On hereditary algebras

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Hereditary algebras form one of the most intensively studied classes of finite dimensional algebras (see [2] for the present state of knowledge). They are important both inside representation theory and for several applications. Our main result underlines this importance even more.

Theorem 1 *Let A be a finite dimensional algebra over a perfect field. Then A is hereditary.*

The **Proof** is based on a series of lemmas.

Lemma 2 *An algebra of global dimension zero is semisimple. A semisimple algebra has global dimension zero.*
An algebra of global dimension zero or one is hereditary.

Lemma 3 *A quotient of a semisimple algebra is semisimple, hence of global dimension zero. By dimension shift [1], a subalgebra of a semisimple algebra has global dimension zero or one.*

Lemma 4 *A finite dimensional algebra is a subalgebra of a semisimple algebra.*

This finishes the proof of the theorem. ■

Based on extensive computations for group algebras of finite groups we conjecture that even more is true.

Conjecture 5 *An algebra over a field of characteristic zero is semisimple.*

References

[1] H.Cartan and S.Eilenberg, Homological algebra, Princeton University Press (1956).

[2] C.M.Ringel, Towards a representation theory for Dynkin diagrams. Talk at ICRA VIII (1996).

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