

exception of groups of type triality  $D_4$ ), of type  $G_2$  or  $F_4$ , then conjecture 2 holds.

More recently, Colliot-Thélène and Scheiderer made the following "Hasse Principle Conjectures". One says that a field  $k$  has virtual cohomological dimension  $\leq n$ , written  $\text{vcd}(k) \leq n$ , if there exists a finite extension  $k'/k$  such that  $\text{cd}(k') \leq n$ . Let  $\Omega$  be the set of all orderings of  $k$ . For  $v \in \Omega$ , let  $k_v$  be the real closure of  $k$ .

**HP Conjecture 1:**  $\text{vcd}(k) \leq 1$ ,  $G$  connected, then the natural map  $H^1(k, G) \rightarrow \prod_{v \in \Omega} H^1(k_v, G)$  is injective.

**HP Conjecture 2:**  $\text{vcd}(k) \leq 2$ ,  $G$  semisimple, simply connected, then  $H^1(k, G) \rightarrow \prod_{v \in \Omega} H^1(k_v, G)$  is injective.

HP Conjecture 1 was proved by Scheiderer in 1996 (after some partial results by Colliot-Thélène and Dueros). In the case of classical groups and groups of type  $G_2$  and  $F_4$ , HP Conjecture 2 was proved by Parimala and E. B. The proof makes extensive use of the theorem of Merkurjev-Suslin.

## M. ROST

### On algebraic cobordism and the common slot lemma for algebras

An important consequence of the recent work of V. Voevodsky is the following:

**Degree formula:** Let  $X, Y$  be proper smooth varieties over a field  $k$  ( $\text{Char}(k) \neq 0$ ) of dimension  $d = p^n - 1$  ( $p$  a prime,  $n \geq 1$ ). Then for any morphism  $f : X \rightarrow Y$  one has

$$\left( \frac{S_d(X)}{p} \right) = (\deg f) \left( \frac{S_d(Y)}{p} \right) \pmod{I_Y}.$$

Here  $I_Y \subset \mathbb{Z}$  is the ideal generated by the degrees of the closed points on  $Y$ . The characteristic number  $S_d(X) \in \mathbb{Z}$  is given by  $S_d(X) = Q_d(c_1(TX), \dots, c_n(TX))$  where  $Q_d$  is the  $d$ -th Newton polynomial. It is known (Milnor) that  $S_d(X) \in p\mathbb{Z}$ .

**Corollary 1:**  $\frac{S_d}{p} \in \mathbb{Z}/I_X$  is a birational invariant of  $X$ .

**Corollary 2:** If  $I_Y \subset p\mathbb{Z}$  and  $S_d(X) \notin p^2\mathbb{Z}$ , then  $\deg f$  is prime to  $p$ .

We discussed an application of Corollary 2 to the common slot lemma for cyclic algebras of degree  $p$ .

A major problem is to compute the number  $S_d(X)$  for certain  $X$ . Here one uses equivariant resolution of singularities and a theorem of Conner-Floyd on fixed point free  $(\mathbb{Z}/p)^n$ -actions.

## J.-P. SERRE

### On a formula of Kac and a theorem of Burnside

Let  $G$  be a semisimple algebraic group over a field  $k$  of characteristic 0. Assume  $G$  is of adjoint type. Let  $g \in G$  be an element of  $G$  of finite order  $m$ , and  $Z_G(g)$  its centralizer.

**Theorem:** One has  $\dim Z_G(g) \geq l + 2 \sum_{i=1}^l \left\lfloor \frac{d_i - 1}{m} \right\rfloor$ , where  $l = \text{rank}(G)$  and the