Periodic Progress Report 1.5.2000 – 31.12.2001

Linear Algebraic Groups and Related Linear and Homological Structures

1 RESEARCH

1.1 Overview of Research Activities/ Conformance with the Work Programme

Remark: The project cooperates with the EU funded TMR network ERB FMRX CT-97-0107 (Title: Algebraic K-Theory, Linear Algebraic Groups and Related Structures), which includes all the teams of this project except the Almaty team. Therefore, there is additional funding for travel and subsistence form there for this project. Also, until Dec. 2000, Bielefeld had substantial national funding, which allowed support of the research of this project as well (short and long term visits of NIS scientist at Bielefeld). Therefore, until now, almost no travel and subsistence money was used from the funds of this project. The nodes Minsk, Tbilisi and St. Petersburg had 7500 Euro funding for these purposes from the TMR network. Also, there was funding by the German Humboldt Foundation: Oleg Izhboldin from St. Petersburg had a Humboldt fellowship for a stay in Bielefeld until his untimely and tragic death in May 2001.
The TMR network will end on June 30, 2002. It is expected to use the funds of this project more intensively after that.

- Work carried out by each of the Contractors
- Has the research been in accordance with the Work Programme? If not, in what respect and why?
- Do you foresee any deviations from the Work Programme for the future? If yes, which ones and why?

1.2 Scientific Results

Team 1: Progress in tasks T1.1, T1.2 was obtained in [IK]: A complete classification of all excellent special orthogonal groups over fields of characteristic 0 was given (cooperation with team 7). Concerning T1.2, some unforeseen cooperation with St. Petersburg was possible and yielded results on commutator properties of simple algebraic groups [GR], while [PA-RE] is still in preparation.

In cooperation with Team 5, progress in T1.6 was obtained in [CG]. For task T1.4, see under team 6.

Team 2: Significant progress on task T2.1 was published in [HI], [H1], [H2]. Concerning tasks T2.2 and T2.3 it needs to be said that Eva Bayer-Fluckiger moved from Besançon to Lausanne. But V. Chernousov (Minsk) has now a visiting position there, and the cooperation still continues. Still, Besançon remains an important place for important research for this project, see their report below.

Team 3: Task T3.1, T3.2: To determine the index of hyperelliptic curves over local fields the curves are divided in two classes (according to the form of the affine equations $Y^2 = g(X)$). The team completed the index of hyperelliptic curves over local fields in the tamely ramified case to one class of equations. A manuscript (Van Geel, Yanchevskii, The index of certain hyperelliptic curves over p-adic fields, preprint) is under preparation. Related to task 3.3 and 3.4 the team started with the investigation of a new problem namely the structure of Ω-division algebras over powerseries fields over real closed fields. (All this in collaboration with team 5 - Minsk).

Team 4: Task T4.1: Significant progress published in [AD]. Task T4.3: Progress will be published in [DI], cf. [I]. Also, [K] has to be mentioned here. For task T4.2 it needs to be said the the person in charge left the institute. However, other collaborators have been found (see the report below).

Team 5: Significant progress in Task T5.A1, T5A2, T5.D1, T5.D2 is published in [RTY]. Concerning tasks T5.B [VGY] and [KU-RO-TI-YA] are in preparation. Additional results in algebraic geometry were obtained [GP], [GU].

Team 6: The essential results concerning tasks T6.1a, T6.2a are published in [BFJS], [G2], [BG1], [BG2], [BJ3]. Progress with respect to task T6.3 was published in [BJG] Significant progress in task T6.4(a) was published in [P3], [PR].

Team 7: Essential results concerning task T7.3.1 are published in [IZ]. The task T8.1 was partially solved in [GR]. For T7.5 the manuscript [Z1] presents some results, and T7.6 is discussed in [PY]. For all other tasks, research has been done and publications are in preparation.

The research is essentially in accordance with the Work Programme. No essential deviations of the work programme have to be made.

Publications

Accepted items are marked by (a), submitted items are marked by (s), preprints are marked by (p), everything else is published:

- Joint Publications of INTAS and NIS project teams

[CG]	Teams 1+5, invited publication in proceedings
[GR]	Teams 1+7, int. journal
[IK]	Teams 1+7, int journal
[RTY]	Teams 1+5, int. journal
[HI]	Teams 2+7, int. journal
[CH-EL-GO]	Teams 5+7, int. journal
[VGY]	Teams 3+5, preprint
[L-R-S]	ed. Conference proceedings, see below under Books, Monographs

- Publications without INTAS-NIS co-authorship of the project teams

  International Journals:

Team 2: [BeFr], [BBR](s), [BO1], [CT], [CK]

Team 3: [DV], [CHDV](a), [GA](a)

Team 4: [AD], [DI](s), [DK](s)

Team 5: [CH-ME], [GU], [VGY](p)

Team 6: [BDFT], [BJ1], [BJ1], [BJ2], [BJG], [BFJS], [DP], [P1], [P2], [CLP](a), [JP](a), [KP](a), [P3](a), [PR](a), [BJ3](s), [BG1](s), [BG2](s), [G2](s), [KS](s)

Team 7: [IZ], [Z], [PY](a)

- Proceedings, invited contributions

  Team 2: [B], [H1], [H2]

  Team 3: [VG],[GP]

  Team 6: [G], [P3]

- Books, Monographs, internal reports, thesis, patents

  Team 1: [L-R-S] (ed. Conference Proceedings)

  Team 2: [B2](p),[B3](p), [BO2](p), [HL](p)

  Team 3: [DLPVG] (ed. Conference Proceedings)

  Team 4: [S], [I], [K] (all: theses)

	ALL PUBLICATIONS			**ONLY: Jointly by INTAS and NIS Project teams**
Scientific Output	**published**	**in press/accepted**	**submitted**	
Paper in an International Journal	22	7	7	5
Paper in a National Journal				
Abstract in proceedings (conferences, workshops)	8			1
Book, Monograph	1			2
Internal Report				
Thesis (MSc, PhD, etc.)	3			

1.3 Impact and Applications (if appropriate)

Here has to be said that the publications in international journals and in proceedings of international conferences already prove the scientific impact of the project results.

2 MANAGEMENT

2.1 Meetings and visits

During the reporting period, the following scientists from NIS have visited the given locations. During theses visits, the research topics of this project have been discussed and promoted.
Two major international conferences took place, which were used for organisatorial purposes like informal network meetings for this project:

1. Conference on Quadratic Forms and Related Topics, LSU, Baton Rouge, USA, March 26 - 30, 2001, with 67 participants, among them 8 from this project representing 5 teams. 6 scientists from this project gave a talk, and 2 contributions from this project were published in the conference proceedings [L-R-S] of this meeting.

2. Conference on K-Theory and Linear Algebraic Groups, Duisburg, Sept. 9 - 15, 2001, with 67 participants, among them 17 from nodes of this this project, representing 6 teams.

   4 scientists from this project gave an invited 1 hour talk here, another one gave a 30 minutes talk.

Name:	from Team	visit at	time	support by/purpose
N. Gordeev	7	Bielefeld	2 months	TMR
I. Panin	7	Bielefeld	2 months	TMR/DFG
T. Pirashvili	6	Bielefeld/Bonn	8 months	TMR/DFG, partially INTAS
A. Dzhumadeldaev	4	Bielefeld	2 weeks	DFG
V. Yanchevski	5	Bielefeld	2 months	DFG/TMR
I. Thichonov	5	Bielefeld	1 month	TMR
V. Chernousov	5	Bielefeld	12 months	DFG/TMR
K. Zainouline	7	Bielefeld	2 months	TMR
V. Yanchevski	1	Ghent	1 month	TMR
S.Yagunov	7	Moscow	1 week	INTAS, talk about T7.6
A. Generalov	7	Becançon	1 week	INTAS, talks
A. Yakovlev	7	Becançon	1 week	INTAS, talks
J. Gubeladze	6	U/Osnabrück, Strasbourg		partially INTAS
M. Jibladze	6	U/Louvain-la-Neuve/Bonn		partially INTAS, T6.3
A. Dzhumadeldaev	4	Pantelleria/Italy	Sep 7-15, 2001	Conference

The visits mentioned below are in the context of research in this project, but financed only partially by this project.

Visits	**Number of scientists**	**Number of person days**
West ⟹East		
East ⟹West	15	930
West ⟹West		
East ⟹East	1	7

2.2 Collaboration

Intensity of Collaboration	high	rather high	rather low	low
West ⟺East	X			
West ⟺West	X			
East ⟺West	X			

2.3 Time Schedule

The project work is essentially in accordance with the Work Programme.
We do not expect a significant deviation form the Work Programme in the future.

2.4 Problems encountered

No serious problems were encountered so far.

Problems encountered	major	minor	none	not applicable
Co-operation of team Members			X	
Transfer of funds			X	
Telecommunication			X	
Transfer of goods			X	
Other			X	

2.5 Actions required

As the project already has been extended in time, there are probably no further actions required.

3 FINANCES (in EURO)

Half of the salaries and half of the money for travel and subsistence has been transferred to the recipients and the local coordinators.
The salaries have been used appropriately. The travel and subsistence money from the first payment has not yet been used totally.
It is planned to transfer the next payment of travel and subsistence funds after the first payment has been used up totally.

Contractor		**Cost Category**						**TOTAL**
#	Name of Contractor	Individ. Grants Labour Cost	Over-heads	Travel and Subsistence	Consum-ables	Equip-ment	Other Costs	(Euro)
1	Bielefeld		666	3334				4000
2	Besançon		666	3334				4000
3	Gent		666	3334				4000
4	Almaty	4400		3890				8290
5	Minsk	5490		5510				11000
6	Tbilisi	7776		3134				10910
7	St. Petersburg	5490		5420				10910
TOTAL (Euro)		23156	1998	27956				53110

4 ANNEXES

Team 1: Bielefeld

Significant Work on tasks T1.1, T1.3 [IK], [PA-RE], and T1.6 [CG] was done:
A complete classification of all excellent special orthogonal groups over fields of characteristic 0 was given, an important property of the generic splitting tower of quadratic forms over fields of any characteristic was given.
Concerning T1.2, some unforeseen cooperation with St. Petersburg was possible and yielded results on commutator properties of simple algebraic groups [GR].
Results on the two torsion of Brauer groups of hyperelliptic curves was obtained [RTY].

Publications

[CG] ⋆ V.I. Chernousov, V.I.Guletskiĭ. 2-*Torsion of the Brauer group of an Elliptic Curve: Generators and Relations.* in: A.K. Louis, U. Rehmann, P. Schneider (eds.), Proceedings of the Conference on Quadratic Forms and Related Topics, Baton Rouge, Extra Volume of Documenta Mathematica 2001, p. 85–120.

[GR] ⋆ N. Gordeev, U. Rehmann. On multicommutators for simple algebraic groups. Journal of Algebra, vol. 245 (2001), 275-296;

[IK] ⋆ Oleg H. Izhboldin and Ina Kersten. Excellent Special Orthogonal Groups, Documenta Math. 06 (2001) 385–412

[L-R-S] ⋆ A.K. Louis, U. Rehmann, P. Schneider (eds.), Proceedings of the Conference on Quadratic Forms and Related Topics, Baton Rouge, Extra Volume of Documenta Mathematica 2001, (250 p.)

[PA-RE] ⋆ I.Panin; U.Rehmann. *Springer's Theorem on Local Rings.* In preparation.

[RTY] ⋆ Rehmann U., Tikhonov S.V., Yanchevskiĭ V.I. Two-torsion of the Brauer groups of hyperelliptic curves and unramified algebras over their function fields. Communications in Algebra. 2001. Vol. 29, No. 9. P. 3971–3981

Team 2: Besançon

Team members and their latest research activities : Eva Bayer-Fluckiger (Directeur de Recherche CNRS, until February 2001) : Ideal lattices and the Minkowski conjecture, trace forms and G-Galois algebras; Karim Becher (Ph.D student, until Sept. 2001) : algebraic theory of quadratic forms, field invariants in the context of quadratic forms; Gregory Berhuy (Postdoc, until Sept. 2001) : scaled and hermitian trace forms; Cédric Bonnafé (Chargé de Recherche CNRS) : Representation theory of finite reductive groups; Anne Cortella (Maître de Conérences) : Central simple algebras with involution, rationality problem for algebraic tori; Christoph Frings (Ph.D student) : the second trace form of central simple algebras in characteristic 2, Serre's conjecture

II for non-perfect fields in characteristic 2; Detlev Hoffmann (Professor) : field invariants in the theory of quadratic forms, embeddability of quadratic forms in Pfister forms, generic splitting of quadratic forms and characterization of Pfister neighbors in characteristic 2; Nikolai Kosmatov (Ph.D student until Sept. 2001, currently Postdoc) : Isotropy of quadratic forms over function fields of Pfister forms; Emmanuel Lequeu (Ph.D student) : self-dual normal bases and G-forms; Mohammad Mahmoudi (Ph.D student) : hermitian u-invariant for forms over division algebras.

The main research activities fall into three interrelated areas : Quadratic and hermitian forms, central simple algebras, algebraic groups.

The algebraic theory of quadratic and hermitian forms is represented by Becher, Hoffmann, Kosmatov, and Mahmoudi. One of the focuses is the topic of field invariants. Here, a complete classification of the possible and realizable values for the u-invariant and the Hasse number $\tilde{u}$ and their interdependence for fields with torsion-free I^3 has been obtained (Hoffmann). A new characterization of fields with finite Hasse number has been obtain using only intrinsic properties of quadratic forms and not invoking any properties of the space of orderings (this is the first such characterization of such a kind) (Hoffmann). An improved lower bound for the number of square classes of a field has been obtained up to which the level conjecture has a positive answer (Becher). (Level conjecture : For a field with a finite number of square classes, is it true that the level is always ≤ 4 ?) There is work in progress on the question of the analogue of the u-invariant for hermitian forms over division algebras with involution (Mahmoudi). The generic splitting of quadratic forms in characteristic 2 has been studied and has lead to a characterization of Pfister neighbors whose totally singular part is of dimension ≤ 3 (Hoffmann, joint work with Laghribi, Louvain-La-Neuve).

Trace forms have been studied in various contexts, in particular the realizability of certain (integral) quadratic or hermitian forms as (quadratic or hermitian) trace forms over fields (rings of integers) (Berhuy), the computation of trace forms of central simple algebras over local and global fields (Berhuy), and of the Arf- and Clifford invariant of the second trace forms of central simple algebras in characteristic 2 (Berhuy, Frings).

In the integral theory, ideal lattices (or Arakelov divisors) in number fields have been studied (Bayer-Fluckiger), leading to a confirmation of the Minkowski conjecture for various types of number fields, thus extending considerably the list of known results.

In the context of central simple algebras, besides the results on their trace forms mentioned above, a characterization of asymmetries in algebras of exponent two has been given (Cortella, joint work with Tignol, Louvain-La-Neuve).

The study of algebraic groups also includes reductive groups over finite fields and their representation theory, and progress has been made towards a confirmation of a conjecture of Lusztig concerning character sheaves in the case of the group $SL_n(\mathbf{F}_q)$ (Bonnafé). Also, a conjecture of Broué concerning the representation theory of a block and its Brauer correspondent for finite reductive groups has been proved in the case of groups with connected center (Bonnafé, joint work with R. Rouquier, Paris VII).

A proof of a conjecture by Le Bruyn concerning stable rationality of generic tori of simple adjoint or simply connected groups has been obtained (Cortella, joint work with B. Kunyavskii, Bar-Ilan University, Israel).

Publications

[HI] ⋆ Detlev Hoffmann, Oleg Izhboldin, Embeddability of quadratic forms in Pfister forms. Indag. Math. 11 (2000), 219–237.

[BeFr] G. Berhuy, Christoph Frings, On the second trace form of central simple algebras in characteristic 2. Manusc. Math. 106 (2001), 1–12

[BBR] D. Bessis, Cédric Bonnafé, R. Rouquier, Quotients de groupes de réflexions complexes, Math. Ann. (to appear)

[BO1] Cédric Bonnafé, Opérateurs de torsion dans $SL_n(q)$ et $SU_n(q)$. Bull. Soc. Math. France 128 (2000), 309–345.

[CT] Anne Cortella, J.-P. Tignol, The asymmetry fo an antiautomorphism. J. Pure and Applied Algebra (to appear)

[CK] Anne Cortella, Boris Kunyavskii, Rationality problem for generic tori in simple groups. J. Algebra 225 (2000), 771–793.

Abstracts in proceedings (indicate invited publications):

[B] Karim Becher, On the number of square classes of a field of finite level. Doc. Math. J. DMV (Extra Volume, Proceedings of the LSU Conference on Quadratic Forms and Related Topics) (2001), 65–84.

[H1] Detlev Hoffmann, Isotropy of quadratic forms and field invariants. Proc. of the Conference on Quadratic forms and their Applications, University College Dublin, Contemp. Math. 272 (2000), 73–102. (Invited publication)

[H2] Detlev Hoffmann, Dimensions of anisotropic indefinite quadratic forms I. Doc. Math. J. DMV (Extra Volume, Proceedings of the LSU Conference on Quadratic Forms and Related Topics) (2001), 183–200.

Books, monographs, internal reports, thesis, patents:

[B2] Karim Becher, Algèbres simples centrales à involution de première espèce (Preprint).

[B3] Karim Becher, Supreme Pfister forms (Preprint).

[BR] Cédric Bonnafé, R. Rouquier, Catégories dérivées et variétés de Deligne-Lusztig.

[BO2] Cédric Bonnafé, Actions of relative Weyl groups I. Prépublications de l'Équipe de Mathématiques de Besançon 41 (2001).

[HL] Detlev Hoffmann, A. Laghribi, Quadratic forms and Pfister neighbors in characteristic 2 (Preprint).

Team 3: Ghent

Collaboration between V.I. Yanchevskii (Minsk) and J. Van Geel (Ghent).

Index of Hyperelliptic curves.

A. We continued some earlier investigations concerning the index of hyperelliptic curves over local fields. Let O_k be a complete discrete valuation ring with finite residue class field, let k be its field of fractions. If k the characteristic of the residue degree is not 2 we proved in [VGY] some results which allow to make an algorithm to calculate the index of hyperelliptic curves given by an (affine) equation of the form $Y^2 = \varepsilon f(X)$, where $f(X) \in O_k[X]$ is an irreducible polynomial of 2 power degree and ε is either a non-square unit in O_k or a uniformising element, π_k, in O_k.
For equations of the form $Y^2 = \pi_k f(X)$ we extended this results to polynomials of any degree provided the field extension generated by a root of $f(X)$ is tamely ramified. This results can be found in [VGY2].
The interest of this problem comes from the study of the natural map

$$Br(k) \to Br(k(C))$$

where $k(C)$ is the function field of the hyperelliptic curve defined by the equation $Y^2 = \varepsilon f(X)$. The kernel of this map is non-trivial exactly when the index of the curve is 2.

B. We started a new research problem with as topic Ω-central simple algebras over $K = \mathbf{R}((t))(X)$. These are central simple algebras of exponent 2 in the Brauer group of K, that are trivial over all real closures of K. We think it must be possible to give a structure theorem for such algebras in terms of their decomposition as a tensor product of quaternion algebras.
Until now we studied the ramification of these algebras in detail. This allowed us to reduced the problem to some particular type of algebras (certain tensor products of 2 quaternion algebras). We investigated several special cases (depending on the number and the degree of the ramification points) and were able to show that in all these case an Ω-algebra is necessarily a quaternion algebra. (The point being that we could prove the existence of a common splitting field for the two quaternion algebras in the tensor product).

Future plans.
We certainly will continue the research mentioned in B. A complete classification of Ω-algebras over K might shed some light a different problems in the theory of division algebras and quadratic forms.

Furthermore we will investigate quadratic form analogues of the results obtained. (Since the Ω-algebras are tensor products of two quaternion algebras their structure is strongly related to so called Albert forms, these are 6-dimensional quadratic forms associated to such algebras.)

Publications

[DV] C. De Volder, *General Blowings-up of* $\mathbf{P}^2$ *and Injectivity of the Gauss maps,* Geometricae Dedicata, Vol. 85, Nr. 1-3, pp237-251, 2001.

[CHDV] S. Chauvin, C. De Volder, *Some Very Ample and Base Point Free Linear Systems on Generic Rational Surfaces*, To appear Math. Nachrichten.

[CODV] M. Coppens, C. De Volder, *Existence of Embeddings for which the Gauss map is an Embedding,* To appear Mathematica Pura et Applicata.

[GA] F. Gardeyn, *Une borne pour l'action de l'inertie sauvage sur la torsion d'un module de Drinfeld,* To appear Archiv der Mathematik.

Abstracts in proceedings (indicate invited publications)

[VG] J. Van Geel, *Galois representations associated to Drinfeld modules* (After F. Gardeyn), proceedings 11èmes Rencontres Arithmétiques de Caen, publication de l' Université de Caen 2001.

Books, monographs, internal reports, thesis, patents

[DLPVG] Editors: Denef, J., Lipshitz, L., Pheidas, T., Van Geel, J., *Hilbert's Tenth Problem: Relations with Arithmetic and Algebraic Geometry*, Proceedings workshop, November 2-5, 1999, Ghent. Contemporary Mathematics volume 270, December 2000

Team 4: Almaty

Participants are A. Dzhumadil'daev (Team Leader), S. Abdykassymova, Sh. Ibraev, I. Irgalieva, A. Kungojin, E. Bekmuchamedov.
Results:
A. Dzhumadil'daev (jointly with S. Abdykassymova): We prove that the cohomology of Leibniz algebras with coefficients in an irreducible module is trivial, if the module is not restricted. The number of irreducible antisymmetric modules with nontrivial cohomology is finite. A Leibniz algebra is called simple, if it has no a proper ideal except ideal generated by squares of its elements. We describe simple Leibniz algebras with Lie factor isomorphic to sl_2 and p^m-dimensional Zassenhaus algebra $W_1(m)$.
A. Dzhumadil'daev (jointly with S. Ibraev): Second cohomology groups of irreducible representations of classical Lie algebras A_2, B_2 and G_2 over an algebraically closed field of characteristic $p > h$ are calculated. Here h is the Coxeter number.
A. Kungojin: Has studied simple n-Lie algebras. Vector products algebras over complex numbers and their modular analogues
E. Bekmuchamedov. Wrote program for calculating cohomology groups in Mathematica
I. Irgalieva has left our team because of miserable salary. Instead of her we get two new members for our group: Uteulieva Kamka and Turetaeva Gulganar. They are post-graduate students (aspirants). One of them (Uteulieva) prepare Ph.D. for defense this summer.

Publications

[AD] S. Abdykassymova, A. Dzhumadil'daev. *Leibniz algebras in characteristic p* , Comptes Rendus Acad. Sci. Paris, Serie I, Vol.332, 2001, 1047-1052.

[DI] A. Dzhumadil'daev, S. Ibraev, *Non split extensions of Lie algebras of rank 2* , Homology, Homotopy and Applications (to appear).

[DK] 3. K. Uteulieva, R. Kerimbaev , *3-homology of degree 3 of special = Lie algebra*, Mathematicheskii Jurnal, 2002 to appear.

[S] *PhD thesis* S.Abdykassymova, Rank one Leibniz algebras in characteristic p, October 2001 (advisors: A. Dzhumadil'daev) (in Russian)

[I] *PhD thesis* S.Ibraev, Nonsplit extensions and cohomologies of modular Lie algebras of classical type, October 2001 (advisors: A. Dzhumadil'daev)(in Russian)

[K] *Diplom thesis* A. Kungojin, n-Lie algebras, June 2001 (advisors: A. Dzhumadil'daev) (in Russian)

Team 5: Minsk

Generators and relations in 2-torsion of Brauer groups of hyperelliptic and elliptic curves defined over an arbitrary field are described of characteristic not equal to two (see [RTY], [CG]).
Let k be a local field, O_k its rings of integers, π a uniformizer in O_k. The index of any hyperelliptic curve defined by an affine equation $y^2 = \pi f(x)$, where $f(x) \in O_k[x]$ such that its root generates a tamely ramified extension of k (see [VGY]).
For simple simply connected quasi-split groups of type ${}^{3,6}D_4, E_6, E_7$ defined over a perfect field F of characteristic $\neq 2, 3$ it is proved that the Rost invariant has the trivial kernel ([Che]). It follows immediately from the result above that if $\operatorname{cd} F \leq 2$ (resp. $\operatorname{vcd} F \leq 2$) then Serre's Conjecture II (resp. the Hasse principle) holds for such a group. For a (C_2)-field, in particular $\mathbf{C}(x, y)$, we prove the stronger result that Serre's Conjecture II holds for all (not necessary quasi-split) exceptional groups of type ${}^{3,6}D_4, E_6, E_7$.
For X being a surface of general type with $p_g = 0$ it is proved that its motivic finite dimensionality implies the explicit decomposition of its motive into a direct sum of the unit motive, b_2 copies of the Lefschetz motive (where b_2 is the second Betti number) and the tensor square of the Lefschetz one. In particular, the motive of the Godeaux surface X is finite dimensional. It gives the motivic proof of the triviality of the Albanese kernel for X. For surfaces $p_g = 0$ and not of general type the motive is finite dimensional, see [GP].

Publications

[CH-EL-GO] ⋆ V.Chernousov, E.Ellers; N.Gordeev. *Gauss Decomposition with Prescribed Semisimple Part: Short Proof.* J.Algebra 229(2000), pp.314-332.

[CG+] ⋆ V.I. Chernousov, V.I.Guletskiĭ. 2-*Torsion of the Brauer group of an Elliptic Curve: Generators and Relations.* in: A.K. Louis, U. Rehmann, P. Schneider (eds.), Proceedings of the Conference on Quadratic Forms and Related Topics, Baton Rouge, Extra Volume of Documenta Mathematica 2001, p. 85–120.

[CH-ME] V. Chernousov, A. Merkurjev, R-equivalence in Spinor groups, *R-Equivalence in Spinor Groups*, Amer. Math. Soc. 14 (2001), 509-534.

[Che] V.I. Chernousov. The kernel of the Rost invariant, Serre's Conjecture II and the Hasse principle for quasi-split groups ${}^{3,6}D_4, E_6, E_7$, preprint.

[RTY+] ⋆ Rehmann U., Tikhonov S.V., Yanchevskiĭ V.I. Two-torsion of the Brauer groups of hyperelliptic curves and unramified algebras over their function fields. Communications in Algebra. 2001. Vol. 29, No. 9. P. 3971–3981

[VGY] ⋆ Van Geel J., Yanchevskiĭ V.I. The index of certain hyperelliptic curves over p-adic fields, preprint.

[KU-RO-TI-YA] B.È. Kunyavskiı̆, L.H. Rowen, S.V. Tikhonov, and V. I. Yanchevskiı̆. Bicyclic algebras over function fields. (in preparation)

[GP] V. Guletskiĭ, C. Pedrini. The Chow motive of the Godeaux surface. Submitted to the Proceedings of the Algebraic Geometry conference in memory of Paolo Francia, September 25 - 29, 2001, Genova. W de Gruyter expositions in mathematics.

[GU] V.I. Guletskiĭ. The middle of the diagonal for surfaces with invariants $p_g = 0$ and $q = 1$. (In Russian). Algebra i Analiz, Vol. 13 (2001), No. 3, pp. 119 - 138.

Team 6: Tbilisi

Results:

T. Pirashvili together with B. Richter proved among other things that classical homology theories like André-Quillen homology, Hochschild homology or cyclic homology has a nice interpretation in terms of functor homology (see [P3] and [PR]).

J. Gubeladze proved in [BFJS] that if a unimodular row can be reduced after elementary transformation in a larger ring, then it can be reduced in smaller one also, provided the ring extension is subintegral. This fact is important for computations of K_1 for monoid algebras. In [BJG], Bruns and Gubeladze study non-linear analogues of projective modules, based on graded retraction of the homogeneous rings of projective toric varieties. In [G2], the main conjecture on nilpotence of the multiplicative action of the natural numbers on higher K-theory of a toric variety has been proved for infinitely many non-trivial (i.e. non-simplicial) cases of toric varieties. In [BG1], theory of Schur multiplier for, based on automorphisms of projective toric varieties has been developed. In [BG2], higher K-theory based on the automorphism groups of projective toric varieties has been developed.

In [BJ3], it is proved that all track categories with Abelian automorphism groups of 1-arrows are linear track extensions of their homotopy categories. In [JP], it is proved that any surjective homomorphism of simplicial Maltsev algebras is a Kan fibration.

In [KS], for a homotopy G-algebra A related with a dg Hopf algebra H by a multiplicative twisting function $\tau : H \to A$ on the tensor product $A \otimes_\tau H$ a twisted multiplication is introduced converting it into a dga.
Papers published during this period:

Publications

[BDFT] H.-J. Baues, W. Dreckmann, V. Franjou and T. Pirashvili. Foncteurs Polynômiaux et foncteurs de Mackey non linéaires. Bull. Soc. Math. France 129 (2001), 237–257

[BJ1] H.-J. Baues and M. Jibladze. Suspension and loop objects and representability of tracks. Georgian Math. J. 8(2001), 683–696.

[BJ2] H.-J. Baues and M. Jibladze. Suspension and loop objects in theories and cohomology. Georgian Math. J. 8(2001), 697–712.

[BJG] W. Bruns and J. Gubeladze, Polytopal linear retractions, Trans. Amer. Math. Soc. 354 (2002), 179–203.

[BFJS] M. Bunge, J. Funk, M. Jibladze and T. Streicher, Distribution algebras and duality. Adv. Math. 156 (2000), no. 1, 133–155.

[DP] T. Datuashvili and T. Pirashvili. On (co)homology of 2-types and crossed modules. J. Algebra. 244(2001), 352–365.

[G] J. Gubeladze, Subintegral extensions and unimodular rows. Geometric and combinatorial aspects of commutative algebra (Messina, 1999), 221–225, Lecture Notes in Pure and Appl. Math., 217, Dekker, New York, 2001.

[P1] T. Pirashvili, Dold-Kan type theorem for Γ-groups. Math. Ann. 318 (2000), no. 2, 277–298.

[P2] T. Pirashvili, Hodge decomposition for higher order Hochschild homology. Ann. Sci. cole Norm. Sup. (4) 33 (2000), no. 2, 151–179.

Papers accepted for publication:

[CLP] J. M. Casas, J.-L. Loday and T. Pirashvili. On Leibniz n-algebras. Forum Math.

[JP] M. Jibladze and T. Pirashvili. On Kan fibrations for Maltsev algebras. Georgian Math. J.

`http://arxiv.org/abs/math.AT/0106143/`

[KP] R. Kurdiani and T. Pirashvili. A Leibniz algebra structure on the second tensor power of Lie algebras. J. of Lie Theory.

[P3] T. Pirashvili. André-Quillen homology via functor homology. Proc. AMS

[PR] T. Pirashvili and B. Richter. Hochschild and cyclic homology via functor homology. K-theory.

Submitted papers:

[BJ3] H.-J. Baues and M. Jibladze, Classification of Abelian track categories, submitted to K-theory.

[BG1] W. Bruns and J. Gubeladze, Polyhedral K_2, preprint (35 p.), submitted.

`http://arXiv.org/abs/math.KT/0104206/`

[BG2] W. Bruns and J. Gubeladze, Higher polyhedral K-groups, preprint, (39 p.) submitted.

`http://arXiv.org/abs/math.KT/0108013/`

[G2] J. Gubeladze, Higher K-theory of toric varieties, preprint (38 p.), submitted.

`http://arXiv.org/abs/math.KT/0104166/`

[KS] T. Kadeishvili and S. Saneblidze, A cubical model of the path space fibration, submitted to J. Pure and Appl. Algebra.

Team 7: St. Petersburg

(a) A field with U-invariant 9 was constructed by O. Izhboldin (T7.3.1) (1); this is the first field with odd U-invariant which disproves the famous long standing Kaplansky conjecture (1953), this result immediately became outstanding in the theory of quadratic forms. One should stress here that the proof involves all the higher technique of motivic cohomology and Chow groups;
(b) the original Gersten conjecture in equi-characteristic case was proved by I.Panin (T7.4) and published as a preprint www.math.uiuc.edu/K-theory/389; the Gersten conjecture in K-theory was solved by Quillen in 1973 in geometric case. Since that time different versions of Gersten conjecture for other cohomology theories were proved by Bloch-Ogus, Gabber and others. But there was no progress in the original Gersten conjecture.
(c) The Conjecture of Grothendieck concerning principal homogeneous spaces over certain classical algebraic groups was solved by K.Zainoulline (T7.5) and published as an article "On Grothendieck Conjecture concerning Principal Homogeneous Spaces for some classical algebraic groups", St.Petersburg Math. J., 12(1), 2001; the results presented in the article extends previous results of J.-L.Colliot-Thelene, Parimala, Shridharan, Rost, A.Suslin, I.Panin and M.Ojanguren on this subject.
(d) Suslin's rigidity theorem (S.Yagunov, I.Panin). Suslin's famous rigidity theorem was extended by I.Panin and S.Yagunov to all orientable cohomology theories on schemes (T7.6) (3). I.Panin and S.Yagunov introduce the notion of orientable cohomology theory on the category of projective smooth schemes, defined a family of transfer maps and, as a consequence of these constructions, proved that taken with finite coefficients such cohomology doesn't change after an extension of algebraically closed fields. This result generalizes the old and remarkable Suslin's theorem about K-theory of algebraically closed fields. Besides K-theory, we treat the following examples of orientable theories: Etale Cohomology, Motivic Cohomology, Algebraic Cobordism.
(e) Multicommutators and multiproducts of conjugacy classes of simple algebraic groups has been studied by N.Gordeev, U.Rehmann (T7.8) (4); the following generalization of the Ore's commutator problem was proved: the group G satisfies the condition C_n if for every sequence $x_1, \dots x_n \in [G,G]$ there exists a sequence $g, g_1, \dots, g_n$ such that $x_i = [g, g_i]$ for every $i = 1, \dots, n$. (If the condition C_1 holds for simple groups it is exactly the Ore's commutator problem). It has been shown that simple algebraic group satisfies the condition C_n up to Zariski closure if and only if $n \leq h+1$ where h is the corresponding Coxeter number.

Publications

[IZ] O.Izhboldin. Fields with U-invariant 9, (to appear in Ann. of Math.);

[IK+] ⋆ Oleg H. Izhboldin and Ina Kersten Excellent Special Orthogonal Groups Documenta Math. 06 (2001) 385–412

[Z] K.Zainoulline. The purity problem for functors with transfers. K-theory, vol. 22 (2001), No.4, 303-333;

[Z1] K.Zainoulline. On Grothendieck Conjecture concerning Principal Homogeneous Spaces for some classical algebraic groups, St. Petersburg Math. J., 12(1), 2001.

[PA-RE+] $\star$ I.Panin; U.Rehmann. *Springer's Theorem on Local Rings.* In preparation.

[PY] I.Panin, S.Yagunov. Rigidity for Orientable Functors, www.math.uiuc.edu/K-theory/489; (to appear in the Journal Pure and Applied Algebra);

[GR+] $\star$ N. Gordeev, U. Rehmann. On multicommutators for simple algebraic groups. Journal of Algebra, vol. 245 (2001), 275-296;