Floer Homology and the Seiberg–Witten Equations (SoSe 2023)

— Homework Sheet 1 —

(due on Friday, April 14)

Grading system: *There will be two exercises per week and 4 points available per exercise. A complete solutions give 4 points, almost complete solutions give 3 points, partial solutions with larger gaps give 2 points, promising ideas give 1 point.*

Exercise 1.1 (Asymptotics of Morse trajectories)**.**

Let (f, ξ) be a Morse pair on a closed n–manifold M. Prove the claim from the lecture that every solution $\gamma\colon \mathbb{R} \to M$ of $\dot{\gamma} + \xi(\gamma) = 0$ has limits

$$\gamma(\pm\infty) = \lim_{t\to\pm\infty} \gamma(t) \in \mathrm{Crit}(f).$$

Exercise 1.2 (Another Morse function on the torus)**.**

On $T^2 = \mathbb{R}^2/2\pi\mathbb{Z}^2$ consider the function defined by $f(x, y) = \cos(x) + \sin(y)$. Study the negative gradient trajectories of f with respect to the flat metric on T^2 induced from $\mathbb{R}^2$. Show that $(f, \nabla f)$ satisfies the Morse–Smale condition and compute its Morse–Floer complex.