APPENDIX

A.1 The periodic additive functions on $\mathbb{Z}\Delta$, with Δ Euclidean

Given a stable translation quiver $\Gamma = (\Gamma_o, \Gamma_1, \tau)$, a function $f : \Gamma_o \longrightarrow \mathbb{Z}$ is said to be an additive function on Γ, provided for every vertex c of Γ, one has

$$f(\tau c) + f(c) = \sum_{b \to c} f(b)$$

(the summation being formed over all arrows ending in c; thus, in case Γ has no multiple arrows, then we just have to sum over all $b \in c^-$). If f is an additive function on Γ, then we define its τ-translate $f \circ \tau$ by $(f \circ \tau)(c) = f(\tau c)$, for $c \in \Gamma_o$. Of course, $f \circ \tau$ is additive, again. Given $t \in \mathbb{N}_1$, an additive function f on Γ is said to be periodic with period t provided $f \circ \tau^t = f$. The set $A(\Gamma)$ of all additive functions on Γ is a subgroup in the group of all functions $\Gamma_o \to \mathbb{Z}$ (with pointwise addition), and the periodic additive functions form a subgroup $A_p(\Gamma)$ of $A(\Gamma)$. Finally, an additive function f on Γ will be said to be positive provided $f \neq 0$, and $f(z) \in \mathbb{N}_o$ for all vertices z.

Now, consider the case $\Gamma = \mathbb{Z}\Delta$, with Δ a finite connected quiver without oriented cycles. Given an additive function f on $\mathbb{Z}\Delta$, let $f_z : \Delta_o \longrightarrow \mathbb{Z}$ be given by the restriction of f to $z \times \Delta_o$, thus $f_z(a) = f(z,a)$ for $z \in \mathbb{Z}$, $a \in \Delta_o$. For a fixed z, we obtain in this way an isomorphism from $A(\mathbb{Z}\Delta)$ onto the group $\mathbb{Z}^{\Delta_o}$ of all integer valued functions on Δ_o, mapping f onto f_z. The following proposition is well-known (for a short proof, we refer to [HPR]).

Proposition. Let Δ be a finite connected quiver without oriented cycles. If there exists a positive, periodic, additive function on $\mathbb{Z}\Delta$, then Δ is Euclidean. Conversely, for Δ Euclidean, $A_p(\mathbb{Z}\Delta)$ is a pure sublattice of $A(\mathbb{Z}\Delta)$ of corank 1, generated by positive functions.

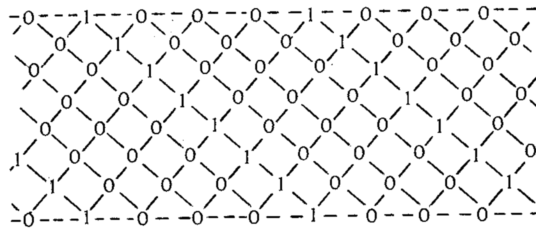
Let Δ be Euclidean, of type $\tilde{\mathbb{T}}_{n_1,\ldots,n_t}$. We want to exhibit t positive additive functions $f_1,\ldots,f_t$, of period $n_1,\ldots,n_t$, respectively, such that these functions f_s, together with their τ-translates $f_s \circ \tau^j$ $(0 \leq j \leq n_s - 1,\ 1 \leq s \leq t)$ generate $A_p(\mathbb{Z}\Delta)$. Actually, we obtain a presentation of $A_p(\mathbb{Z}\Delta)$, using as relations that

$$\sum_{j=0}^{n_s-1} f_s \circ \tau^j = \sum_{j=0}^{n_{s'}-1} f_{s'} \circ \tau^j .$$

Actually, in the cases $\tilde{\mathbb{A}}_n$ and $\tilde{\mathbb{D}}_n$, we just deal with typical special cases:

$\tilde{\mathbb{A}}_{4,3} = \tilde{\mathbb{T}}_{4,3}$

f_1



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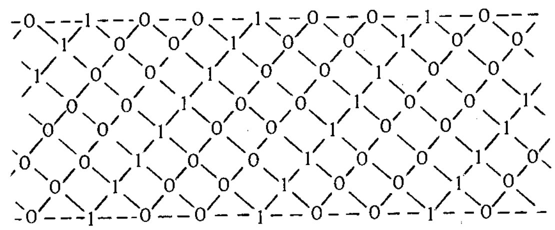
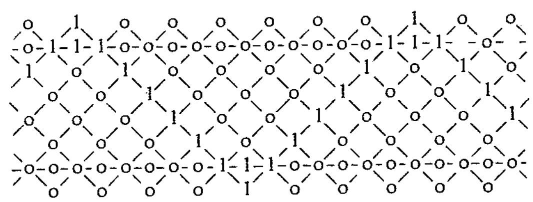


$\tilde{\mathbb{D}}_9 = \tilde{\mathbb{T}}_{7,2,2}$

f_1



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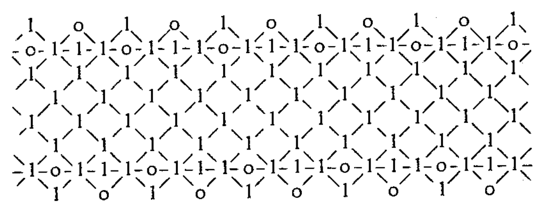

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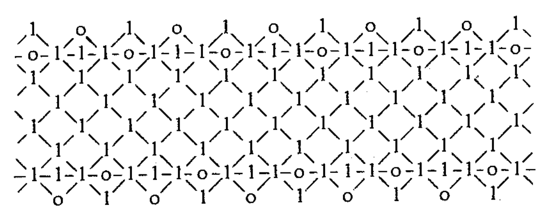
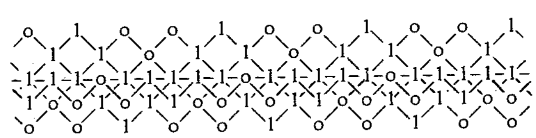


$\tilde{\mathbb{E}}_6 = \tilde{\mathbb{T}}_{3,3,2}$

f_1



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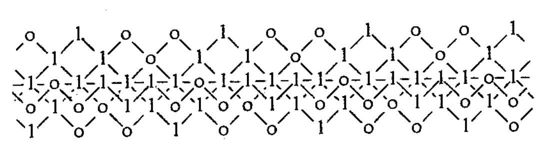

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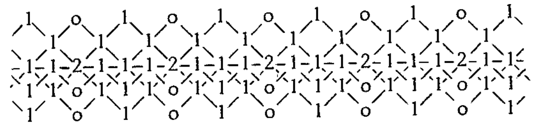
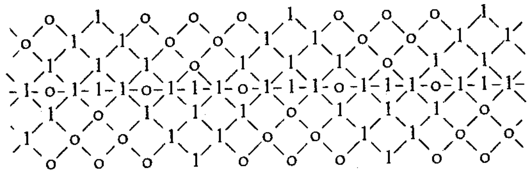


$\widetilde{\mathbb{E}}_7 = \widetilde{\mathbb{T}}_{4,3,2}$

f_1



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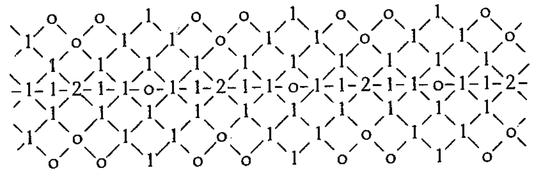

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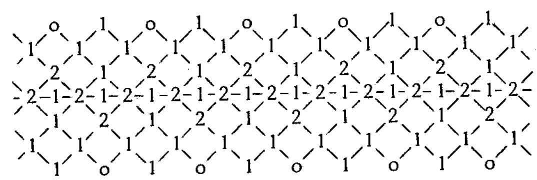
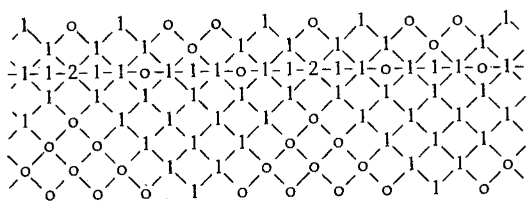


$\widetilde{\mathbb{E}}_8 = \widetilde{\mathbb{T}}_{5,3,2}$

f_1



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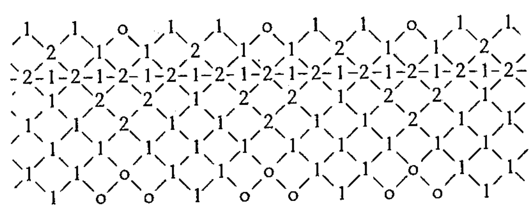

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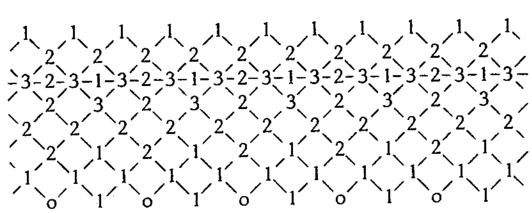
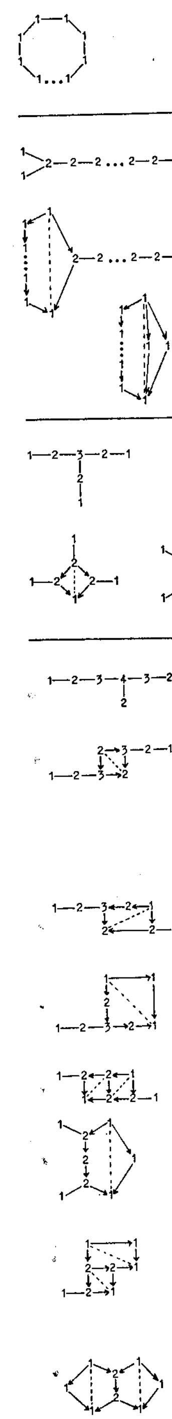
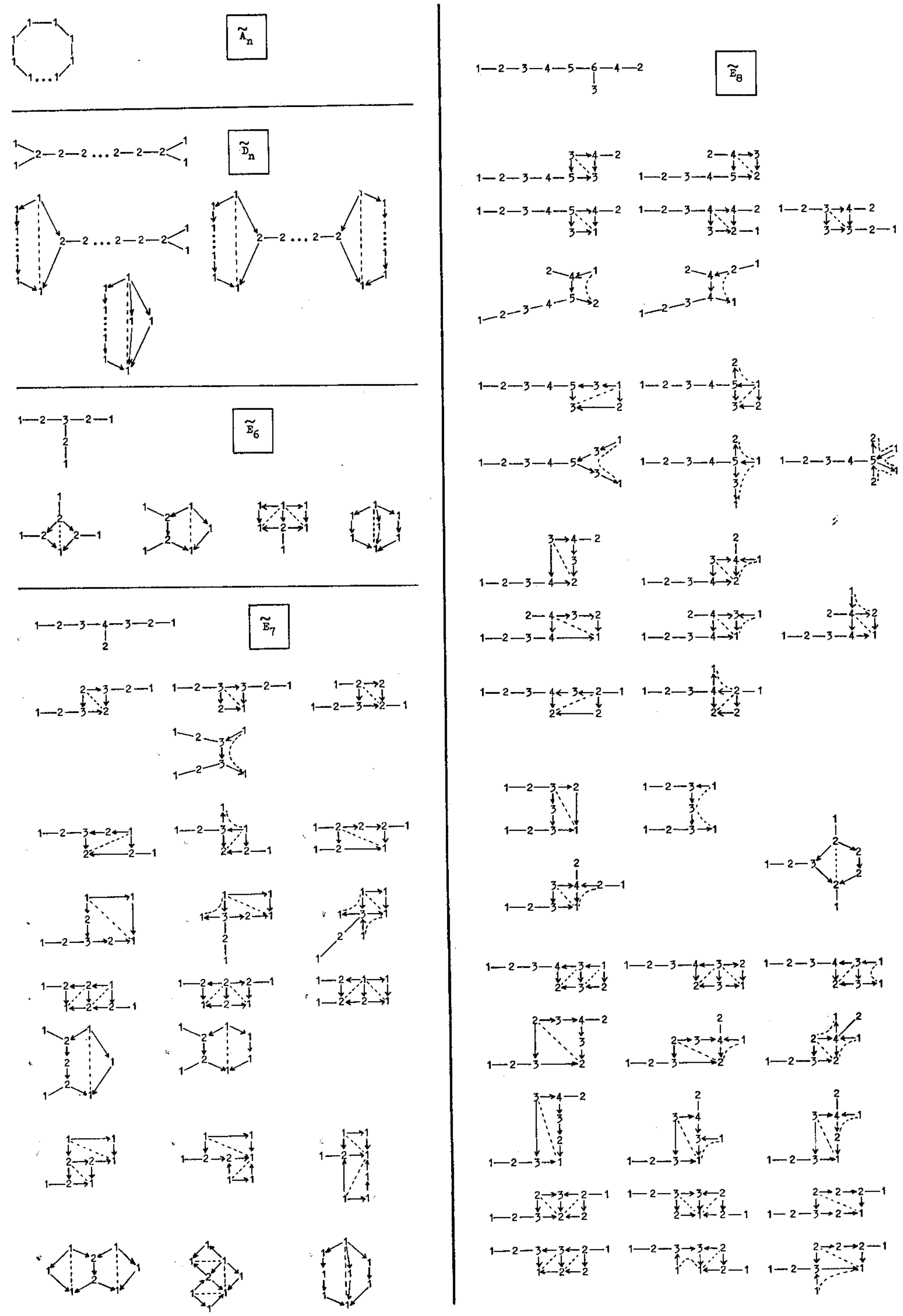
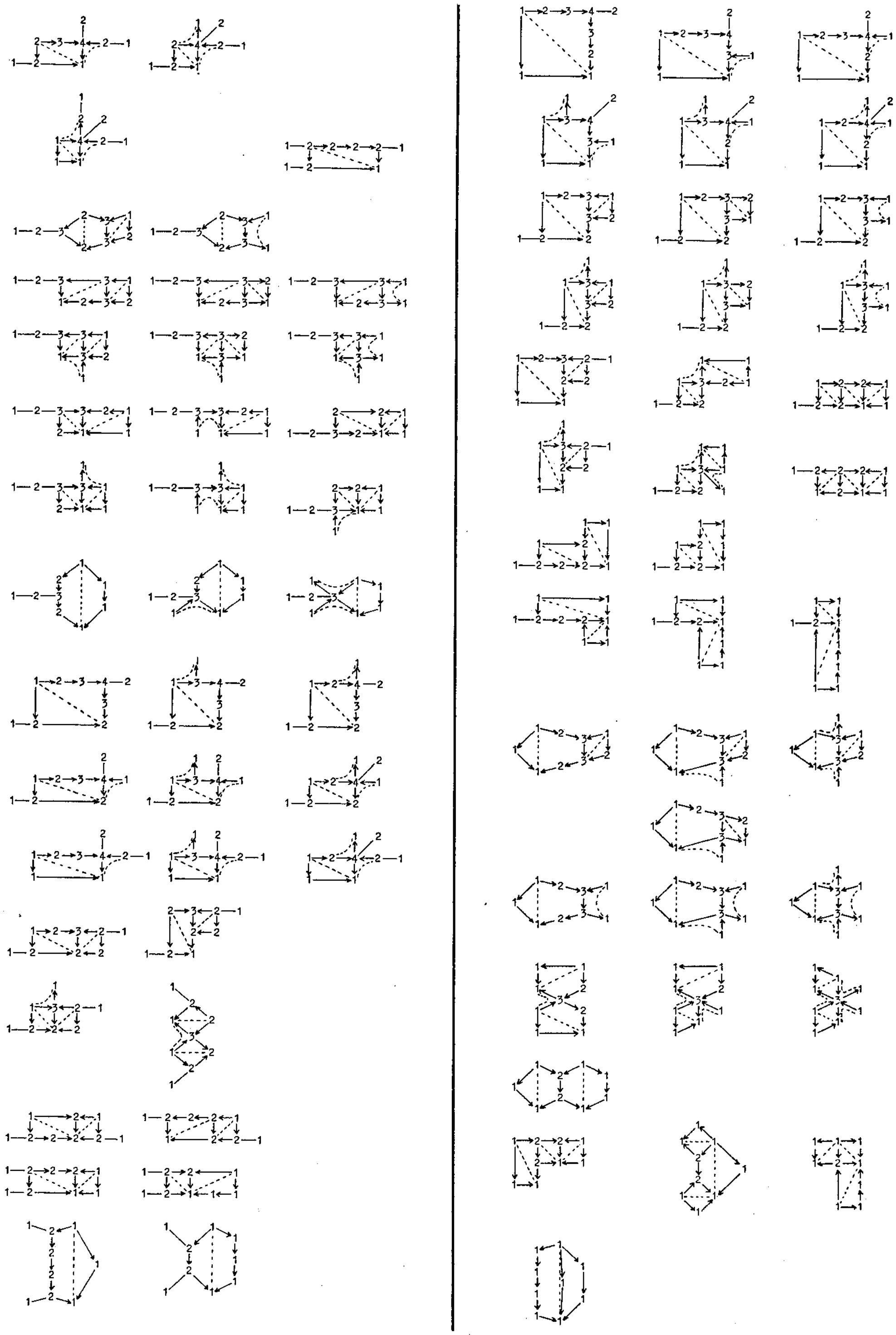


A.2 The frames

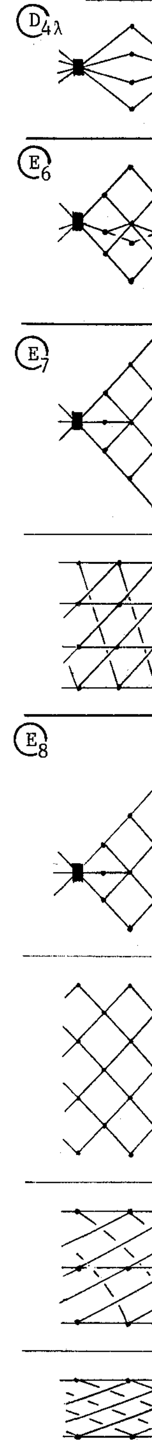


A.2 The frames of the tame concealed algebras (see 4.3.4)

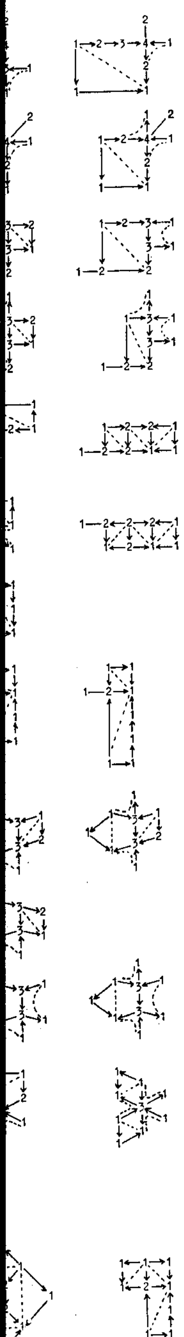




A.3 The tu



A.3 The tubular patterns (see p. 323)



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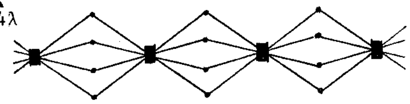

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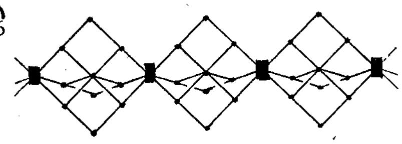
E_6



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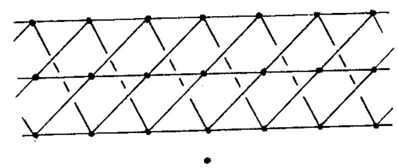

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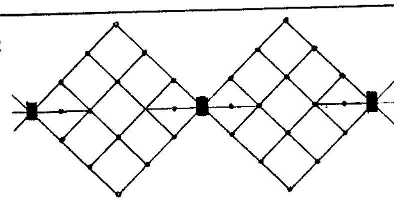
E_7



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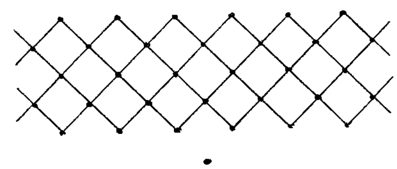

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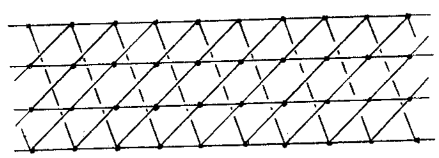

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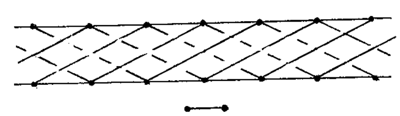

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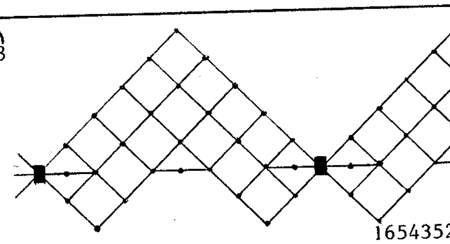
E_8



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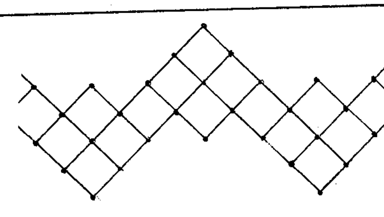

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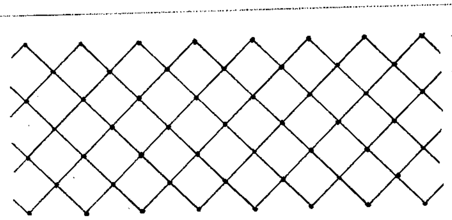

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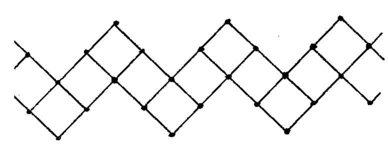

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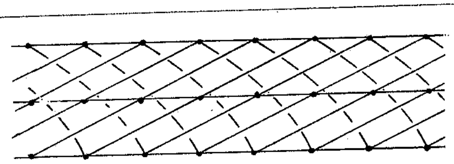

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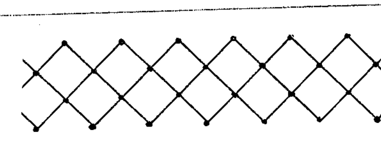

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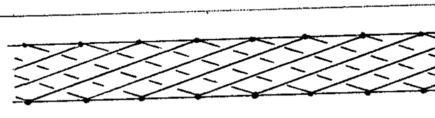

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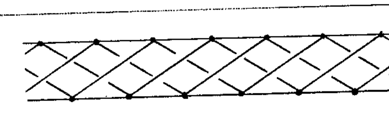

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