# Floer Homology and the Seiberg–Witten Equations (SoSe 2023)

## — Homework Sheet 5 —

(due on Friday, May 12)

**Exercise 5.1** (The orbit category)**.**

Let $G$ be a compact Lie group and $H, K \subset G$ two closed subgroups. Determine all $G$–maps $f\colon G/H \to G/K$ where $H, K \subset G$ are closed subgroups.

*(Hint: Write $f(eH) = aK$ with $a \in G$. What can you say about $a$?)*

**Exercise 5.2** (An invariant Morse function)**.**

Let $SO_2$ act on $S^2 \subset \mathbb{R}^3$ by rotation around the $z$–axis. Consider the $SO_2$–invariant functions

$$f\colon S^2 \to \mathbb{R}, \quad f(x, y, z) = 1 - z^2.$$

(a) Show that $\mathrm{Crit}(f)$ is a disjoint union of $SO_2$ orbits and that the Hessian of $f$ is non-degenerate on the normal spaces of critical orbits.

(b) Determine the gradient trajectories of $f$ and identify the $SO_2$–Conley index of all $SO_2$ invariant compact isolated subsets of $S^2$.