Abstract

Combinatorial aspects of tilting theory

Tilting theory plays a vital part in various aspects of the representation theory of artin algebras. Usually it is applied for comparing the module category $\operatorname{mod}\Lambda$ with $\operatorname{mod}\Gamma$ where $\Gamma = \operatorname{End} T$ and T is a tilting module over Λ. More generally tilting objects can be defined in abelian categories $\mathcal{A}$. Not much is known in general in this situation unless $\mathcal{A}$ is a hereditary category. In these talks we plan to give a brief introduction to the general aspects of tilting theory as well as recalling the classification of hereditary categories with tilting objects.

The main purpose of the talks however will lie in discussing the underlying combinatorial structure on the set of all tilting objects in $\mathcal{A}$. We denote by $\mathcal{T}_{\mathcal{A}}$ the set of isomorphism classes of square-free tilting objects in $\mathcal{A}$. For a tiling object $T \in \mathcal{A}$ we set

$$T^{\perp} = \{X \in \mathcal{A} \mid \operatorname{Ext}^i_{\mathcal{A}}(T, X) = 0 \text{ for all } i > 0\}.$$

For $T, T' \in \mathcal{T}_{\mathcal{A}}$ we set $T \leq T'$ provided $T^{\perp} \subset T'^{\perp}$. This gives $\mathcal{T}_{\mathcal{A}}$ the structure of a partially ordered set. We will give a description of the Hasse diagram of this partially ordered set. We will discuss under additional assumptions the local structure of a vertex of the Hasse diagram, as well as the existence of minimal and maximal elements. We will also deal with the question of connectedness of the Hasse diagram.

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