Periodic Progress Report 1.1.2002 – 31.12.2002

Linear Algebraic Groups and Related Linear and Homological Structures

1 RESEARCH

1.1 Overview of Research Activities/ Conformance with the Work Programme

Remark: The project cooperates with the EU funded TMR network ERB FMRX CT-97-0107, which formally ended at June 30, 2002, and also with its successor network RTN HPRN-CT-2002-00287, which started on October 1, 2002, and will last for 48 months (Title of both networks: Algebraic K-Theory, Linear Algebraic Groups and Related Structures) Both networks contain all the teams of this project except the Almaty team. Therefore, there is additional funding for travel and subsistence form there for this project.
We consider it a success for the scientific work of this INTAS project together with the TMR network, that it was possible to achieve the funding for a successor RTN network, allowing to embed the scientific work of the relatively small Intas group into the larger RTN network, which consists of 10 western European and three eastern nodes.
There is also funding from other sources like the Humboldt foundation in Germany: A particular big succes is, that Teimuraz Pirashvili, the local coordinator for our Tbilisi node, was awarded a highly valued Humboldt prize (cf. http://www.avh.de) in order to spend a year in Bielefeld for joint research.
During the reporting period, Ahmed Laghribi was working in Bielefeld as a Humboldt fellow in the context of the research of this project, he is continuing his work now (in 2003 and further) as a Postdoc fellow of the RTN network.
Also, one of the young scientists from our St. Petersburg team has applied for a Humboldt fellowship which, in case of being granted, will allow him to spend a full year on that basis in Bielefeld as well. This was certainly induced by the contacts arranged through this Intas project. Plans in this direction are underway for a young scientist from Minsk as well.

- Work carried out by each of the Contractors
- Has the research been in accordance with the Work Programme? If not, in what respect and why?
- Do you foresee any deviations from the Work Programme for the future? If yes, which ones and why?

1.2 Scientific Results

Team 1: Research has been done in the context of T1.1, T1.2, two publications are in preparation. Significant work is being done concerning T1.4: Since Teimuraz Pirashvili was awarded a Humboldt Price, which allows him to continue his joint research with Waldhausen at Bielefeld for another year, the work in T1.4

could be intensified. Publications [CELP], [JP], [KP], [Prop], [PR] represent significant progress for this subtask.

Team 2: Advances in the generic theory of quadratic forms in characteristic 2 (joint work of Hoffmann and Laghribi) (T2.1). Advances in the generic theory of quadratic forms in characteristic $\neq 2$ (Roussey): Questions on isotropy of quadratic forms over function fields of quadrics defined over fields equipped with a valuation. Scientific significance : Generalization of results by Izhboldin. (T2.1).

As E. Bayer and V. Chernousov are not working any more at Besançon, Task T2.2 has been replaced by related topics, but of similar importance for the whole work programme: Research in the theory of algebras with involution (joint work of Cortella and Tignol), in the theory of Witt groups (Grenier-Boley, Mahmoudi), the theory of algebraic groups (Bonnafé), in K-theory (joint work of Hoffmann and Becher). See the list of publications below.

Task T2.3 (Workshop) will be performed in June 2003. (Since persons changed in Besançon, the program had to be modified.)

Team 3: (1) The collaboration between the research team of Ghent university (Van Geel, Wylleman) and the university of Minsk (Yanchevskii, Bazyleu, Tikhonov) was concentrated around problems related to the Brauer group and the structure of division algebras over fields of transcendence degree one over different local fields. this in accordance with tasks T3.1-T3.2-T3.3 of the workprogram. (Task T3.4, applications to the u-invariant has still to be worked out).

In Ghent Van Geel was mainly involved in this work. In October Wylleman, a research student, started in Ghent to work for a PH-D and was involved partly with the project.

(2-3) The research has been in accordance with the work program. We do not expect large changes in the future.

1.2 Progress in several directions has been made on T3.1, T3.3. (See annexe, and publications [1], [2], [3]). Significant progress has been made on task T3.2 see publications [4].

Team 4: Progress on T4.1.2 was published in [Dzh-Abd], cf. also [S]. Partial results for T4.2.1 are published in [Ut1],[Ut2]

Results for T4.3 are published in [Dzh-Ibr], [Ibr-Tur], see also [I].

Material stongly related to the project tasks is published in [Dzh-Jac], [Dzh01], [Dzh-Dav], [Dzh-Lof], [Dzh-Nov-Jor], and [Dzh-N-com].

Team 5: Significant progress in Task T5.B1, T5.B2, T5.B3 is published in [BGY1], [BGY2], [BGY3] (this in collaboration with team 3 - Gent). Essential result in T5.A3, T5.C3 is obtained in [Y]. Progress with respect to T5.C1 is published in [TY]. Additional results in algebraic geometry were obtained in [GP1], [GP2].

Team 6: Essential results concerning task T6.1 and T6.2 are published in [PR],[Prop], [BJ]. See also the talk of Pirashvili on Bourbaki seminare about development on functor homology (task T6.1). Gubeladze made a significant progress in the K-theory of toric varieties (task T6.3). In particular he showed that contrary to some conjectures the Grothendieck groups of locally trivial sheaves and coherent sheaves on such varieties rationally are NOT isomorphic [G2]. He also made a progress on the so colled nilpotency conjecture on K-theory of toric variaties [G1]. Results concerning task T6.4 are in [KS1],[KS2].

Team 7: Task T7.6 and its further developments are presented in [PY1, YA, PA, PY2]. Some important technique, which can be applied to T7.2., was obtained in [SU]. Significant progress in T7.4. reflected in publications [BGPW, PZ]. Some new results on T7.8. are presented in [GE]. For all other tasks, research has been done and publications are in preparation.

The research is essentially in accordance with the Work Programme. No essential deviations of the work programme have to be made.

Publications

Accepted items are marked by (a), submitted items are marked by (s), preprints are marked by (p), everything else is published:

- Joint Publications of INTAS and NIS project teams

[PA-RE]p	Teams 3+5 (preprint)
[BGY1]	Teams 3+5, nat. journal
[BGY2]	Teams 3+5, nat. journal
[BGY3]	Teams 3+5, nat. journal
[VY]	Teams 3+5, int. journal

- Publications without INTAS-NIS co-authorship of the project teams

- International Journals:

  Team 1: [HL]s,

  Team 2: [BBR], [BR]a, [B1]a, [B2]a, [B3]a, [CT1]

  Team 4: [Dzh-Abd], [Dzh-Ibr], [Dzh-Lof], [Dzh01], [Dzh-Dav], [Dzh-Nov-Jor], [Dzh-N-com]a, [Dzh-Hadamar]a, [Dzh-Rep]s, [Dzh-Wron]s,

  Team 5: [Y]a, [TY]a, [TY1]a

  Team 6: [BJ], [BG], [CLP], [CELP], [JP], [KP], [Prop], [P], [PR], [P1]a

  Team 7: [PY1], [GE](s), [BGPW], [GE]s

  [BGPW](p), [PA]

- National Journals:

  Team 4: [Ut1], [Ut2], [Ibr-Tur]

  Team 5: [TY1], [GP2]s

- Proceedings, invited contributions

  Team 4: [Dzh-Jac]

  Team 5: [GP1]

- Books, Monographs, internal reports, thesis, patents

  Team 1: [HR]p, [R]p

  Team 2: [BH]p, [CT2]p, [GBM]p

  Team 4: [Dzh-Rep], [Dzh-Wron], [S], [I], [Ut], [K]

  Team 5: [GP1]p

  Team 6: [G1]p, [G2]p, [J]p, [KS1]p, [KS2]p,

  Team 7: [SU](p), [PY2](p), [YA](p), [PZ] [P2]

	ALL PUBLICATIONS			**ONLY: Jointly by INTAS and NIS Project teams**
Scientific Output	**published**	**in press/accepted**	**submitted**	
Paper in an International Journal	23	10		1
Paper in a National Journal	7		1	3
Abstract in proceedings (conferences, workshops)	8 2		1	
Book, Monograph				
Internal Report, Thesis, preprint	11			1

1.3 Impact and Applications (if appropriate)

Here has to be said that the publications in international journals and in proceedings of international conferences already prove the scientific impact of the project results.

2 MANAGEMENT

2.1 Meetings and visits

During the reporting period, the following scientists from NIS have visited the given locations. During theses visits, the research topics of this project have been discussed and promoted.
Several major international conferences took place, which were partially used for organisatorial purposes like informal network meetings for this project, and also, several scientists have participated as invited speakers:

1. Conference "Quadratic Forms" (Math. Research Inst. Obervolfach, Germany, May 12 - 18, 2002)

2. Conference "Algebraic K-Theory", (Math. Research Inst. Obervolfach, Germany, August 4 - 10, 2002)

3. "Algebraic K-theory and its applications" (Trieste, Italy, July 20 - 27, 2002);

Name:	from Team	visit at	time	support by/purpose
D. Hoffmann	2	St. Petersburg	1 week	INTAS
V. Yanchevski	5	Ghent	1 month	INTAS
V. Yanchevski	5	Ghent	1 month	INTAS
D. Bazyleu	5	Ghent	1 month	INTAS
S. Tikhonov	5	Ghent	1 month	INTAS
J. Van Geel, V. Yanchevski	3,5	Lens	3 days	INTAS, report on work
I. Panin	7	Bielefeld	1 week	INTAS
K. Zainoullin	7	ielefeld	1 week	INTAS
S. Yagunov	7	Bielefeld	2 months	TMR
S. Yagunov	7	Bielefeld	3 months	INTAS
V. Guletski	5	St. Petersburg	1 month	INTAS/TMR
T. Pirashvili	6	Bielefeld	6 months	Humboldt
T. Pirashvili	6	Strasbourg/Nantes	1 month	INTAS
J. Gubeladze	6	Osnabrück	1 month	INTAS and others
A. Dzhumadildaev	4	Conf. Vienna	1 week	INTAS
A. Dzhumadildaev	4	Conf. Moscow	1 week	INTAS

The visits mentioned below are in the context of research in this project, but financed only partially by this project.

Visits	**Number of scientists**	**Number of person days**
West $\Longrightarrow$East	1	7
East $\Longrightarrow$West	9	544
West $\Longrightarrow$West	1	3
East $\Longrightarrow$East	2	37

2.2 Collaboration

Intensity of Collaboration	**high**	**rather high**	**rather low**	**low**
West $\Longleftrightarrow$East	X			
West $\Longleftrightarrow$West	X			
East $\Longleftrightarrow$West	X			

2.3 Time Schedule

The project work is essentially in accordance with the Work Programme.
We do not expect a significant deviation form the Work Programme in the future.

2.4 Problems encountered

No serious problems were encountered so far.

Problems encountered	**major**	**minor**	**none**	**not applicable**
Co-operation of team Members			X	
Transfer of funds			X	
Telecommunication			X	
Transfer of goods			X	
Other			X	

2.5 Actions required

It may be desirable to extend the project till December 31, 2003 or even till March 31, 2004. Since the project work has been supported not only by INTAS, but also from other sources, more than originally planned has been and will be done.

3 FINANCES (in EURO)

90 % of the salaries and 50 % of the money for travel and subsistence has been transferred to the recipients and the local coordinators.
The salaries have been used appropriately. The travel and subsistence money from the first and second payment has not yet been used totally, but will be used up during the remaining project time.

Contractor		**Cost Category**						**TOTAL**
#	Name of Contractor	Individ. Grants Labour Cost	Over-heads	Travel and Subsistence	Consum-ables	Equip-ment	Other Costs	(Euro)
1	Bielefeld		666	3334				4000
2	Besançon		666	3334				4000
3	Gent		666	3334				4000
4	Almaty	7920		3890				11810
5	Minsk	9882		5510				15392
6	Tbilisi	13994		3134				17128
7	St. Petersburg	10206		5420				15626
TOTAL (Euro)		42002	1998	27956				71956

4 ANNEXES

Team 1: Bielefeld

Research has been done in the context of T1.1, T1.2, several publications are in preparation ([PA-RE], [HR], [R]. Significant work is being done concerning T1.4: Since Teimuraz Pirashvili from Tbilisi was awarded a Humboldt Prize, which allows him to continue his joint research with Waldhausen at Bielefeld for another year, the work in T1.4 could be intensified. Publications [CELP], [JP], [KP], [Prop], [PR] represent significant progress for this subtask.

Publications

[PA-RE] ⋆ I. Panin, U. Rehmann. *Springer's Theorem on Local Rings.* In preparation.

[HR] J. Hurrelbrink, U. Rehmann. *Indices of Quadratic Forms.* In preparation.

[R] U. Rehmann *Galois Cohomological Description of Certain Orthogonal Groups* In preparation.

[HL+] D. Hoffmann, A. Laghribi : Quadratic forms and Pfister neighbors in characteristic 2, Preprint No. 103 (Preprint Server "Linear Algebraic Groups and Related Structures", http://www.mathematik.uni-bielefeld.de/lag), 33 pages, 2003

Team 2: Besançon

Team members and their latest research activities :
Cédric Bonnafé (Chargé de Recherche CNRS) : Representation theory of finite reductive groups; Anne Cortella (Maître de Conérences) : Central simple algebras with involution, rationality problem for algebraic tori; Nicolas Grenier-Boley (Ph.D student of D. Hoffmann) : Isomorphism theorems and exact sequences of Witt groups of hermitian forms over central simple algebras with involution. Detlev Hoffmann (Professor) : field invariants in the theory of quadratic forms, embeddability of quadratic forms in Pfister forms, generic splitting of quadratic forms and characterization of Pfister neighbors in characteristic 2; Emmanuel Lequeu (Ph.D student of Eva Bayer-Fluckiger) : self-dual normal bases and G-forms; Mohammad Mahmoudi (Ph.D student of Eva Bayer-Fluckiger) : hermitian forms over central simple algebras and their Witt groups. Simona Nespolino (Ph.D student of D. Hoffmann, currently Louvain-La-Neuve) : Hermitian lattices with trivial automorphism group. Sylvain Roussey (Ph.D student) : Quadratic forms over function fields of quadrics defined over base fields with valuation.
The main research activities fall into four interrelated areas : Quadratic and hermitian forms, central simple algebras, algebraic groups, K-theory.
The algebraic theory of quadratic and hermitian forms is represented by , Grenier-Boley, Hoffmann, Mahmoudi and Roussey. The generic splitting of quadratic forms

has been studied in two contexts. In characteristic $\neq 2$, Roussey has studied function fields of quadratic forms over fields equipped with a (complete or 2-henselien) valuation and the question of isotropy of quadratic forms over such function fields, and he generalized results by Izhboldin who treated the particular case of a power series field (thesis work in progress). In characteristic 2, based on earlier work by Laghribi, the theory of generic splitting has been developed further, systematically including the case of singular forms (which has hitherto been almost always excluded). In particular, Hoffmann's theorem of the separating 2-power has been generalized to characteristic 2, as well as a theorem by Fitzgerald characterizing Pfister forms in terms of the hyperbolicity of a form over the function field a subform which is in a certain sense "large" enough. This has lead to a characterization of Pfister neighbors whose totally singular part is of dimension ≤ 4 (Hoffmann, joint work with Laghribi, Bielefeld).

In the integral theory, hermitian lattices over the integers in imaginary quadratic number fields have been studied, in particular the construction of such lattices with trivial automorphism group and of smallest possible rank ≥ 2 for a given discriminant of the number field (Nespolino, thesis work in progress).

At the interface of the theory of hermitian forms and the theory of central simple algebras, the question of the isomorphism of Witt rings of hermitian forms over fields with an automorphism of order 2 or over algebras with involution has been studied and has led to generalizations of Harrison's theorem (Grenier-Boley, thesis work in progress). An exact octagon of Witt groups of hermitian forms over algebras with involution has been constructed, generalizing earlier results by David Lewis and by Parimala-Sridharan-Suresh (Grenier-Boley and Mahmoudi, thesis work in progress).

In the context of central simple algebras, a new invariant — the so-called asymmetry — of an anti-automorphism of a central simple algebra has been defined, which leads to the notion of the discriminant of an anti-automorphism. The notion of asymmetry in the case of a split algebra has existed before. The above notion is a generalization which yields an important invariant to decide when an anti-automorphism is an involution, and it leads naturally to the notion of determinant of an anti-automorphism, a notion which previously has been known in the special case of involutions and which has been of great importance there. The study of asymmetries also leads to considering involutions on algebras which are not simple nor even semi-simple, but for which a Skolem-Noether type theorem could be established in the split case (Cortella, joint work with Tignol, Louvain-La-Neuve).

The study of algebraic groups also includes reductive groups over finite fields and their representation theory, and progress has been made towards a confirmation of a conjecture of Lusztig concerning character sheaves in the case of the group $SL_n(\mathbf{F}_q)$ (Bonnafé). Also, a conjecture of Broué predicting a Morita equivalence of geometric origin between certain blocks of finite reductive groups has been proved. (Bonnafé, joint work with R. Rouquier, Paris VII). Finally, the Kazhdan-Lusztig cells have been calculated for certain parameters in type B (Bonnafé, jointly with L. Iancu).

In Milnor's K-theory, the p-symbol length (p a prime number) of fields has been studied (i.e. the smallest n such that each element in K_2F/p can be written as a sum of $\leq n$ symbols). In particular, the best upper bounds for fields with $|F^*/F^{*p}| = p^n < \infty$

have been obtained (Hoffmann, joint work with K. Becher, Konstanz).

Publications

[BBR] D. Bessis, C. Bonnafé, R. Rouquier : Quotients et extensions de groupes de réflexion, Math. Ann. 323 (2002), 405-436

[BR] C. Bonnafé, R. Rouquier : Categories derivées et variétés de Deligne-Lusztig, Publ. Math. de l'IHES, 55 pages (accepted)

[B1] C. Bonnafé : A note on centralizers of unipotent elements, Italian Journal of Pure and Applied Mathematics, 9 pages (accepted)

[B2] C. Bonnafé : Elements unipotents réguliers des sous-groupes de Levi, Journal Canadien de Mathématiques, 30 pages (accepted)

[B3] C. Bonnafé : Actions of relative Weyl group I, Journal of Group Theory, 40 pages (accepted)

[CT1] A. Cortella, J.-P. Tignol : The asymmetry of an anti-automorphism, J. Pure Applied Algebra 167 (2002), 175-193

[BH] K. Becher, D. Hoffmann : On symbol lengths in Milnor K-theory, preprint, 17 pages, 2002.

[CT2] A. Cortella, J.-P. Tignol : Skolem-Noether for endomorphism algebras of modules over principal ideal domains, preprint, 2002.

[GBM] N. Grenier-Boley, M.G. Mahmoudi : Exact sequences of Witt groups, preprint, 18 pages, 2003.

[HL] D. Hoffmann, A. Laghribi : Quadratic forms and Pfister neighbors in characteristic 2, Preprint No. 103 (Preprint Server "Linear Algebraic Groups and Related Structures", http://www.mathematik.uni-bielefeld.de/lag), 33 pages, 2003 (submitted to Trans. Amer. Math. Soc.)

Team 3: Ghent

The collaboration between the research team of Ghent university (Van Geel, Wylleman) and the university of Minsk (Yanchevskii, Bazyleu, Tikhonov) is concentrated around problems related to the Brauer group and the structure of division algebras over fields of transcendence degree one over different local fields.

A. *Pfister's question on the structure of division algebras over* $\mathbf{R}((t))(X)$.

The question of Pfister asks whether or not central division algebras of exponent 2 over $\mathbf{R}((t))(X)$ which are trivial over all real closures of $\mathbf{R}((t))(X)$ (we called such algebras Ω-algebras) are of index 2. (Pfister conjectures a positive answer. The problem turns up naturally in the study u-invariant of function fields of transcendence degree two over real fields. The determination of this important (quadratic form) invariant of fields is still open.) A case by case study of the ramification divisor of such algebras enable us to prove the existence of quadratic splitting fields (implying that the index is 2) for large classes of algebras. For these classes the quadratic splitting fields are determined explicitly. (The ramification divisor can be considered as product of irreducible polynomials, all have to be sums of squares since we consider Ω-algebras. We divide this divisor up in 3 parts, the product of quadratic irreducible polynomials such that its roots are squares in the root field, the product of quadratic irreducible polynomials such that its roots are non-squares in the root field, and the product of polynomials of 2 power degree. This division leads to the case by case study).
We reported on these partial results in short notes [1],[2], [3].
Using these results B and Yanchevskii were able to extend these results by considering automorphsims of the algebra that change the type of the ramification. During the last visit of the Minsk team to Ghent (November 2002) these methods were explored further and some new ideas following from the discussion formed the bases for the research we are continuing now.
We are preparing larger research articles on this work.

B. *Pythagoras number of function fields of curves over hereditary pythagorian fields.*
Tikhonov and Yanchevskii were able to determine the pythagoras number of function fields of conics over hereditary pythagorian fields. The results are based on the study of unramified quaternion algebras.
During his visit to Ghent in May 2002, Yanchevskii discussed this work and we investigated the possibility of determining the pythagoras number for function fields of elliptic curves as well. The method relied on the explicit knowledge of the Brauer group of the curve therefore we restricted the base field to $\mathbf{R}((t))$ (instead of a general pythagorian field). We were able to calculate the pythagoras number in several cases (including the case where the curve has good reduction). Yanchevskii and Tikhonov finished the remaining cases later. During the visit of the Minsk team in November 2002 we started to work on the problem for hyperelliptic curves. In the case the curves have good reduction (in the sense that the hyperelliptic equation has good reduction), the problem can be solved.

We will report on this work in research notes before bringing it together in a larger research paper.

A question that came up during theses investigations is "how the kernel of the natural map of the Brauer group of $\mathbf{R}((t))(X)$ to $\mathbf{R}((t))(C)$, with C an hyperelliptic curve, depends on the curve. (If the curve is defined over p-adic field a theorem of Roquette and Lichtenbaum says that the cardinality of the kernel equals the index of the curve.). L. Wylleman a research student in Ghent and S. are working out the partial results we obtained.

C. Yanchevskii and Van Geel finished a generalization of their results on the index of hyperelliptic curves over local p-adic fields (Manuscripta math. 96, 317-333, (1998)). The methods to determine the index could be extended to equations of the form $Y^2 = \pi f(X)$ where $f(X)$ is a monic irreducible polynomial over the ring of integers in the local field and such that the root field of $f(X)$ is tamely ramified. In the manuscripta paper strong restrictions on $f(X)$ were needed.
This results was submitted and accepted for publication in the Journal of the Belgian mathametical society, [4].

Publications

[BGY1+] D. Bazyleu, J. van Geel, V. Yanchevskii. One special case of the local Pfister conjecture. Vesti. Nat. Acad. Nauk Belarusi (2002) n 2, 14-16

[BGY2+] D. Bazyleu, J. van Geel, V. Yanchevskii. Ω-algebras over rational function fields with special constant fields. Dokl. Nat. Acad. Nauk Belarusi **46** (2002) n 6, 21-22

[BGY3+] D. Bazyleu, J. van Geel, V. Yanchevskii On the local Pfister conjecture for Ω-algebras with special constant fields. Dokl. Nat. Acad. Nauk Belarusi **47** (2003) n 1, 19-21.

[1] [1]

[VY] Van Geel, Yanchevskii, The index of certain hyperelliptic curves over p-adic fields, to appear Journal of the Belg. Math. Society - Simon Stevin.

Team 4: Almaty

Participants are A. Dzhumadil'daev (Team Leader), S. Abdykassymova, Sh. Ibraev, I. Irgalieva, A. Kungojin, E. Bekmuchamedov.
Results of Almaty team:

- Identities for Jacobian algebras as n-Lie algebras are described. All derivations of Jacobian algebras are found. Two constructions of simple n-Lie algebras by strong n-Lie-Poisson algebras are given. [Dzh-Jac]
- Simple Leibniz algebras of rank 1 are classified. All irreducible modular antisymmetric module with non-trivial Leibniz cohomology should be restricted. [Dzh-Abd] (T4.1.2-3)
- Jacobson formula for right-symmetric algebras in characteristic p is obtained. [Dzh01]
- Factor complex for Lie cohomology and Leibniz cohomology is constructed. [Dzh-Dav]
- Basis for free right-symmetric algebras is constructed. Basis elements can be indexed bu rooted trees. [Dzh-Lof]
- Basis for free Novikov algebras is constructed. Basic elements can be charactersised by Young diagrams, but filling rules are different. [Dzh-Lof]
- Nonsplit extensions of Lie algebras of rank 2 in characteritis p are classified. All cocycles and second cohomology groups are found. [Dzh-Ibr], [Ibr-Tur] (T4.3)
- Second and third cohomologies of nilpotent subalgebras of Cartan Type Lie algebras are calculated. Descriprion of cocycles is given in terms of highest weights of L_0-components and highest vectors in exterior cubs of sl_n-modules and sp_n-modules. [Ut1], [Ut2] (T4.2.1)
- Identities for Novikov algebra under Jordan product are found. Novikov-Jordan algebra is commutative, has no identitiy of degree 3 and satisfies one identity of degree 4. Some classes of simple Novikov-Jordan algebras are constructed. [Dzh-Nov-Jor]
- N-commutators for vector fields algebras are constructed. For $Vect(n)$ N-commutator, an alternating sum of $N!$ compositions of any N vector fields, is once again a vector field if $N = n^2 + 2n - 2$. In particular, divergenecless vector fields on S^2 has nontrivial 5-commutator. [Dzh-N-com]

These results was obtained in collaboration of S. Abdykassymova, S. Ibraev, A. Dzhumadil'daev, K. Uteulieva G.A. Turetaeva with computer assistance of A. Kungojin and E. Bekmuchamedov. By these results was defenced three Ph.D. thesises, S. Abdykassymova [S], S. Ibraev [I], K. Uteulieva [Ut], and one master thesis A. Kungojin [K].

All members of Almaty team except K.Uteulieva and G.A. Turetaeva has received partial salary.

Publications

[Dzh-Abd] S. Abdykassymova, A.S. Dzhumadil'daev, *Leibniz algebras in characteristric p,* Comptes Rendus Acad. Sci. Paris, Série I, **332**(2001), 1047-1052.

[Dzh-Ibr] S. Ibraev, A.S. Dzhumadil'daev *Nonsplit extensions of Lie algebras of rank 2*, Homology, Homotopy and Applications, **3**(2001), No.3

[Dzh-Lof] A.S. Dzhumadil'daev, C. Lofwall, *Trees, free right-symmetric algebras, free Novikov algebras and identities*, Homology, Homotopy and Applications, **3**(2001), No.3 or 4.

[Dzh01] A.S. Dzhumadil'daev, *Jacobson formula for right-symmetric algebras in characteristic p*, Communications in Algebra, **29**(2001), no. 9, 3759-3771.

[Dzh-Dav] A.S. Dzhumadil'daev, A. Davydov *Factor-complex for Leibniz cohomology*, Communications in Algebra, **29**(2001), no.9, 4197-4210.

[Dzh-Jac] A.S. Dzhumadil'daev,*Identities and derivations for Jacobian algebras*, 2002, Contemp. Math., **315**(2002), 245-278.

[Dzh-Nov-Jor] A.S. Dzhumadil'daev, *Novikov-Jordan algebras,* Commun. Algebra, **30**(2002), No.11, 5207-5240.

[Dzh-N-com] A.S. Dzhumadil'daev, *N-commutators on vector fields,* Nuclear instruments and methods in physics research, section A, 2003. to appear.

[Dzh-Hadamar] A.S. Dzhumadil'daev, *On Hadamard inverse of a matrix,* to appear Expositiones Mathematicae, 2003.

[Dzh-Rep] A.S. Dzhumadil'daev, *Representations of n-Lie algebras*, 2003, submitted.

[Dzh-Wron] A.S. Dzhumadil'daev, *Wronskians as n-Lie multiplications,* 2003, submitted.

[Ut1] Kerimbaev R. Uteulieva K.*Trivialnye homologii tret'ego poryadka podalgebry $\mathcal{L}_1$ kontaktnoi algebry Lie*, Izvestia Nation. Acad. Kazakhstan, ser. fiz-mat.- 2001, No. 1.- Str. 50-55.

[Ut2] Uteulieva K.N. *O gomologiyah tretei stepeni dlya nilpotentnoi podalgebry $\mathcal{L}_1$ special'noi algebry Lie*, Matem. zhurnal, 2002, t.2, N1, str. 94-102.

[Ibr-Tur] S.S. Ibraev, G.A. Turetaeva,*2-kogomologii coprisoedinennogo modulya algebr Lie tipa A_n* , Matem.zhurnal, t.2, N1 2002, str. 43-48

[S] *PhD thesis* S.Abdykassymova, Rank one Leibniz algebras in characteristic p, October 2001 (advisor: A. Dzhumadil'daev) (in Russian)

[I] *PhD thesis* S.Ibraev, Nonsplit extensions and cohomologies of modular Lie algebras of classical type, October 2001 (advisor: A. Dzhumadil'daev)(in Russian)

[Ut] K. Uteulieva, Thret'i gomologii stepeni 3 dlya algebr Lie kartanovskich tipov, march, 2002 (advisor: A.S. Dzhumadil'daev) (in russian).

[K] *Diplom thesis* A. Kungojin, n-Lie algebras, June 2001 (advisor: A. Dzhumadil'daev) (in Russian)

Team 5: Minsk

It is proved that index of Ω-algebras over rational function field over real formal power series field with special ramification is equal to 2 (see [BGY1], [BGY2], [BGY3]).
It is proved that the reduced Whitehead group of a division algebra D, the reduced unitary Whitehead group and Whitehead Spinor group over a field of virtual cohomological dimension 2 are trivial. For corresponding algebraic groups the groups of R-equivalence classes are computed (see [Y]).
Let K be a field of zero characteristic, $Br\ K$ its Brauer group. The bounds of indices of elements of $Br\ K$ as function of their exponent are obtained for function fields of projective spaces and projective curves defined over $P_{n,r}$-fields (see [TY]).
Pythagoras numbers of function fields of conics defined over hereditarily-pythagorean fields are computed (see [TY1]).
It is proved that for a surface X with $p_g = 0$ its motive $M(X)$ is finite dimensional if and only if the Chow group of 0-cycles of X is finite dimensional in the sense of Mumford, i.e. iff the Bloch Conjecture holds for X. (see [GP1], [GP2]).

Publications

[BGY1] D. Bazyleu, J. van Geel, V. Yanchevskii. One special case of the local Pfister conjecture. Vesti. Nat. Acad. Nauk Belarusi (2002) n 2, 14-16

[BGY2] D. Bazyleu, J. van Geel, V. Yanchevskii. Ω-algebras over rational function fields with special constant fields. Dokl. Nat. Acad. Nauk Belarusi **46** (2002) n 6, 21-22

[BGY3] D. Bazyleu, J. van Geel, V. Yanchevskii On the local Pfister conjecture for Ω-algebras with special constant fields. Dokl. Nat. Acad. Nauk Belarusi **47** (2003) n 1, 19-21.

[Y] V. Yanchevskii. Whitehead groups and groups of R-equivalence classes of linear algebraic groups of non-commutative classical type over some virtual fields Algebraic Groups and Arithmetic (Mumbai), Tata Inst. Fund. Res. Studies in Math. (to appear).

[TY] S. V. Tikhonov, V. I. Yanchevskii. Indices of central simple algebras over function fields of projective spaces defined over $P_{n,r}$-fields. Mat. Sbornik. 2002. **193** n 11. P.125-138.

[TY1] B. È. Kunyavskii, L.H. Rowen, S.V. Tikhonov, and V.I. Yanchevskii. Bicyclic algebras over function fields (accepted in Trans. Amer. Math. Soc.).

[TIY1] S. V. Tikhonov, V. I. Yanchevskii. Pythagoras numbers of function fields of conics defined over hereditarily-pythagorean fields. Dokl. Nat. Acad. Nauk Belarusi (to appear).

[GP2] V. Guletskii, C. Pedrini. Motivic finite dimensionality and 0-cycles on algebraic surfaces. (In Russian). Submitted in the Proceedings of the Belarus Acad. of Sciences

[GP1] V. Guletskii, C. Pedrini. Finite dimensional motives and the Conjectures of Murre and Beilinson. Preprint 2003, http://www.math.uiuc.edu/K-theory/0617/

Team 6: Tbilisi

Results:
Essential results concerning task T6.1 and T6.2 are published in [PR], [Prop], [BJ]. See also the talk of Pirashvili on Bourbaki seminare about development on functor homology (task T6.1). Gubeladze made a significant progress in the K-theory of toric varieties (task T6.3). In particular he showed that contrary to some conjectures the Grothendieck groups of locally trivial sheaves and coherent sheaves on such varieties rationally are NOT isomorphic [G2]. He also made a progress on the so colled nilpotency conjecture on K-theory of toric variaties [G1]. Results concerning task T6.4 are in [KS1],[KS2]. Publications:
Papers published during this period:

Publications

[BJ] H.-J. Baues, M. Jibladze. Classification of abelian track categories. K-Theory 25 (2002), no. 3, 299–311.

[BG] W.Bruns, J.Gubeladze, Polytopal linear retractions. Trans. Amer. Math. Soc. 354 (2002), no. 1, 179–203.

[CLP] J. M. Casas, J.-L. Loday, T. Pirashvili, Leibniz n-algebras. Forum Math. 14 (2002), no. 2, 189–207.

[CELP] J.M. Casas, G. Ellis, M. Ladra, T. Pirashvili, Derived functors and the homology of n-types. J. Algebra 256(2002), 583–598.

[JP] M.Jibladze, T.Pirashvili. On Kan fibrations for Maltsev algebras. Georgian Math. J. 9 (2002), no. 1, 71–74.

[KP] R. Kurdiani, T. Pirashvili, A Leibniz algebra structure on the second tensor power. J. Lie Theory 12 (2002), no. 2, 583–596.

[Prop] T. Pirashvili, On the prop corresponding to bialgebras. Cah. Topol. Gom. Diffr. Catg. 43 (2002), no. 3, 221–239.

[P] T. Pirashvili, Polynomial functors over finite fields (after Franjou, Friedlander, Henn, Lannes, Schwartz, Suslin). Sminaire Bourbaki, Vol. 1999/2000. Astrisque No. 276 (2002), 369–388.

[PR] T. Pirashvili, Richter, B. Hochschild and cyclic homology via functor homology. K-Theory 25 (2002), no. 1, 39–49.

[G1] J. Gubeladze. The nilpotence conjecture in K-theory of toric varieties. Preprint

[G2] J. Gubeladze. Toric varieties with huge Grothendieck group. Preprint.

[J] M. Jibladze. Linear extensions and nilpotence for Maltsev theories. arXive: math.CT/0203084.

[KS1] T. Kadeishvili, S. Sanebilidze. The twisted Cartesian model for the double path space fibration. Preprint

[KS2] T. Kadeishvili, S. Sanebilidze. Cochain operations defining Steenrod $\cup_i$-products in the bar construction. Preprint

[P1] T. Pirasvili. Vanishing line for the descent spectral sequence. Arch. Math. (To appear).

[P2] T.Pirashvili. Sets with two associative operation. arXive:math.CO/0301248

Team 7: St. Petersburg

a) (Suslin) It was shown in [SU] that the second terms of the Grayson spectral sequence (converging to K-theory)and the Motivic Cohomology spectral sequence introduced by Bloch-Lichtenbaum coincide. This gives a new, more clear look at the construction of the Motivic cohomology spectral sequence and may serve as a powerful tool for the investigation of the Motivic cohomology and K-theory (T7.2)
b) (Panin, Yagunov) Suslin's famous rigidity theorem was extended by I.Panin and S.Yagunov to all orientable cohomology theories on schemes (T7.6) [PY1]. As a further development of the introduced technique they constructed Bekker-Gottlieb transfer maps for nonorientable representable cohomology theories on schemes. This enabled them to generalize the rigidity theorem to the class of all representable theories [YA]. For instance, this proves validity of the rigidity theorem statement for such important functors as Hermitian K-theory and higher Witt functors.
c) The technique developed for the solution of task (T7.6) may be successfully used for a much wider class of problems in Algebraic Geometry, which can be reformulated in topological terms. In particular, I.Panin obtained a far generalization of the Riemann-Roch theorem [PA], for cohomology theories on schemes. I.Panin and S.Yagunov [PY2] generalized such a classical topological result as a Poincaré duality theorem to the case of orientable cohomology theories on schemes.
d) I.Panin, jointly with P. Balmer, S.Gille, and C.Walter [BGPW] (all are not in the INTAS) proved Gersten conjecture (T7.4) for Witt groups of regular local rings. Panin and Zainoulline [PZ], using the geometric representation lemma of Ojanguren–Panin, received a general form of the Gersten resolution for étale cohomology.
e) N.Gordeev and E.Ellers (not in the INTAS) made the following contribution [GE] to task (T7.8). Let $G = \widetilde{G}(K)$ where $\widetilde{G}$ is a simple and simply-connected algebraic group that is defined and quasi-split over a field K. Properties of intersections of Bruhat cells with conjugacy classes were investigated, in particular, the question, when is $B\dot{w}B \cap C \neq \emptyset$. (where C is a conjugacy class of G and $B\dot{w}B$ is a Bruhat cell of G). For groups of classical type we find conditions for conjugacy classes and Bruhat cells under which they intersect $B\dot{w}B$ nontrivially.

Publications

[SU] A.Suslin. On the Grayson Spectral Sequence. (preprint) `www.math.uiuc.edu/ K-theory/0588/` 2002.

[PY1] I.Panin, S.Yagunov. Rigidity for Orientable Functors Journal of Pure and Applied Algebra Vol.172(2002), 49–77.

[YA] S.Yagunov. Rigidity II non-Orientable case. preprint (preprint) `www.math.uiuc.edu/ K-theory/0610/` 2002.

[PA] I.Panin. Riemann-Roch theorem for oriented cohomology (preprint) `www.math.uiuc.edu/ K-theory/0552/` 2002.

[PY2] I.Panin, S.Yagunov. Poincare Duality for Algebraic Varieties. (preprint) `www.math.uiuc.edu/ K-theory/0576/` 2002.

[BGPW] P.Balmer, S.Gille, I.Panin, and C.Walter. The Gersten conjecture for Witt groups in the equicharacteristic case. Documenta Math. 7 (2002), 203-217.

[PZ] I.Panin, K.Zainullin. Variations on the Bloch-Ogus theorem, Prepublications de l'IHES, Mathematiques-16, 2002.

[GE] N.L.Gordeev, E.W.Ellers. Intersection of Conjugacy Classes with Bruhat Cells in Chevalley groups, 25 p. submitted to Group Theory 2002.