Multiplicative Bases. Bautista's proof of Brauer-Thrall II relies on the existence of a special basis for minimal representation-infinite algebras: a multiplicative Cartan basis. The existence proof for such a basis is the main aim of the second paper which we have to mention here: the joint work of Bautista, Gabriel, Roiter and Salmer´on with the title Representation-finite algebras and multiplicative bases, which has appeared in the journal Inventiones Mathematicae [B-S]. We assume again that Λ is a k-algebra with k an algebraically closed field. In addition, we require that Λ is basic: this means that the factor algebra Λ of Λ modulo its radical is a product of copies of k (this additional requirement can always be achieved by replacing Λ by a Morita equivalent algebra Λ_0; note that for Morita equivalent algebras, the module categories are equivalent). We say that a k-basis of Λ is a **multiplicative Cartan basis** provided the following three conditions are satisfied:

(1) If b_1, b_2 belong to B, then b_1b_2 is either zero or belongs to B,
(2) B contains a complete set B‘ of orthogonal primitive idempotents.
(3) The non-idempotent elements of B generate the radical rad Λ of Λ.

If B is a multiplicative Cartan basis, let B“ be the set of all non-idempotent elements of B. Then it is easy to see that B is the disjoint union of B‘ and B“, that the non-zero elements in $(B")^t$ form a basis of $(\text{rad }\Lambda)^t$, for t = 1,2,... , and that for any element $b \in B$, there are idempotents $e_1, e_2 \in B$ such that $b = e_1be_2$. The latter conditions just means that the basis B consists of homogeneous elements with respect to the Cartan decomposition $\Lambda = \sum_{i,j} e_i \Lambda e_j$ (this is the reason why we prefer to call such a basis a Cartan basis, in contrast to the terminology of the paper which speaks of a "normed" multiplicative basis). The multiplicativity property (1) asserts that one deals with a combinatorially defined algebra; however in applications also the remaining properties turn out to be of great importance: after all, any group algebra has, for trivial reasons, a basis with the multiplicative property (1), but usually will not have a multiplicative Cartan basis.

The main result of [B-S] asserts that any representation-finite, as well as any minimal representation-infinite algebra, has a multiplicative basis. The paper is quite long, however it still is very condensed: the main method used is the socalled cleaving procedure, a sort of partial covering theory, and in the numerous applications of this method usually only the main ingredients are provided, whereas the actual verifications are left to the reader.

In the Mathematical Reviews, I wrote in 1987: The result has a rather long history ... The general result was announced by Roiter in 1981, however his proof was incomplete and partly incorrect. The first complete seems to have been given by Bautista in his lectures at U.N.A.M. (Mexico) in the spring of 1983, but was not published. Since Bautista's proof was based on the ideas of Roiter, the result may be referred to as a theorem of Roiter and Bautista. Gabriel complained about this comment and insisted that he also had a complete proof at the same time, thus I wrote a corresponding addendum for the Reviews.

I remember very well the spring 1983: I was visiting U.N.A.M. at that time and listened to the lectures of Raymundo. What he presented was clearly his and Salmeron's work, of course (as mentioned before) based on the old draft of Roiter. The final paper of 4 authors differs from the preceding version of Raymundo and Salmeron: Various structure theorems deal there with small factor algebras of Λ which have to be considered as being nasty: there are three essentially different kinds, called penny-farthing, dumbbell and diamond, and the essential observation asserts that such factor algebras have only minor overlappings. We should also mention the topological considerations in sections 8 and 10, dealing with the simplicial complex of a ray category and showing the vanishing of a second cohomology group.

The structure theory presented in [B-S] has obviously scared away all other mathematicians: One would expect to find a big variety of papers which are based on this marvelleous investigation. But this is not the case! This is really a pity, for several reasons: one should try to squeeze the arguments in order to obtain a more comprehensive version; one needs corresponding results in the species case (thus working with k-algebras, where k is not algebraically closed); and one should try to understand the module theoretic behaviour in case one deals with slightly larger[4] overlappings of critical parts: when do we still have tameness? when do we get wild clusters, just as islands which can still be separated from the surrounding.

[4] *I should add that Bautista himself stresses the parallel streamlines and the progressive interrelation. He wrote to me: I cannot claim priority on this work. It is true that I had a preliminary version of a proof, but this was on the bases of joint work with Salmer´on, on discussions in Kiev in May 1982 with Roiter and Ovsienko... This version was never published, may-be in the future I will look again at this old manuscript.*