, postgraduate student of A. Suslin).

(Budget: 6920 ECU) A salary shall be paid to 4 persons. Total of salaries: 3180 ECU. 9 persons should have the possibility of a 1 weeks visit at one of the western labs or one of the conferences mentioned above (each visit 415 ECU). Alternatively, fewer persons could be visiting for some longer time. A. Merkurjev shall travel from St. Petersburg to Bielefeld for 10 days in July 1995 in order to organize the joint conference with I. Kersten and U. Rehmann on "Linear algebraic groups and related structures",

4. Lab Minsk (Belorussia), 3 participants:

Local coordinator: V. I. Yanchevskii (Head of Dept. and member of Acad. Sci. of Belarus, linear algebraic groups and reduced K-theory), V. I. Chernousov (arithmetic of algebraic groups), A. S. Rapinchuk (arithmetic of algebraic groups).

(Budget: 2770 ECU) V. Iantchevski will visit Bielefeld in Spring 1995 for 1 month in order to perform joint research with U. Rehmann. Total amount: 1740 ECU. An amount of 1030 ECU will be spent by the Minsk group on Computer equipment.

5. Lab Alma-Ata (Kazakhstan), 3 participants:

Local coordinator: A. Dzumadeldaev (Lie algebras and algebraic groups), K. Abdukhalikov (group theory, linear groups and Lie algebras), Rashid Kerimbaev (defining relations and cohomologies of Lie algebras)

(Budget 2770): A salary shall be paid to 2 persons. Total of salaries: 624 ECU. For travel and subsistance 1246 ECU is required (possibly one 10-14 days stay of A. Dzumadeldaev at Strassbourg). An amount of 900 ECU will be spent by the Alma-Ata group on Computer equipment.

6. Lab Tiblisi (Georgia), 6 participants.

Local coordinator: T. Pirashivili (K-theory, Hochschild homology), H. Inassaridze (K-theory of normed rings), M. Jibladze (topological Hochschild homology), T. Kandelaki (K-theory of C^*-algebras, and Banach algebras), D. Pataraia (simplicial descriptions of plus-construction), M. Uridia (Unitary K-theory for exact categories).

(Budget: 5540): A salary shall be paid to 6 persons. Total of salaries: 1246 ECU. For travel and subsistance 3740 ECU is required 1 visit (Pirashvili) of conference 2. (623 ECU), 5 short term (623 ECU each) visits at Strassbourg and at Bielefeld to do research. Alternatively, fewer persons could be visiting for some longer time.

Final remark

In the original proposal, the following Western participants were involved in the whole project. It is understood that if any of the NIS participants wants to use the travel and subsistance money of this project in order to perform joint research in the area of this project with one of these scientists that he should do so, provided the project coordinator and the respective local coordinator both agree to this.

Germany (7 participants) U. Rehmann (Bielefeld; linear algebraic groups, algebraic K-theory), H. Abels (Bielefeld; linear algebraic groups, transformation groups), I. Kersten (Bielefeld; linear algebraic groups, Brauer groups, Azumaya algebras), M. Rost (Bonn; quadratic forms); R. Schwänzl (Osnabrück; topology, K-theory), R. Vogt (topology, K-theory), F. Waldhausen (Bielefeld; topology, K-theory).

Belgium (1 participant) J.-P. Tignol (Louvain-la-Neuve; linear algebraic groups, Azumaya algebras, quadratic forms).

Denmark (4 participants) H. H. Andersen (linear algebraic groups), M. Bökstedt (algebraic K-theory), J. Dupont. I. Madsen (Aarhus),

France (7 participants) E. Bayer (Besançon; linear algebraic groups, quadratic forms). J.-L. Colliot-Thélène (University of Paris Sud, Orsay; algebraic geometry, algebraic K-theory), B. Kahn (Paris VII; algebraic K-theory, quadratic forms), M. Karoubi (Paris VII; K-theory, topology). C. Kassel (Strasbourg; Lie algebras, K-theory). J.-L. Loday (Strasbourg; cyclic homology, Lie algebras, K-theory), R. Thomason (Paris VII; algebraic K-theory, algebraic geometry),

Italy (4 participants) L. Barbieri-Viale (Genova; Applications of algebraic K-theory to algebraic geometry, motivic cohomology), C. Pedrini (Genoa; Applications of algebraic K-theory to intersection theory on singular varieties), L. Robbiano (Genova; Commutative algebra and applications to computer algebra), A. Vistoli (Bologna; Algebraic groups, intersection theory in algebraic geometry).

The Netherlands (1 participant) W. v. d. Kallen (Utrecht; K-theory).

United Kingdom (2 participants) John Jones (Warwick; K-theory). Andrew Ranicki (Edinburgh; L-theory, applications of K-theory to topology)