Floer Homology and the Seiberg–Witten Equations (SoSe 2023)

— Homework Sheet 3 —

(due on Friday, April 28)

Exercise 3.1 (Additivity and multiplicativity of the Conley index)**.**

Let ϕ and ψ be local flows on locally compact metrizable spaces X and Y.

(a) Finish the proof of the additivity property $C(S_1 \amalg S_2, \phi) = C(S_1, \phi) \vee C(S_2, \phi)$ for disjoint isolated invariant sets S_1 and S_2 of ϕ.

(b) Let S and T be compact isolated invariant sets of ϕ and ψ, respectively. Show that $S \times T$ is isolated invariant of the product local flow $\phi \times \psi$ on $X \times Y$ with $C(S \times T, \phi \times \psi) = C(S, \phi) \wedge C(T, \psi)$.

Exercise 3.2 (Poincaré compactification)**.**

Let $A\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map, considered as a vector field on $\mathbb{R}^2$. Consider the map $h^{\pm}\colon \mathbb{R}^2 \to S^2$ by sending $(x, y) \in \mathbb{R}^2$ to the unique intersection point of S^2 with the line segment from $(x, y, \pm 1)$ to the origin. You can take for granted that $h^{\pm}$ are smooth embedding onto the upper and lower hemispheres.

(i) Show that there is a unique vector field ξ_A on S^2 that simultaneously extends $h^+_* A$ and $h^-_* A$, and that the equator is invariant under the flow of ξ_A.

(ii) Describe the orbits of ξ_A for the following choices of A:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

In the cases that the equator is isolated invariant, determine its Conley index with respect to the flows generated by ξ_A and $-\xi_A$.