Representations of quivers over the algebra of dual numbers.

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Abstract: The representations of a quiver Q over a field k (the kQ-modules, where kQ is the path algebra of Q over k) have been studied for a long time, and one knows quite well the structure of the module category $\operatorname{mod} kQ$. It seems to be worthwhile to consider also representations of Q over arbitrary finite-dimensional k-algebras A. The lecture will draw the attention to the case when $A = k[\epsilon]$ is the algebra of dual numbers (the factor algebra of the polynomial ring $k[T]$ in one variable T modulo the ideal generated by T^2), thus to the Λ-modules, where $\Lambda = kQ[\epsilon] = kQ[T]/\langle T^2\rangle$.

The algebra Λ is a 1-Gorenstein algebra, thus the torsionless Λ-modules are known to be of special interest (as the Gorenstein-projective or maximal Cohen-Macaulay modules). They form a Frobenius category $\mathcal{L}$, thus the corresponding stable category $\underline{\mathcal{L}}$ is a triangulated category. Actually, this category $\mathcal{L}$ is the category of perfect differential kQ-modules and $\underline{\mathcal{L}}$ is the corresponding homotopy category. As we will see, the homology functor $H\colon \operatorname{mod}\Lambda \to \operatorname{mod} kQ$ yields a bijection between the indecomposables in $\underline{\mathcal{L}}$ and those in $\operatorname{mod} kQ$ and the kernel of H is a finitely generated ideal of the category $\mathcal{L}$ which will be described explicitly.

This is a report on joint investigations with Zhang Pu (Shanghai).