Summer term 2023
Prof. Dr. Michael Röckner and Sebastian Grube

Exercises to Introduction to Stochastic Partial Differential Equations I

Sheet 3
Total points: 3+3*
Submission before: Friday, 28.04.2023, 12:00 noon

([Parts of] Exercises marked with "" are additional exercises.)*

Problem 1 (Presentation). (1 extra point for presentation)

Let $(X, ||\cdot||)$ be a Banach space, $\mathcal{B}(X)$ the Borel σ-field of X and $(\Omega, \mathcal{F}, \mu)$ a measure space with finite measure μ. Prepare to present the proof that $(L^1(\mu; X), ||\cdot||_{L^1})$ is a Banach space (cf. p.190 in the script).

Therefore you can adapt the proof of the Riesz–Fischer theorem in, e.g., [Röc21, Theorem 9.12][1].

Problem 2. (3 Points)

Show that the constructed functions $f_m : \Omega \to E$, $m \in \mathbb{N}$, in the proof of Lemma A.1.4. are simple $\mathcal{F}/\mathcal{B}(E)$-measurable functions.

Problem 3 (Presentation). (1 extra point for presentation)

Prepare to present the proof of Proposition B.0.2.

You may use, e.g., [Wer18, from p.309][2] or the mentioned source.

Problem 4 (Presentation). (1 extra point for presentation)

Prepare to present the proof of Proposition B.0.10.

Regarding **Problem 1, 2** and **3**: The presentation should be understandable and concise. Emphasise what exactly needs to be proved and try to identify the critical points in the (given) proofs. Structure the main ideas if possible (e.g. *Step 1: ..., Step 2: ...*). Please give the presentation using the blackboard. Of course, you can use your notes.

Literatur

[Röc21] Michael Röckner. *Maß- und Integrationstheorie.* Lecuture notes, 2021.

[Wer18] Dirk Werner. *Funktionalanalysis.* Springer-Verlag, Berlin, 2018.

[1] available on `https://www.math.uni-bielefeld.de/~roeckner/teaching.html`
[2] available for free within Bielefeld University's network (also via VPN) on `https://rd.springer.com`