**Internal symmetries of root posets.**

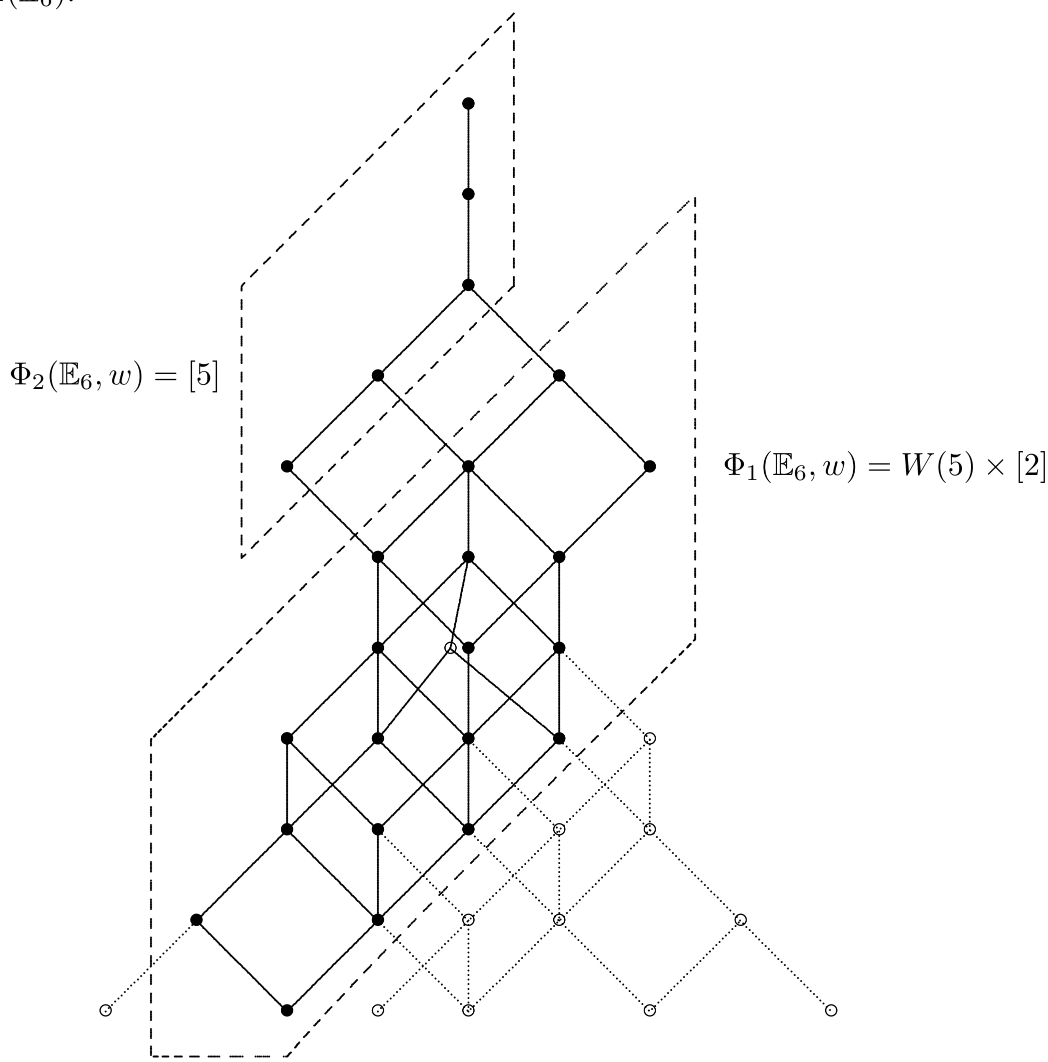
Given a Dynkin diagram $\Delta$, let $\Phi_+(\Delta)$ be the corresponding root poset (its elements are the positive roots, and given positive roots $a, b$, the relation $a \le b$ means that $b - a$ is non-negative). Given a vertex $x$ of $\Delta$ and $t \in \mathbb{N}_1$, let

$$\Phi_t(\Delta, x) = \{a \in \Phi_+(\Delta) \mid a_x = t\}.$$

The posets $\Phi_1(\Delta, x)$ with $\Delta$ simply laced and $x$ a leaf of $\Delta$ will be called *basic lattices.*

**Claim.** *Any poset $\Phi_t(\Delta, x)$ is a product of basic lattices* (thus also self-dual).

Example: Let $\Delta = \mathbb{E}_6$ and $w$ a vertex which is neither the branching vertex, nor a leaf. Here are the posets $\Phi_1(\mathbb{E}_6, w)$ and $\Phi_2(\mathbb{E}_6, w)$ inside the root poset $\Phi_+(\mathbb{E}_6)$.




We denote by $[n]$ the chain of cardinality $n$, it is the only basic lattice which occcurs for $\mathbb{A}_n$. And $W(n) = \Phi_1(\mathbb{D}_n, x)$, where $x$ is the leaf on a short arm of $\mathbb{D}_n$.