ON THE PROOF OF THEOREM DN

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`This text is a complement to the lectures at IAS, 2005.`

Theorem DN follows from the Conner-Floyd theorem and Morel/Levine [3].

As for Morel/Levine, I refer here to Proposition 13.22, Theorem 13.23 and Theorem 13.24 in [3]. Let X and Y be as in Theorem DN and denote by $[X]$, $[Y] \in L$ their (topological) cobordism classes.

Then Theorem 13.24 *obviously* implies Theorem DN in the special case when $[X] = [Y]$.

In fact, one needs less: It suffices that $[X] - [Y]$ is contained in some ideal of the Lazard ring which is generated by p and some elements $x_i \in L$ with $0 < \dim x_i < d$ where $d = p^n - 1$ (and $\dim X = rd$). This is so, because the numbers $t_{d,r}$ vanish *obviously* on such an ideal.

However the assumptions on the action of $G = p^n$ on X and Y, together with the argument using multi-fold connected sums along the fix points, plus the Conner-Floyd theorem, show that $[X] - [Y]$ is contained in such an ideal.

The last argument works in the oriented bordism theory (for odd p) or in the complex bordism theory.

The Conner-Floyd theorem in oriented bordism theory is perhaps bit simpler. There is a complete proof in [2]. However it works only for odd p and it shows only that $[X] - [Y]$ is contained in such an ideal in the oriented bordism ring $\pi_*(\mathrm{MSO})$. However, one has

$$\pi_*(\mathrm{MSO}) \otimes \mathbf{Z}[\frac{1}{2}] = L/L_{\mathrm{odd}} \otimes \mathbf{Z}[\frac{1}{2}]$$

where L_{odd} is the ideal generated by odd-dimensional elements ([5]). It follows that all the numbers $t_{d,r}$ factor through $\pi_*(\mathrm{MSO})$, because $d = p^n - 1$ is even.

The Conner-Floyd theorem in complex bordism theory is proven in [4]. In order to see why the "Conner-Floyd conjecture" considered by Mitchel implies the Conner-Floyd theorem as we used it, you have to consult [1].

References

[1] P. E. Conner and E. E. Floyd, *Differentiable periodic maps*, Academic Press Inc., Publishers, New York, 1964, Ergebnisse der Mathematik und ihrer Grenzgebiete, N. F., Band 33.

[2] E. E. Floyd, *Actions of $(Z_p)^k$ without stationary points*, Topology **10** (1971), 327–336.

[3] M. Levine and F. Morel, *Algebraic cobordism I*, preprint, 2002, http://www.math.uiuc.edu/K-theory/0547.

[4] S. A. Mitchell, *A proof of the Conner-Floyd conjecture*, Amer. J. Math. **106** (1984), no. 4, 889–891.

[5] R. E. Stong, *Notes on cobordism theory*, Princeton University Press, Princeton, N.J., 1968, Mathematical notes.

Date: March 6, 2005.

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