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# UNIVERSITÄT

Ergänzungsreihe 93 – 005

**The Gabriel quivers of the  
sincere simply connected algebras**

by

**Axel Rogat and Thomas Tesche**

QA 050  
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## THE GABRIEL QUIVERS OF THE SINCERE SIMPLY CONNECTED ALGEBRAS

- Axel Rogat and Thomas Tesche -

Let  $A$  be an associative finite-dimensional algebra over an algebraically closed field  $k$ .  $A$  is said to be *representation-finite* (or of *finite representation type*), if there is only a finite number of isomorphism classes of indecomposable (finite-dimensional)  $A$ -modules, or, equivalently, if the corresponding Auslander-Reiten-quiver  $\Gamma$  contains only a finite number of vertices.  $A$  is called *representation-directed*, if  $\Gamma$  is finite and contains no oriented cycle.  $A$  is called *simply connected*, if it is connected, *basic* (i.e.  $A/\text{rad}(A)$  is isomorphic to  $k^n$ ), representation-finite, and if  $\Gamma$  is simply connected. In that case, the orbit graph  $O(A)$  of  $A$  is a tree. We call  $A$  *sincere*, if there exists an indecomposable  $A$ -module, where all simple  $A$ -modules occur as composition factors. Every sincere representation-directed algebra is simply connected.

For any quiver a *relation* is a finite linear combination of paths of length  $\geq 2$ . Any basic finite dimensional algebra  $A$  is of the form  $kG/I$  for some uniquely determined finite quiver  $G = G(A)$  and an ideal  $I$  generated by relations whose paths are of globally bounded length.  $G(A)$  is called the *Gabriel quiver* of  $A$ , by  $n(A)$  we denote its number of vertices (i.e. the number of isomorphism classes of simple  $A$ -modules).

All these details are well-known and can be found e.g. in Claus Michael Ringel's book [R].

Simply connected algebras gain their importance from the theory of covering spaces (see e.g. [BoG]). In fact, if a representation-finite algebra  $B$  has a faithful indecomposable module  $U$ , then it admits a Galois covering  $\tilde{B}$  which has an indecomposable module  $\tilde{U}$  whose push-down is  $U$  and whose support is simply connected. These facts are given in [G], [BGRS], [Bo3]. Thus - up to Galois coverings - the supports of the modules  $\tilde{U}$  occurring this way are the blueprints of all indecomposables over all representation-finite algebras.

The treatment of simply connected algebras is suitable to numerical computation, as the isomorphism class of an indecomposable  $A$ -module  $M$  is uniquely determined by its dimension vector  $\underline{\dim}M$ . Moreover,  $M$  is sincere if and only if all components of  $\underline{\dim}M$  are non-zero. Also, if  $M$  is an indecomposable  $A$ -module, the components of  $\underline{\dim}M$  are bounded by 6, which enables us to easily find out the representation type of  $A$ .

In [Bo1], Klaus Bongartz gave a list of 24 infinite families containing all sincere simply connected algebras with  $n(A) \geq 72$  (this bound was refined to 14 in [R]). The inductive method of constructing representation-finite algebras using the notion of graded trees as introduced in [BoG] made it possible for Bongartz to classify the remaining exceptional algebras (i.e. those with  $n(A) \leq 13$ , which do not appear in the list of the so-called regular

algebras), based on a computer program (1982). His results consisted of huge illegible lists of gradings, and they were never published.

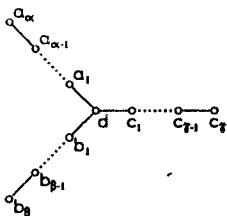
Whereas the structure of the infinite families is easy to understand and of considerable theoretical interest (see e.g. [Bo1], [Bo2], [F]), the exceptional algebras seemed to be an exotic object. Nevertheless, detailed knowledge about them is very useful to answer nasty questions about indecomposable modules.

In order for Peter Dräxler to confirm that any indecomposable of a representation-finite algebra is an amalgamation of indecomposables without multiple composition factors ([Dr2]), he independently classified the exceptional algebras (1987), using the different approach of one-point (co-)extensions as proposed by Ringel in [R]. His list [Dr1] is in accordance with Bongartz's, and it offers numerical combinatorial data determining the so-called frame of  $G(A)$ .

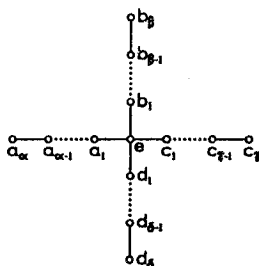
Our purpose was not only to verify the above-mentioned results, but most of all to present the sincere simply connected algebras in graphical form, i.e. by their Gabriel quivers with relations - thereby obtaining the foregoing blueprints -, and to sort them following visual properties. Moreover, using these data, we could answer the question by Christine Riedtmann (in the positive) whether any indecomposable over a simply connected algebra admits a codimension 1 degeneration. We also checked again von Höhne's guess that such an algebra has a unique minimal sincere indecomposable. Whereas C. Riedtmann's problem has been solved in the meantime by theoretical means, the observations of Dräxler and von Höhne still wait for an explanation.

In our computer programs, we chose the method suggested in [BoG]. If  $A$  is a sincere directed algebra,  $O(A)$  is a tree with at most four endpoints. It is easy to show that there is just a small number of trees which are orbit graphs of sincere representation-directed algebras, namely the following types:

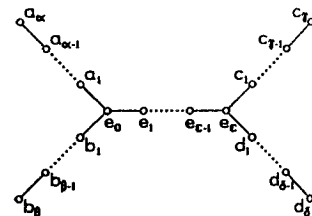
$$(\alpha, \beta, \gamma) \\ (\alpha \geq \beta \geq \gamma)$$



$$(\alpha, \beta, \gamma, \delta) \\ (\alpha \geq \beta \geq \gamma \geq \delta)$$



$$(\alpha, \beta, \gamma, \delta, \epsilon) \\ (\alpha \geq \beta, \gamma \geq \delta, \alpha \geq \gamma)$$



Let us give a short description to recall the inductive method. Given a tree  $T$ , leave out one point  $m$  and examine the obtained smaller trees  $T_i$ . In order to find all representation-finite gradings for  $T$ , adjust the representation-finite gradings to all  $T_i$  and join them via  $m$  as the point of maximal grading (checking the representation-finiteness separately). Unfortunately, a sincere algebra is not necessarily an amalgamation of smaller sincere algebras in the above sense. However, it suffices to generate the data for the greater class of algebras having a vertical section (i.e. the orbit graph occurs in the Auslander-Reiten quiver as points whose gradings differ at most by one). This class is closed under the above construction and contains all sincere algebras. Eventually, of the whole of the constructed algebras, we had to filter out the sincere ones.

This construction suggests sorting the algebras by  $O(A)$  and  $n(A)$  first. Starting with the Gabriel quiver of a sincere simply connected algebra, if you change the direction of an arrow not involved in a relation or of *all* arrows in one and the same relation, the resulting algebra will still be sincere and simply connected. Therefore, we only present the underlying non-oriented graph of each such orientation class. Thus, our subdivision is finer than the one given in [Dr1]. In the following, we will speak of algebras instead of their orientation classes, hoping no one will get confused.

For algebras having the same orbit graph, the following criterions were used:

- number of commutativity relations
- size of the  $i^{th}$  commutativity relation
- length of the  $i^{th}$  commutativity relation
- number of arrows in  $i$  commutativity relations
- number of zero relations
- length of the  $i^{th}$  zero relation
- number of arrows in  $i$  zero relations
- number of arrows in  $i$  relations (commutativity or zero)
- number of vertices with  $i$  neighbours
- number of twigs of type  $A_i$

Using these criterions in successive order, one obtains a few groups of at most four uncomparable algebras. To achieve a linear order, thus separating these groups, we had to apply additional criterions which are difficult to recognize at first sight and lengthy to formulate, and which we do not want to bore you with.

In general, *commutativity relations* are differences of two paths having the same starting and ending points, *zero relations* are given by a single path. For a simply connected algebra, the ideal  $I$  (where  $A = kG(A)/I$ ) is generated by relations of the above two types. Moreover, the number of minimal generators of  $I$  and the starting and ending points of the relations are uniquely determined, whereas there can be several choices for the paths. However, these cases are rare (e.g. (5, 2, 1), no. 700, or (4, 3, 1), no. 80), so that we do not want to specify a precise choice. Given a starting and an ending point,

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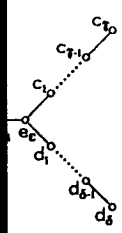
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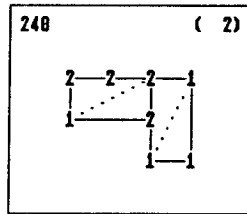
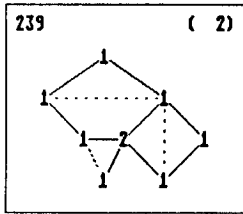


there are at most two commutativity relations or one zero relation between them. The size of a commutativity relation is its number of vertices, the length is the number of vertices in the longer path. For each algebra we consider a 'vector', whose entries are the commutativity relations sorted in lexicographically descending order (following size, length and some of the above mentioned 'obscure' criteria), analogously for the zero relations. All this enables us to talk about 'numbers' of relations and an  $i^{th}$  relation.

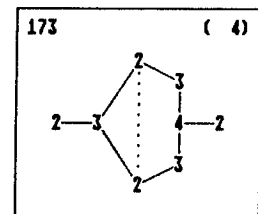
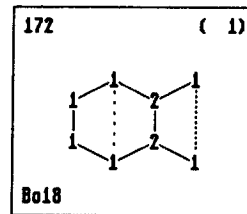
An  $A_n$  twig is a walk starting at a vertex with more than two neighbours, having no branching point and ending in a vertex with only one neighbour.

To illustrate how our criteria work we give some examples of pairs of algebras of type  $(4, 2, 1)$ . In each case we have mentioned the first of our criteria distinguishing the two algebras.

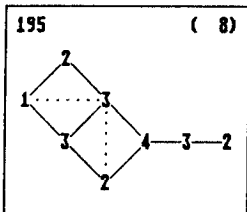
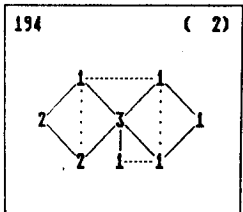
- size of the  $i^{th}$  commutativity relation:



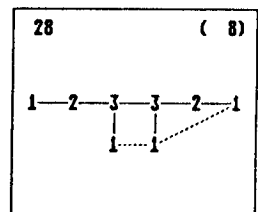
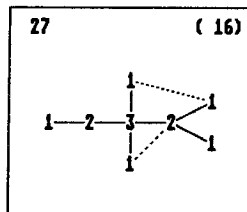
- length of the  $i^{th}$  commutativity relation:



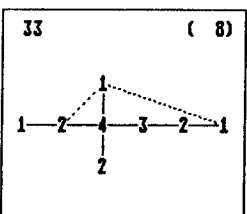
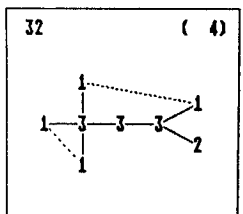
- number of arrows in  $i$  commutativity relations:



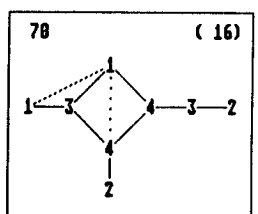
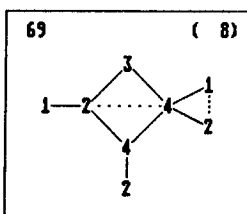
- length of the  $i^{th}$  zero relation:



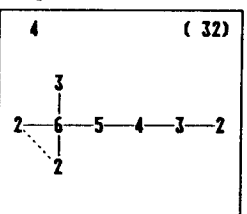
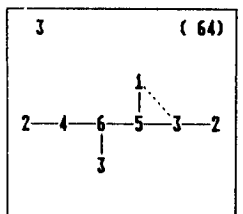
- number of arrows in  $i$  zero relations:



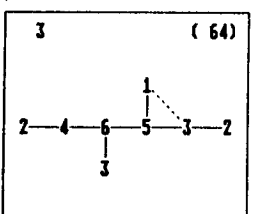
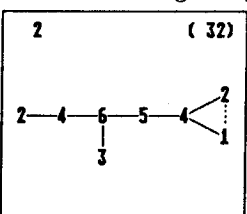
- number of arrows in  $i$  relations:



- number of vertices with  $i$  neighbours:



- number of twigs of type  $A_1$ :



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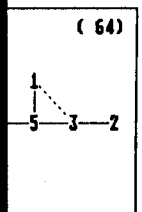
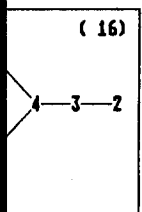
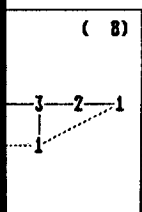
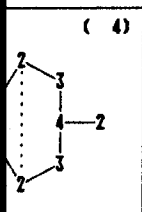
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For each sequence of pictures, the corresponding orbit graph is indicated. You find the continuous numbering in the upper left corner, the number of elements in the orientation class in the upper right. The vertices show a maximal root, i.e. a root with maximal total dimension and among these with a maximal component. If the algebra is regular, this is indicated by the corresponding number (as proposed in [R]) in the lower left corner. Notice that for  $n(A) \leq 9$  some of those families coincide, while some do not appear, so we thought it might be better to list all algebras. For a better distinction, the zero relations are more narrowly dotted than the commutativity relations.

The following table shows the numbers of algebras and orientation classes belonging to the different orbit graphs. The entries have the following format:

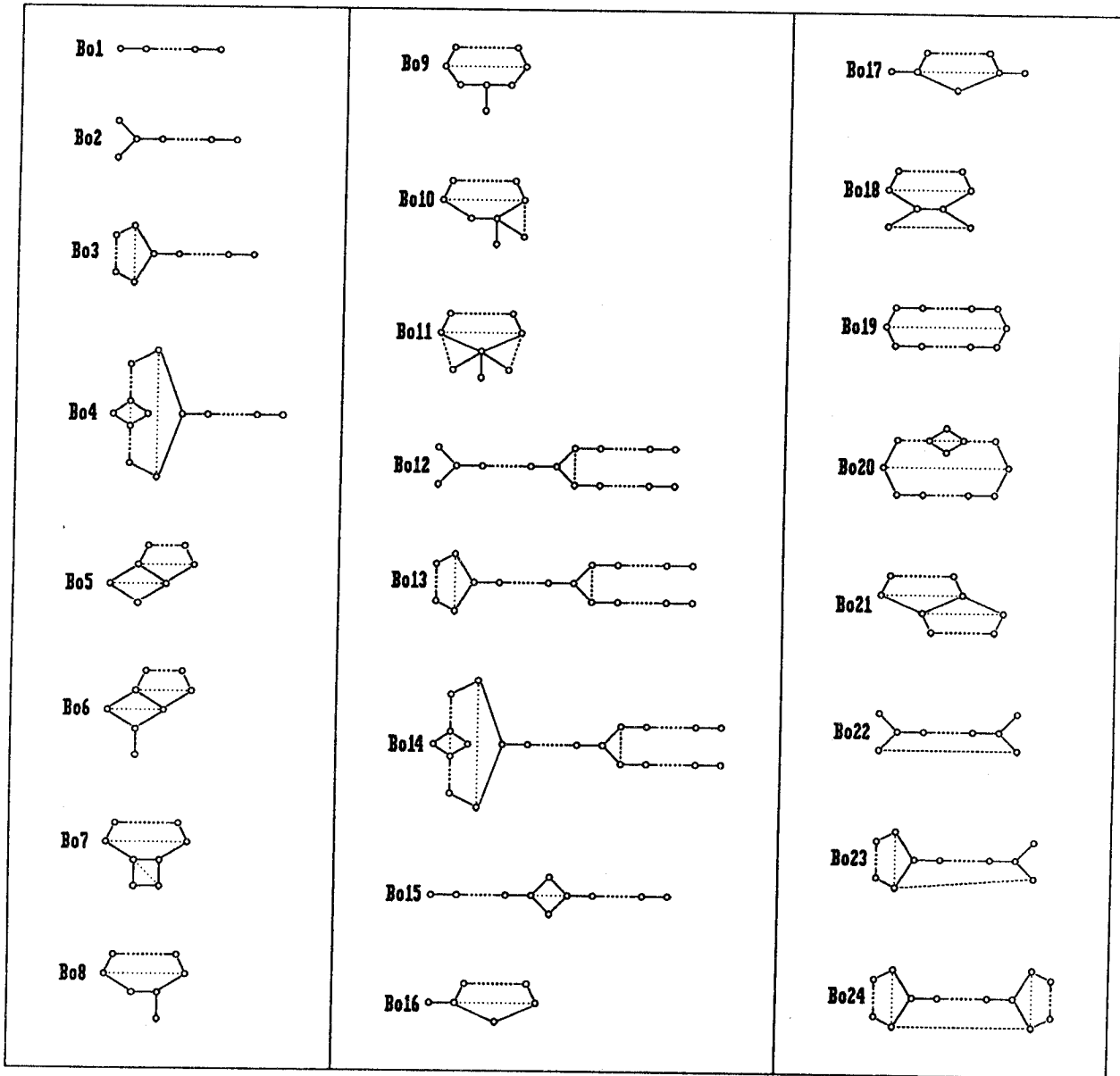
orbit graph  
 number of orientation classes of exceptional algebras / number of orientation classes of regular algebras  
 number of exceptional algebras / number of regular algebras

(2,2,1) 5/7 41/20	(3,2,1) 45/16 380/45	(4,2,1) 235/18 1978/40	(5,2,1) 933/21 5795/56	(6,2,1) 283/25 3746/65	(7,2,1) 30/29 1788/75	(8,2,1) 4/34 832/86	(9,2,1) 1/39 256/98
(3,3,1) 86/2 368/5	(4,3,1) 168/2 697/5	(5,3,1) 13/2 48/5	(4,4,1) 9/1 104/1	(5,4,1) 1/1 16/1	(2,2,2) 9 28	(3,2,2) 18 55	(4,2,2) 25 77
(5,2,2) 1 1	(3,3,2) 1 1	(4,3,2) 1 1	(2,2,1,1) 1 1	(3,2,1,1) 1 1	(4,2,1,1) 1 1	(2,1,1,1,1) 3/1 5/2	(2,1,1,1,2) 8/2 18/3
(2,1,1,1,3) 20/2 59/4	(2,1,1,1,4) 9/3 20/5	(2,1,1,1,5) 3/3 5/6	(2,1,1,1,6) 1/4 1/7	(2,1,2,1,1) 2 3	(2,1,2,1,2) 4 7	(2,1,2,1,3) 2 3	(2,1,2,1,4) 1 1
(3,1,1,1,1) 3/1 6/2	(3,1,1,1,2) 1/2 1/3	(3,1,2,1,1) 2 3	(3,1,2,1,2) 1 1	(4,1,1,1,1) 4/1 7/2	(4,1,1,1,2) 1/2 1/3		

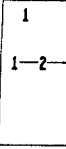
This means there are exactly 16,344 exceptional algebras.

Finally, we give you the list of the 24 families of regular algebras with the numbering suggested by [R]. The corresponding orbit graphs are:

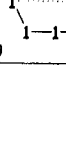
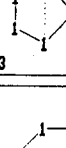
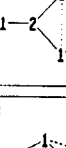
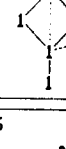
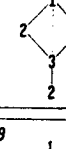
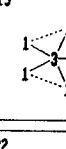
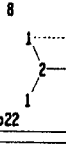
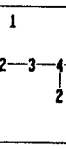
$A_n$	Bo1
$(\alpha, 1, 1)$	Bo2, Bo3, Bo4, Bo12, Bo13, Bo15
$(\alpha, 2, 1)$	Bo5, Bo6, Bo7, Bo8, Bo9, Bo10, Bo11, Bo16, Bo18, Bo21, Bo22, Bo23, Bo24
$(\alpha, 3, 1)$	Bo17
$(\alpha, \beta, 1)$	Bo19
$(1, 1, 1, 1, \epsilon)$	Bo4, Bo14
$(\alpha, 1, 1, 1, \epsilon)$	Bo20 (including $\epsilon=0$ )



(2, 2)



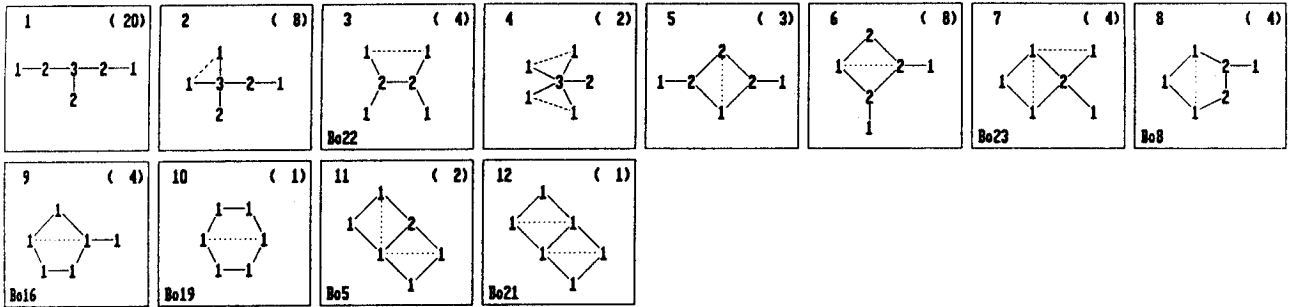
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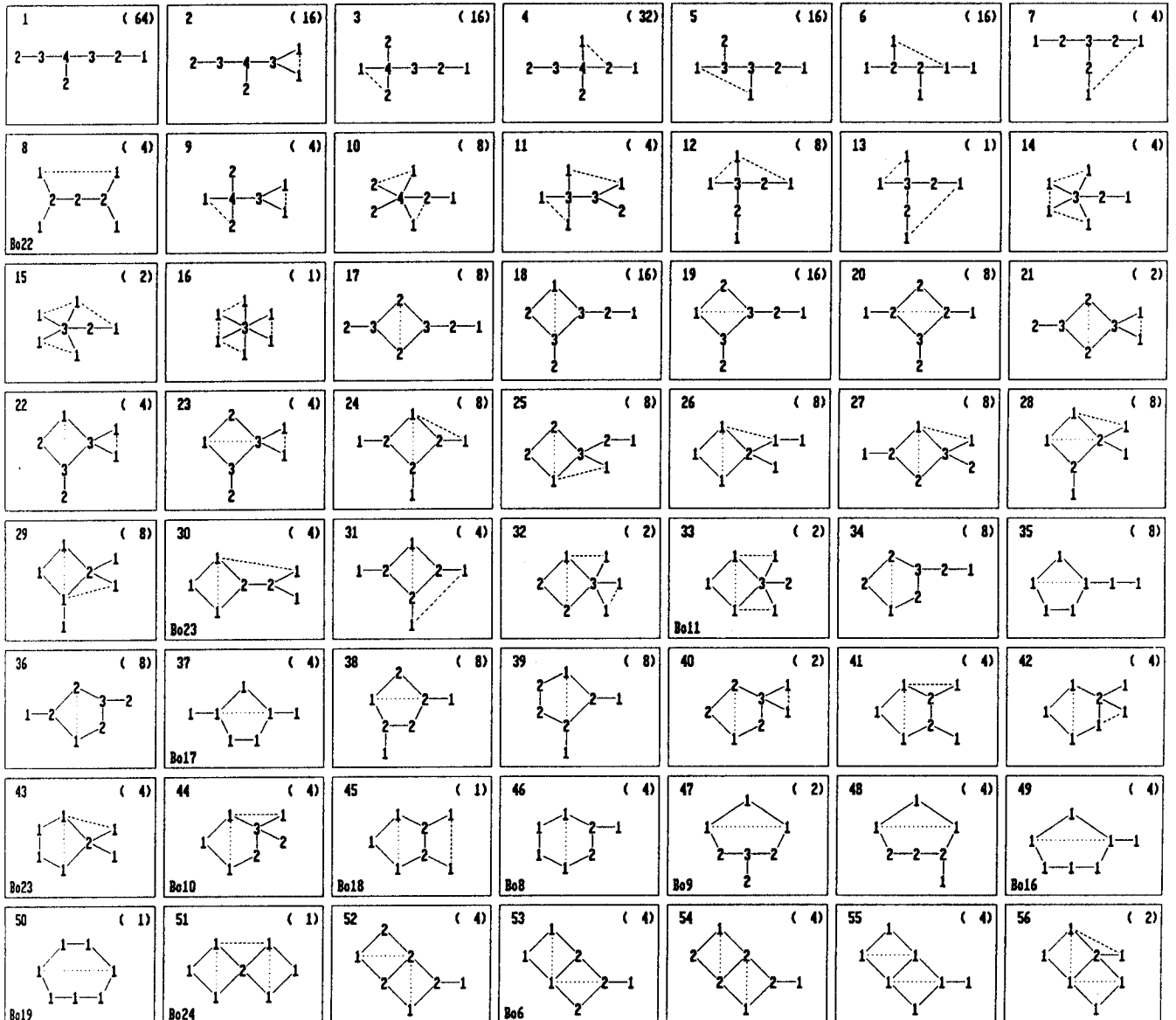
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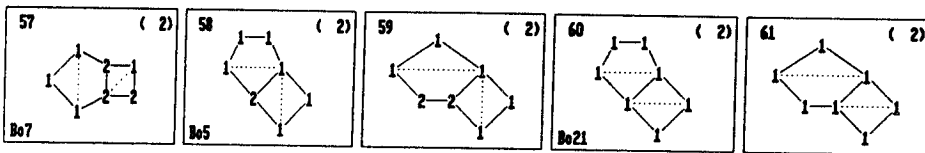
(2, 2, 1)



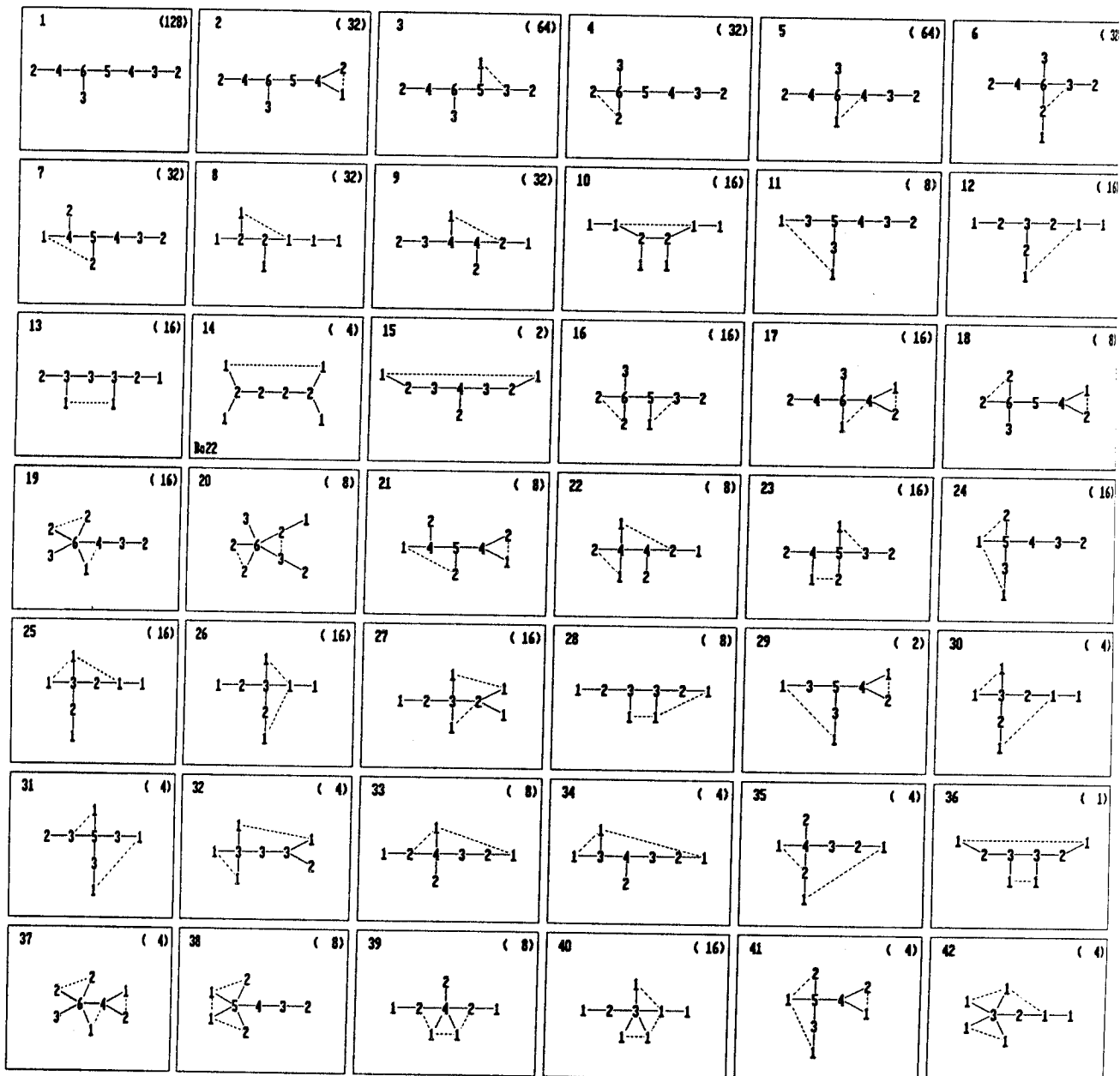
(3, 2, 1)



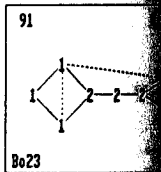
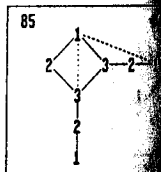
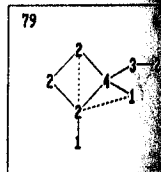
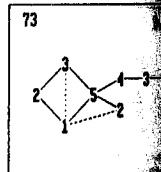
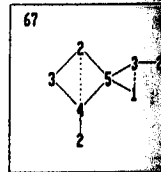
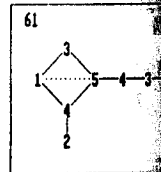
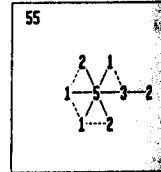
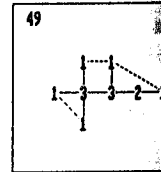
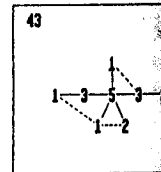
(3, 2, 1)



(4, 2, 1)

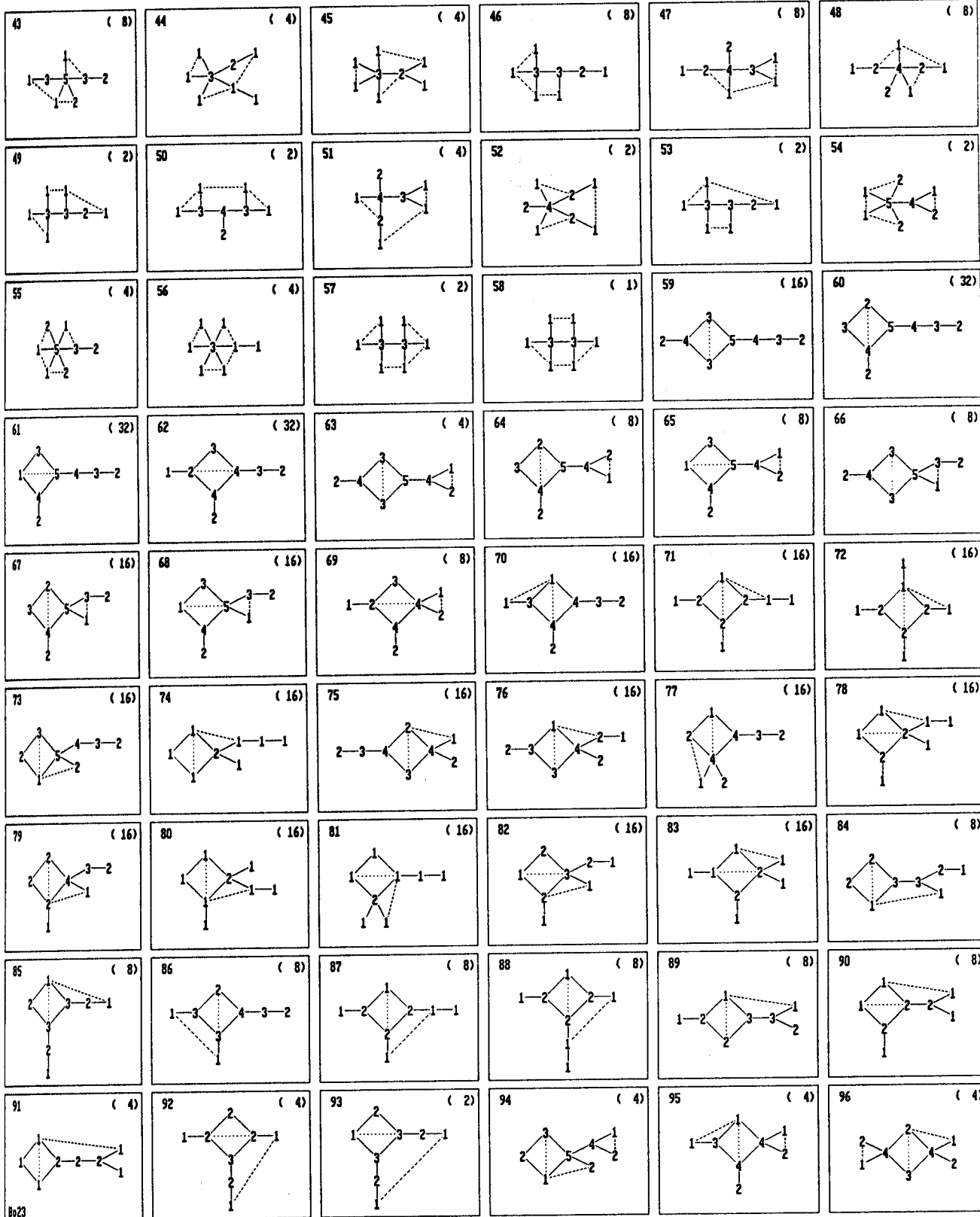


(4, 2, 1)



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(4, 2, 1)



Bo23

6 (32)

12 (16)

18 (8)

24 (16)

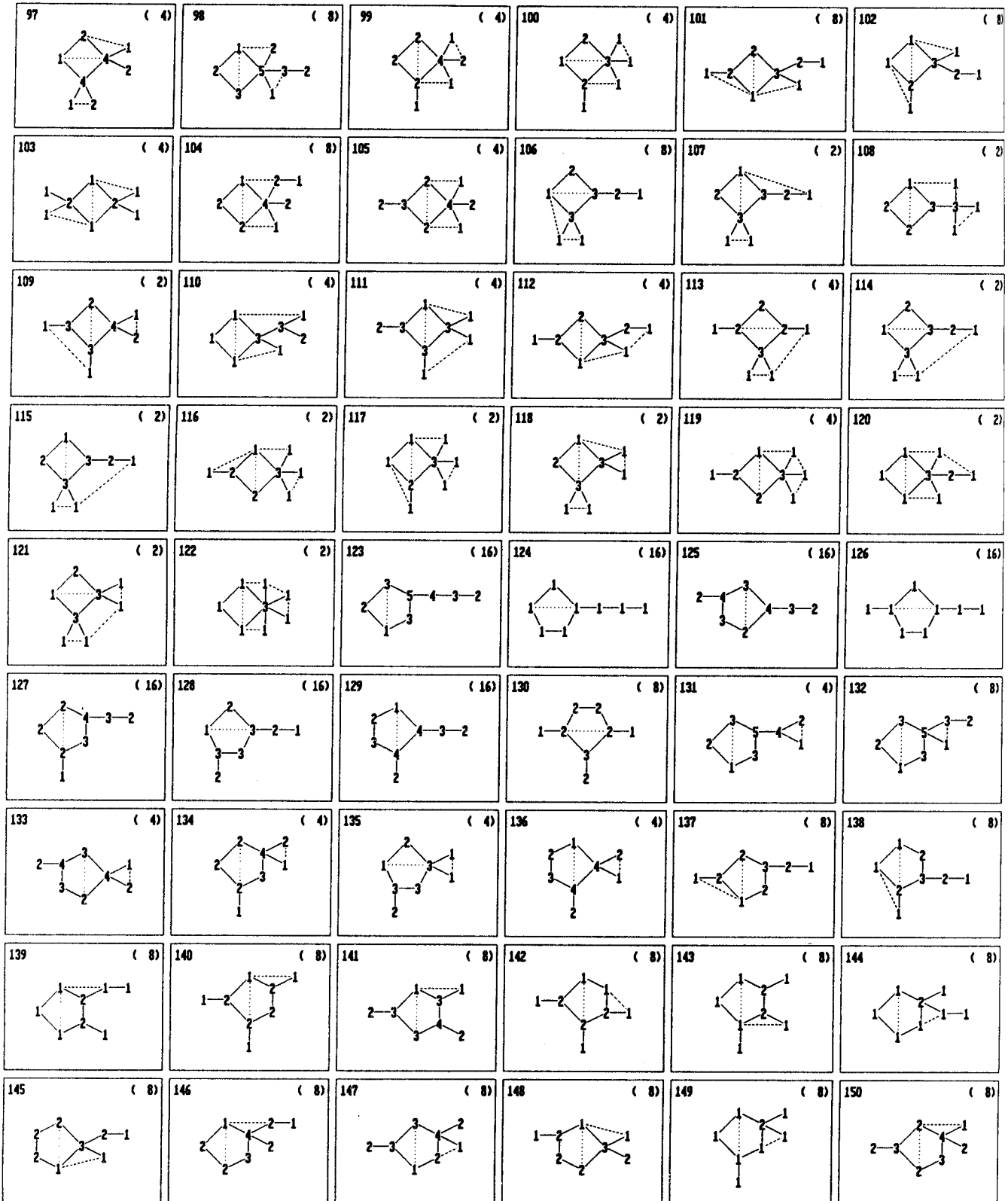
30 (4)

36 (1)

42 (4)

(4, 2, 1)

(4, 2, 1)



151

157

163

Bo11

169

175

181

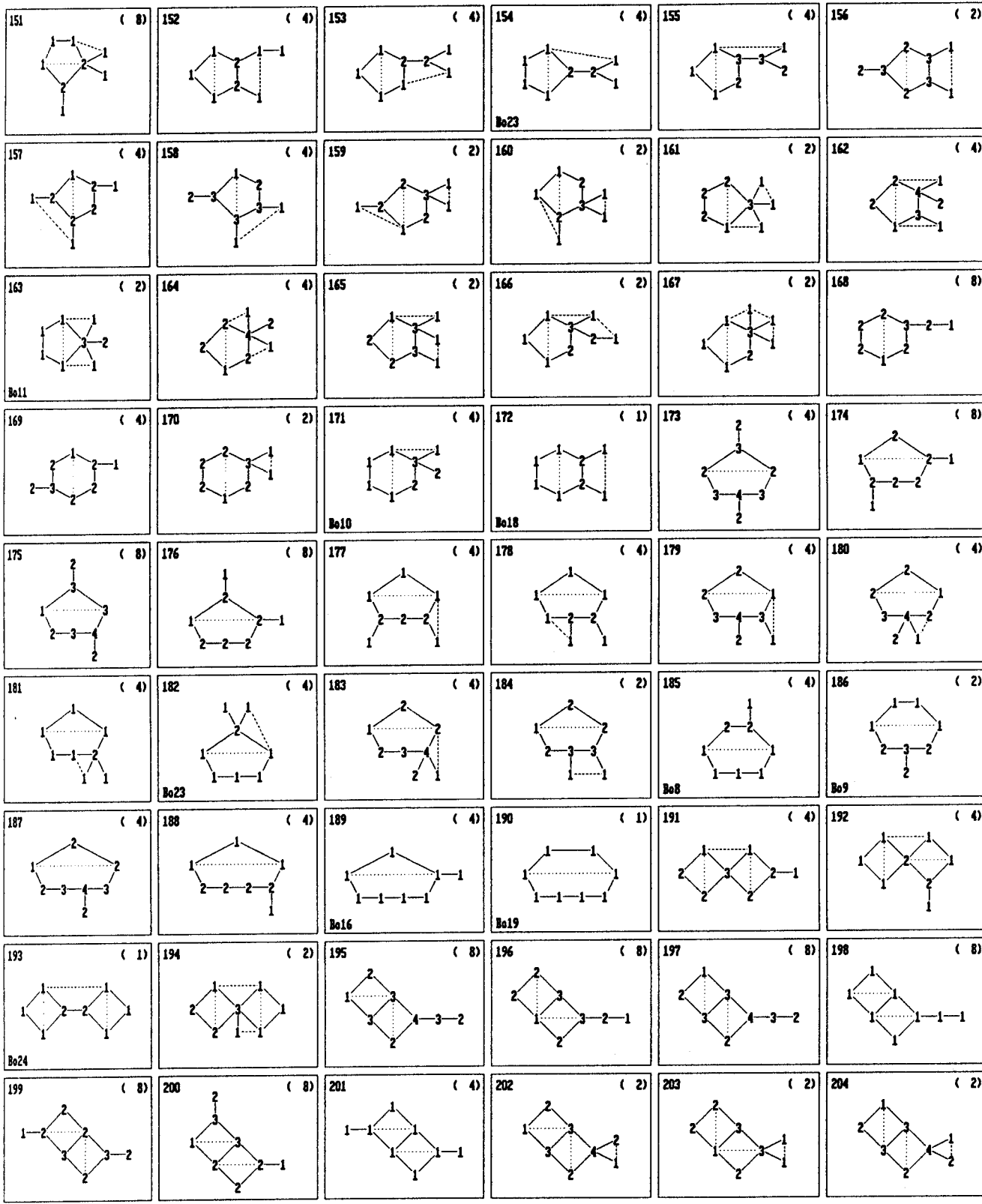
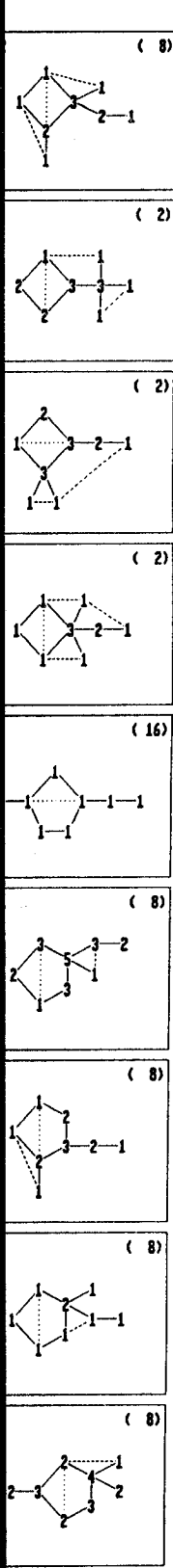
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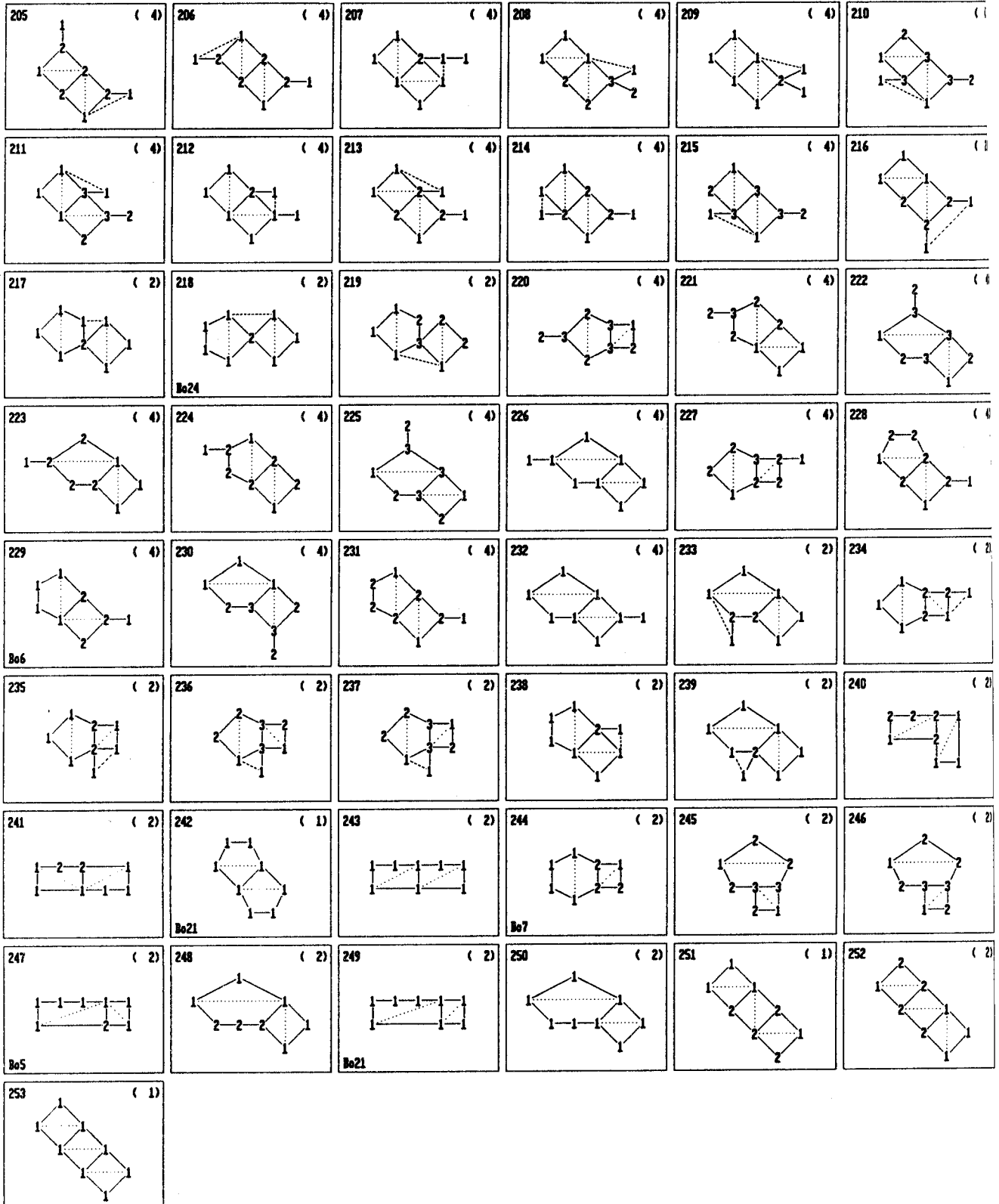
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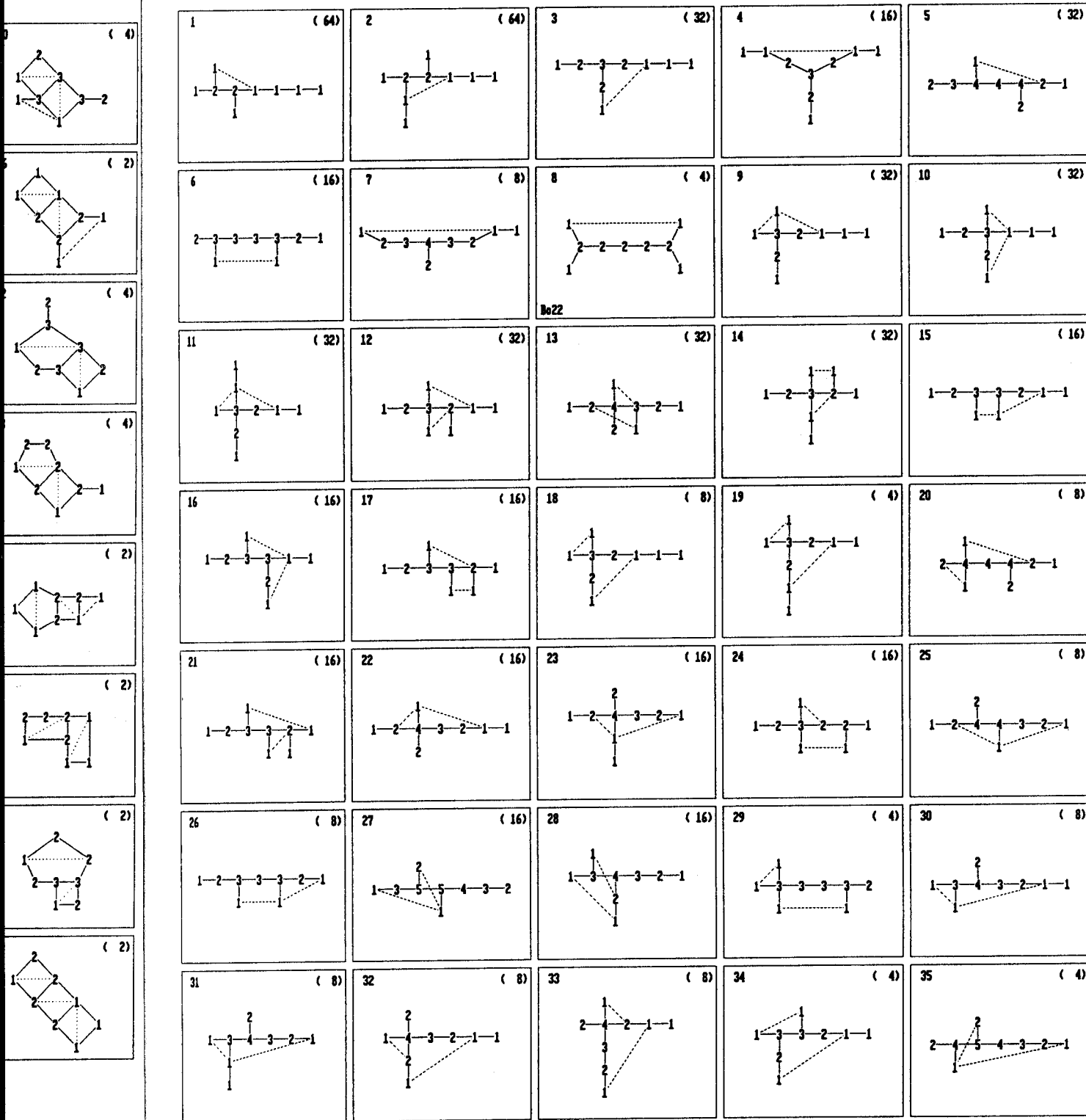
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(5, 2, 1)



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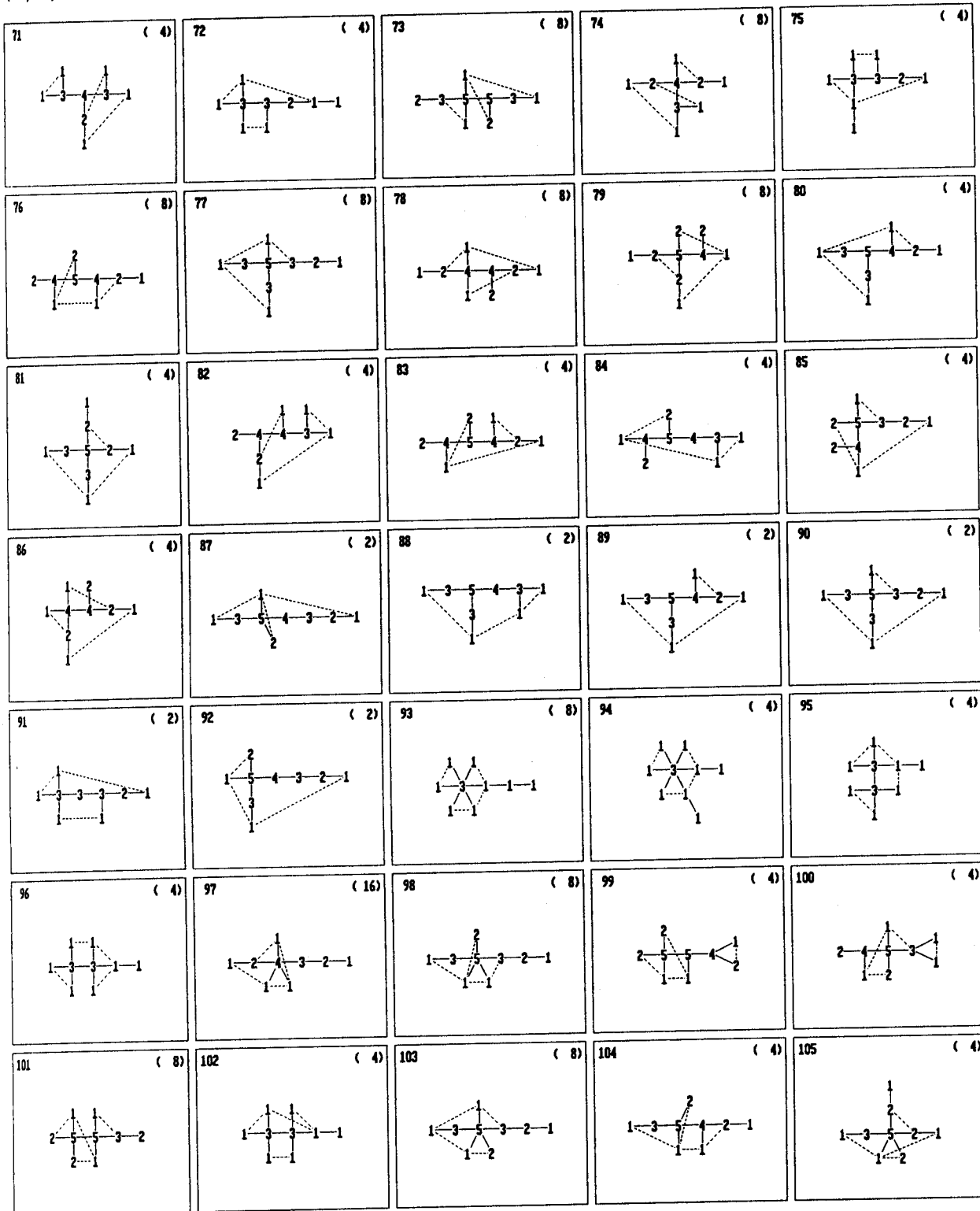
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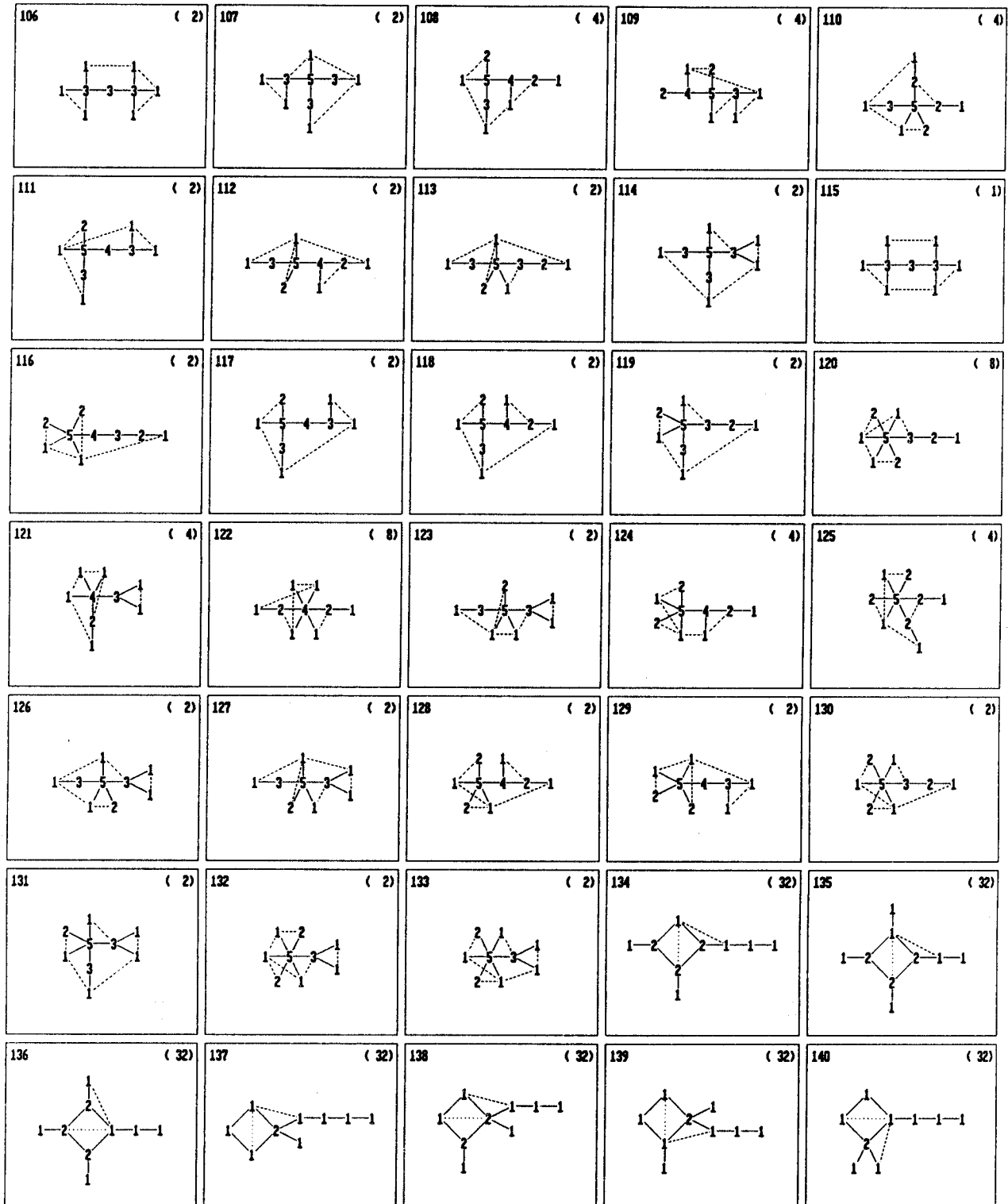
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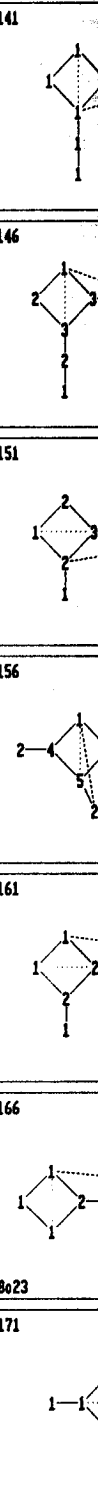
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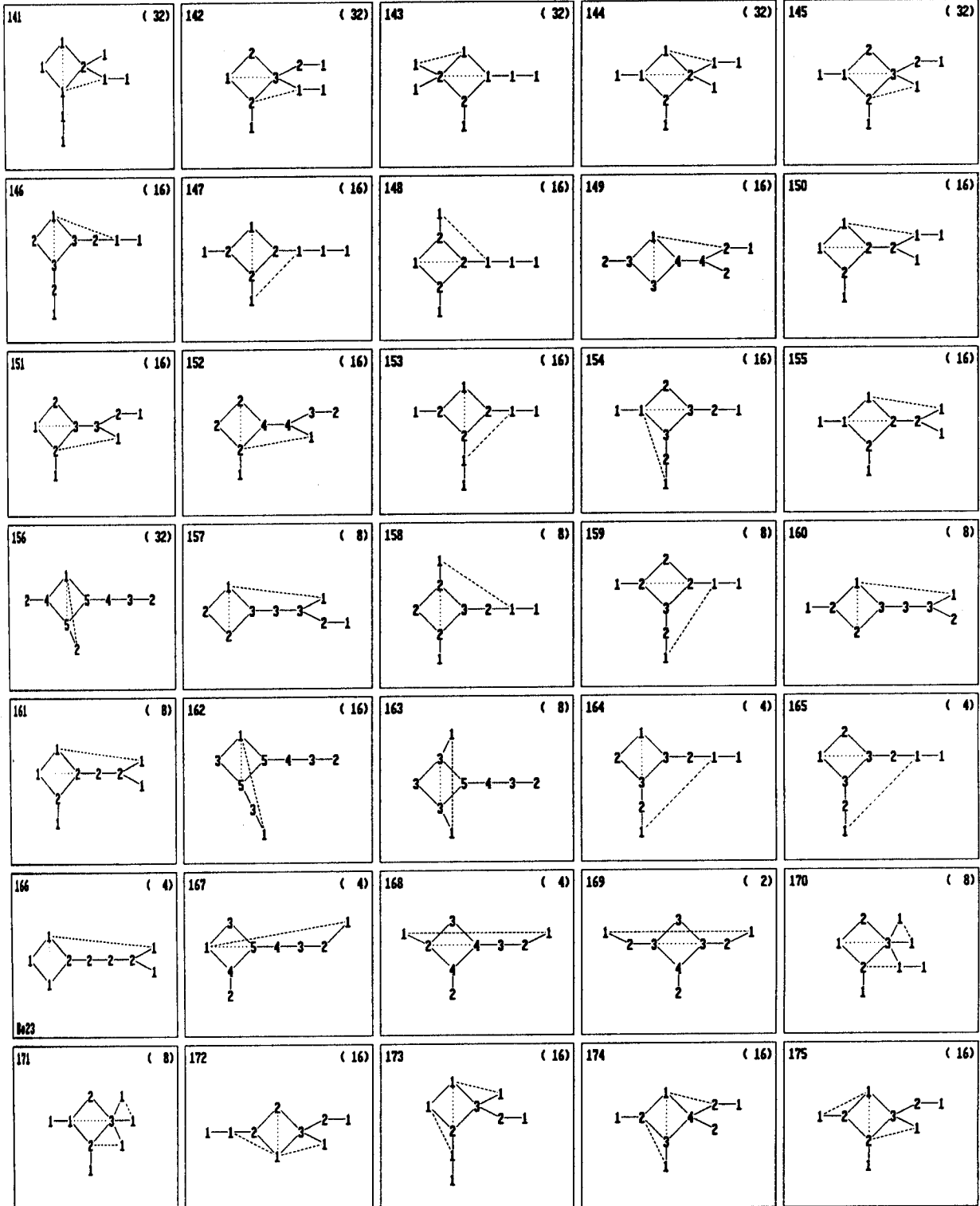
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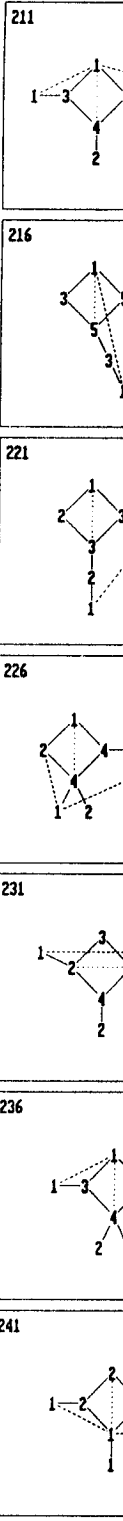
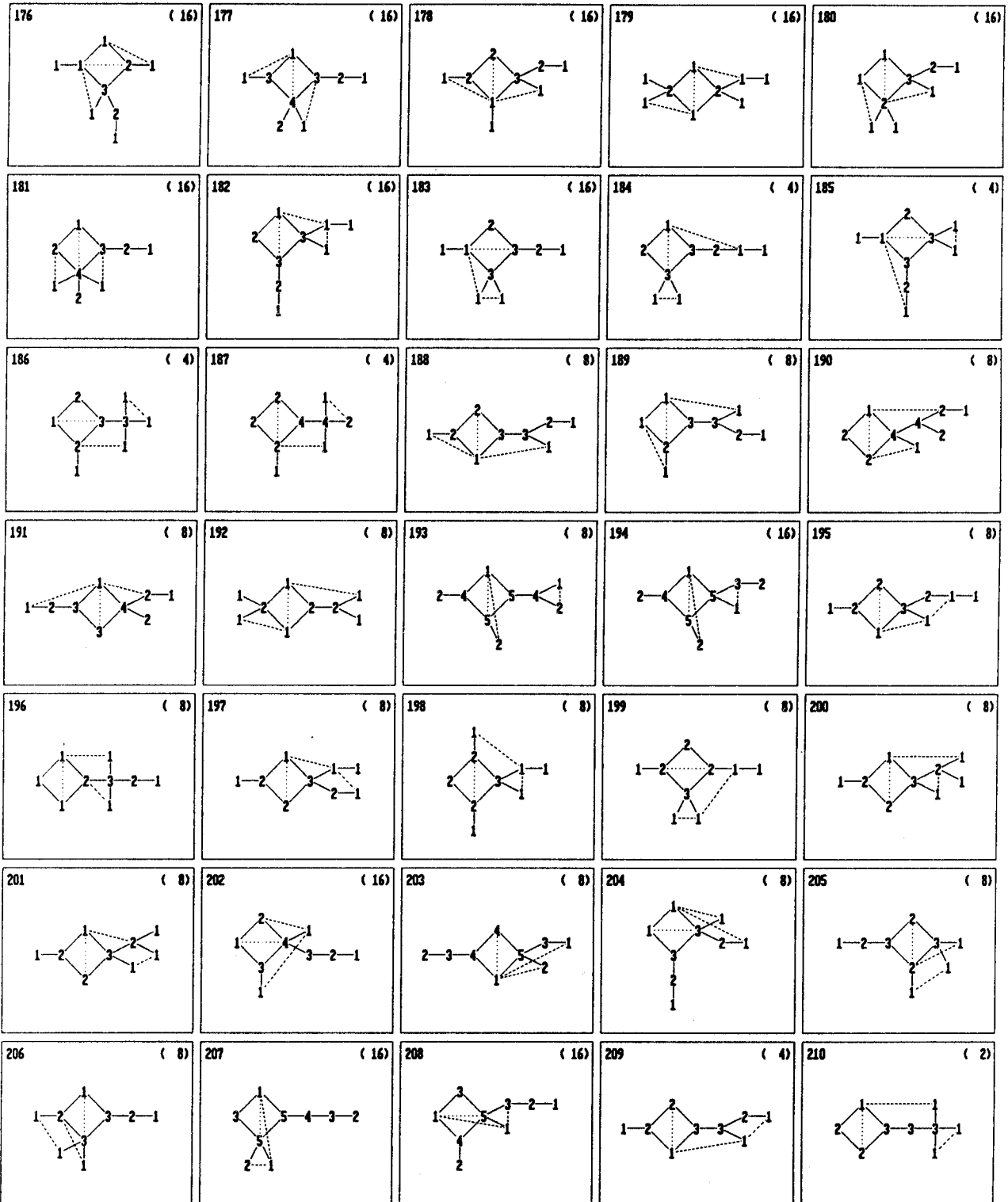


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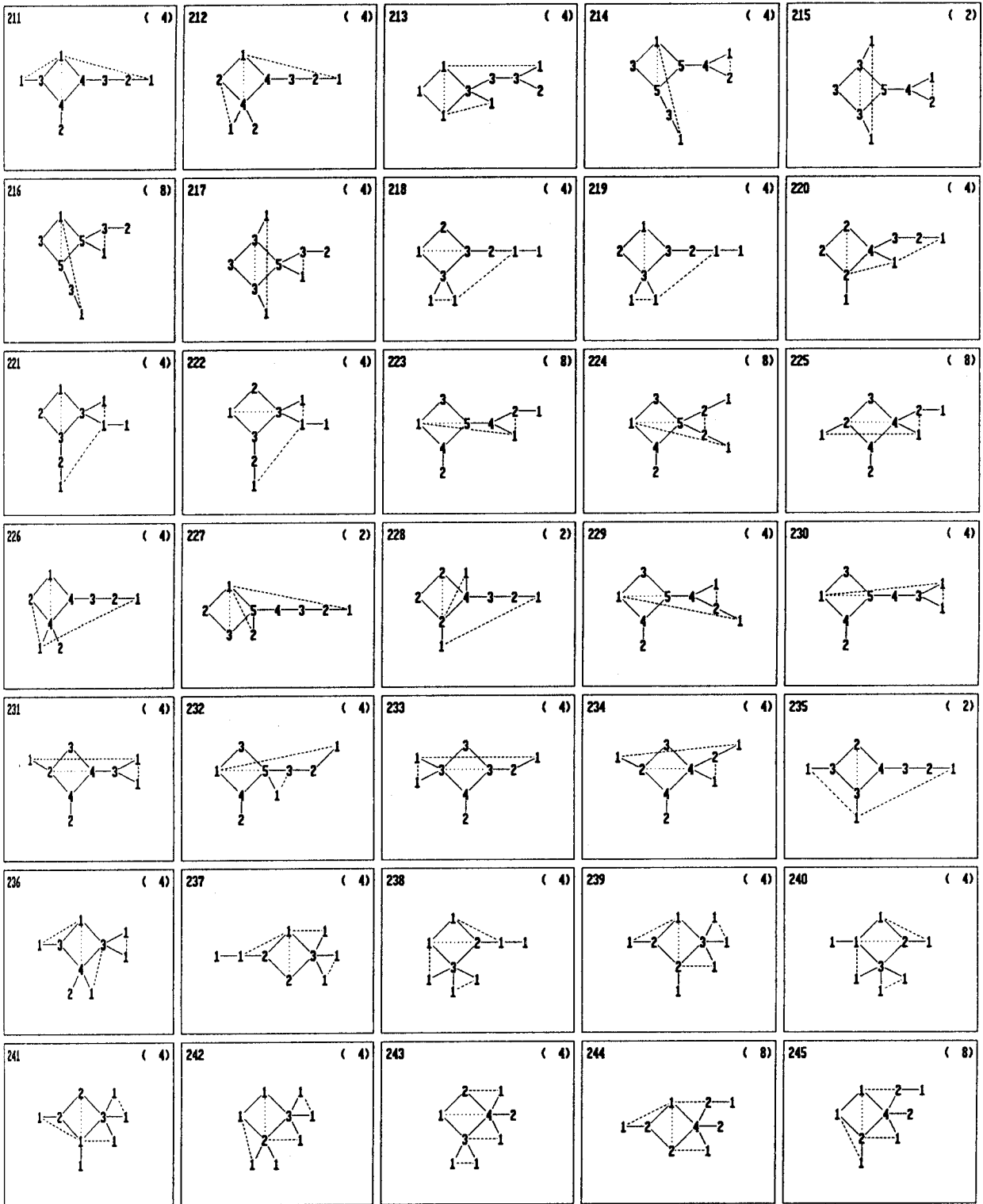
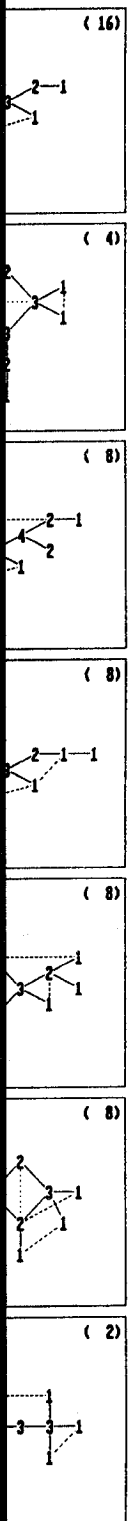


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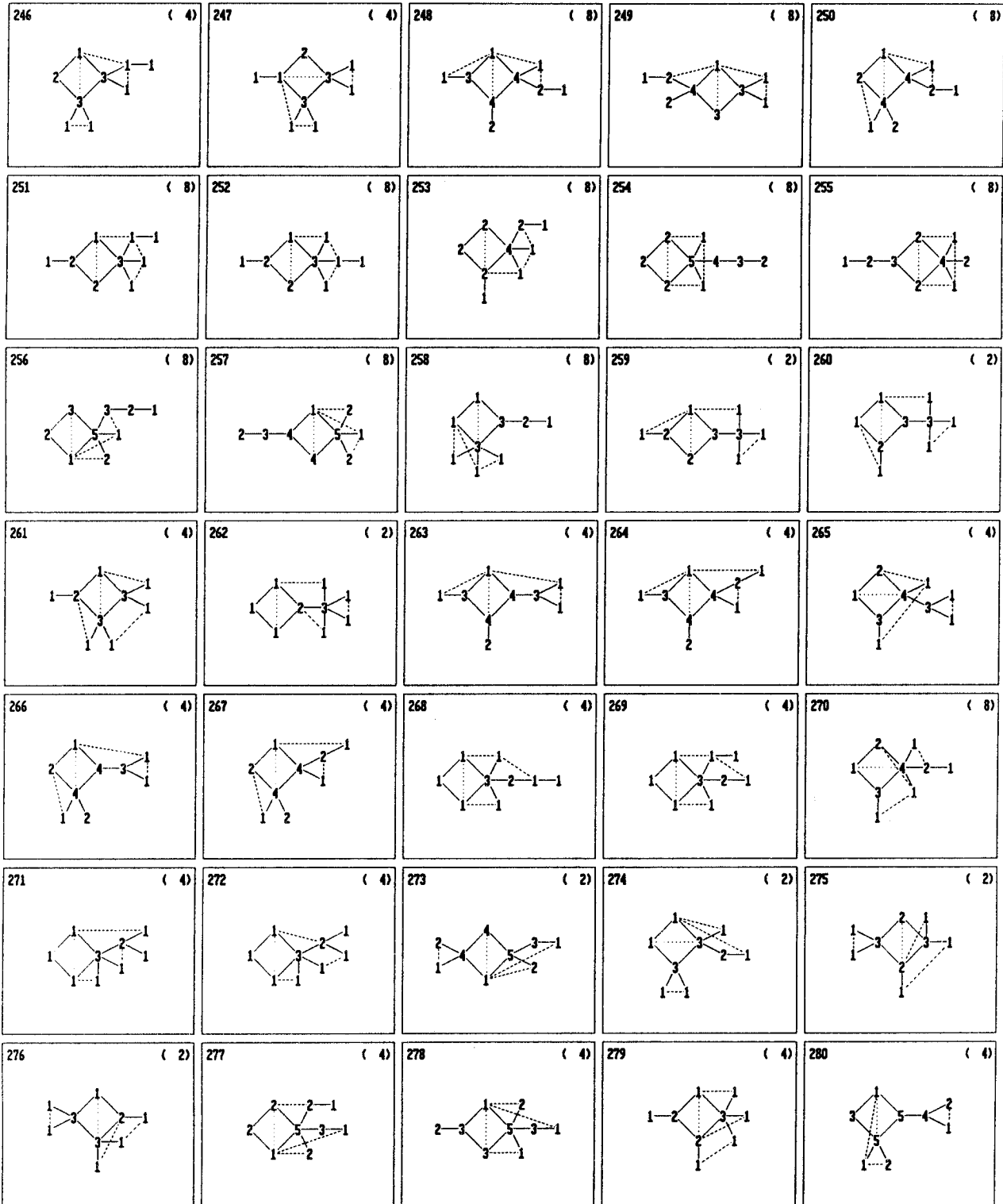
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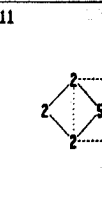
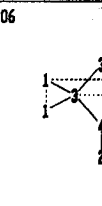
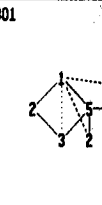
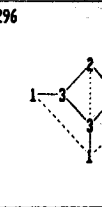
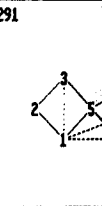
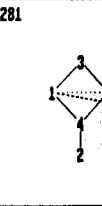
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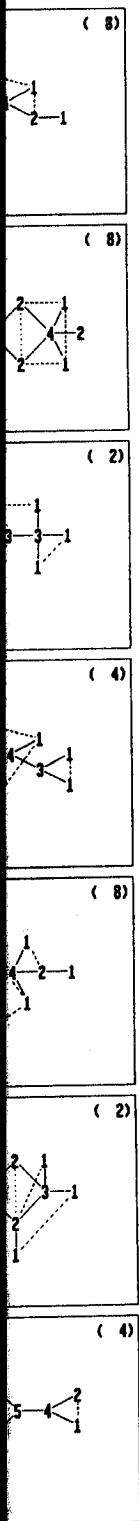


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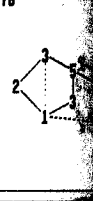
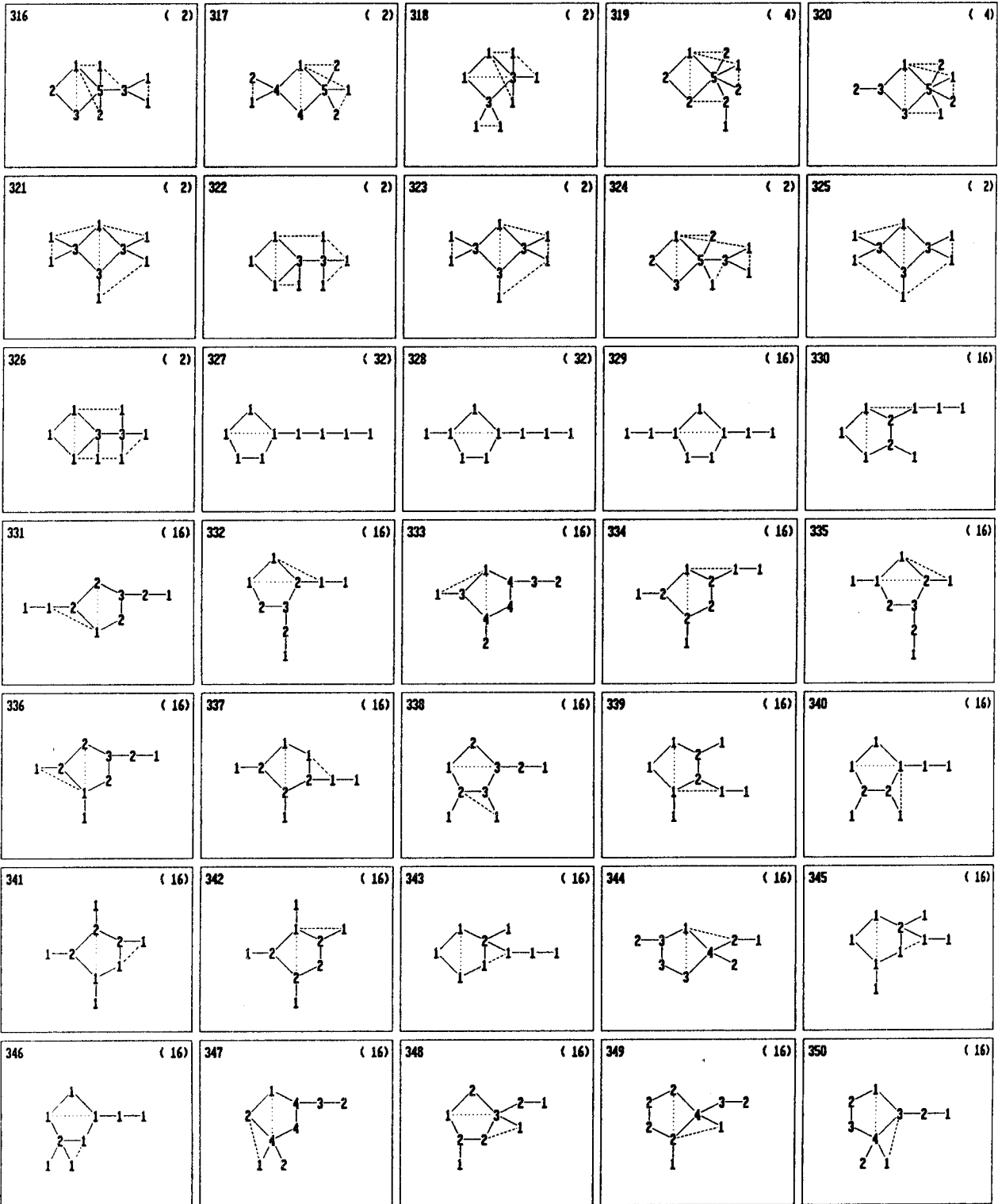
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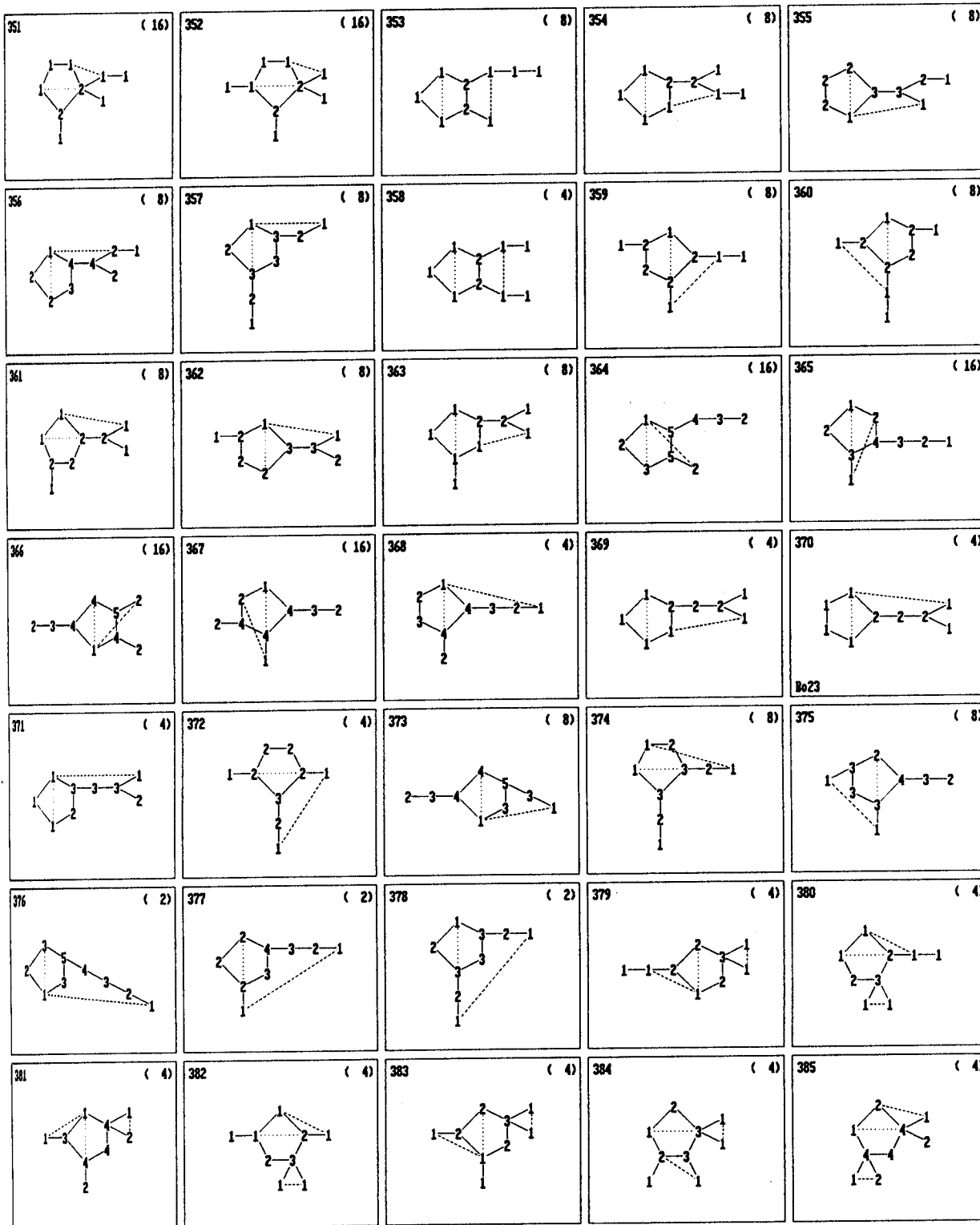
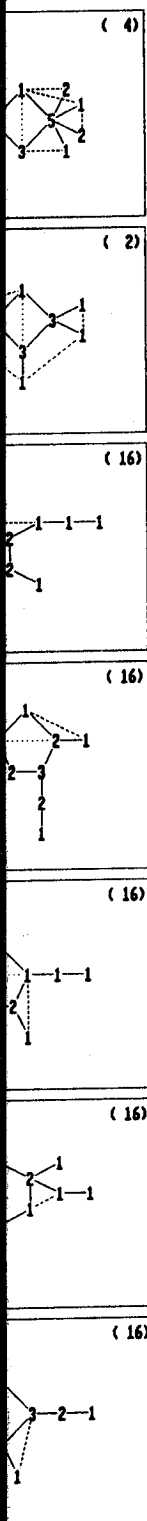
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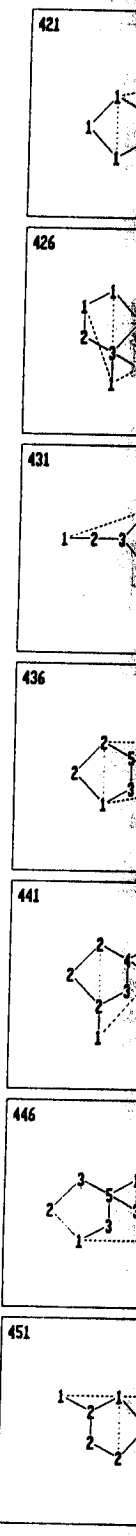
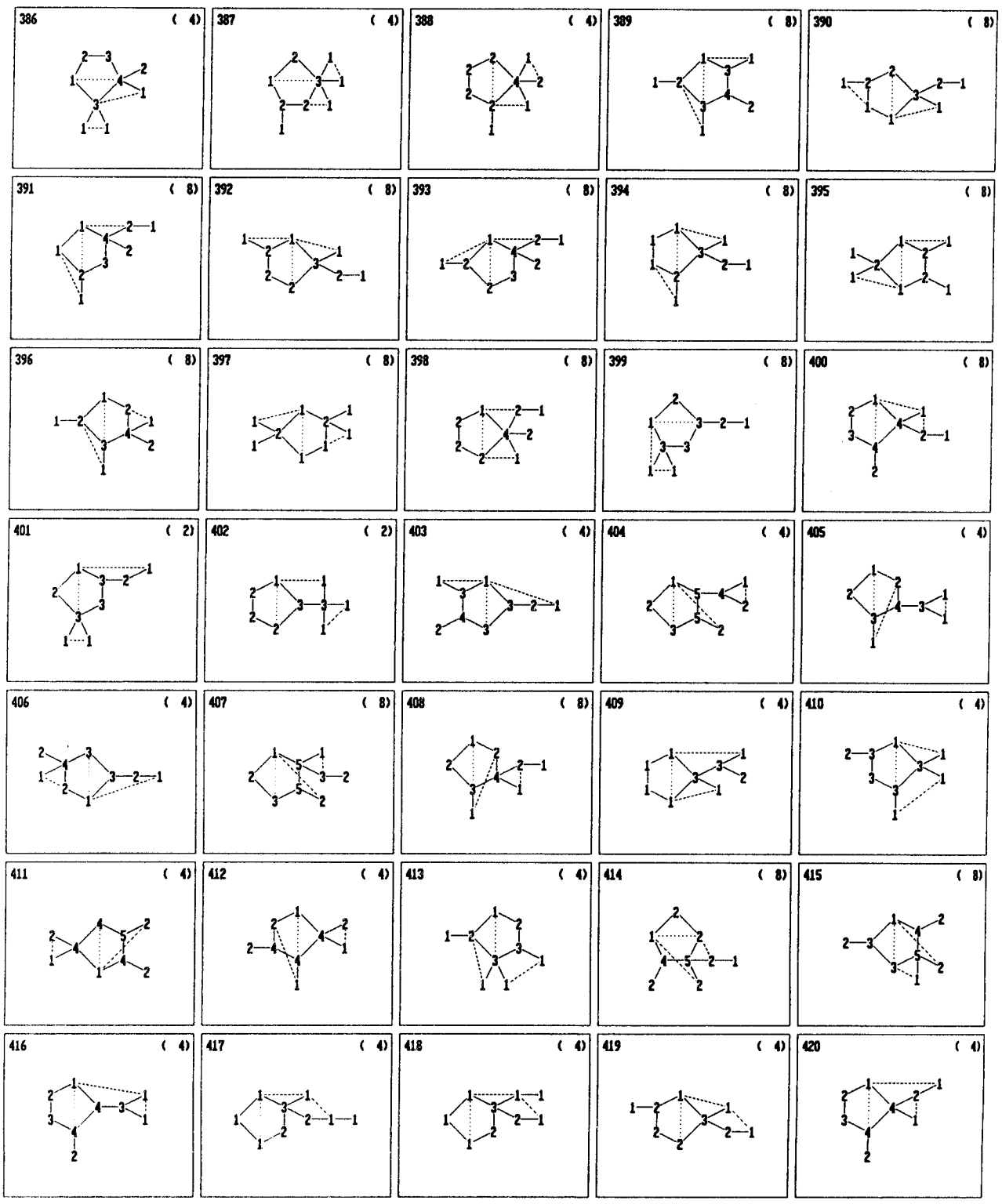


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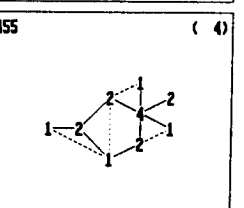
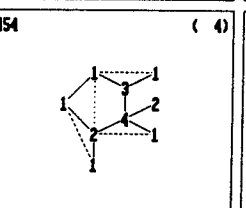
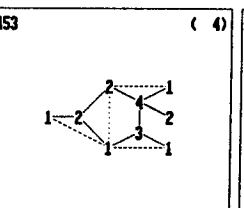
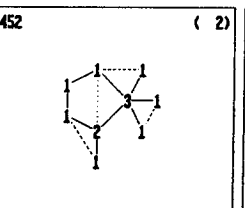
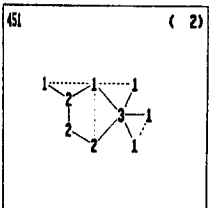
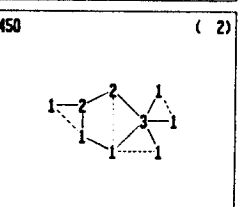
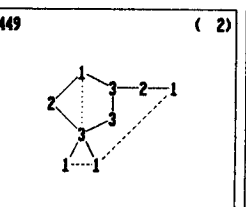
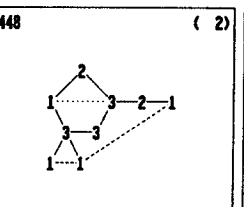
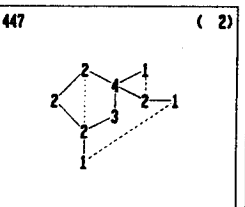
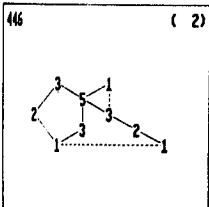
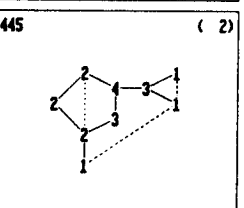
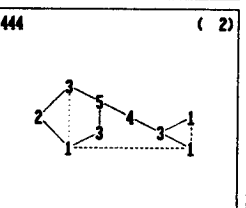
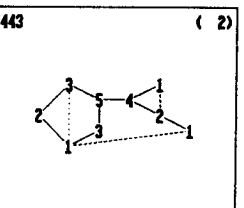
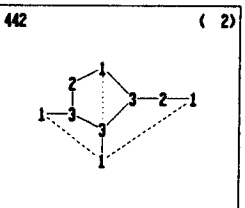
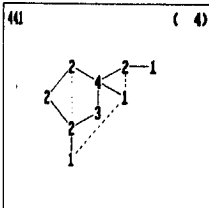
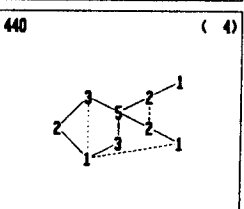
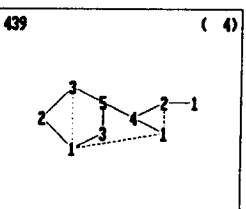
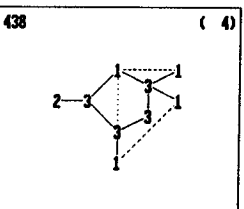
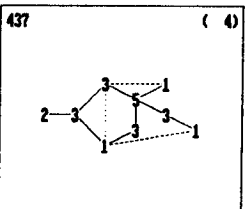
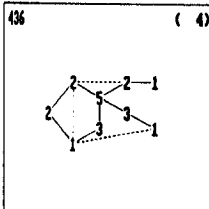
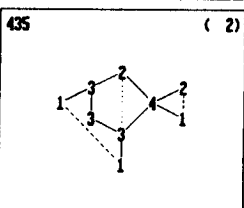
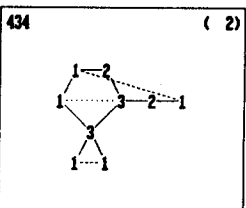
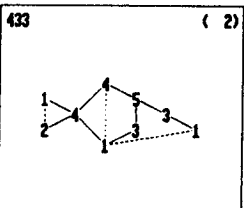
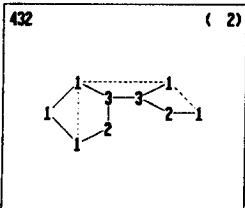
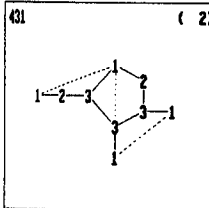
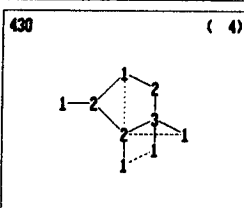
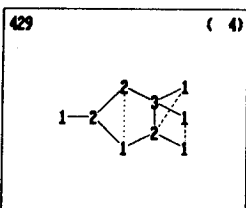
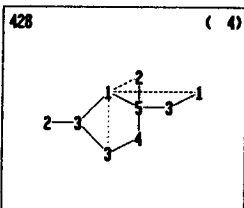
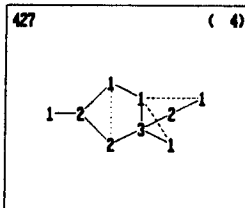
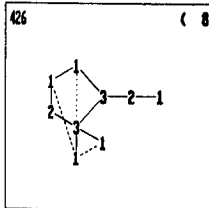
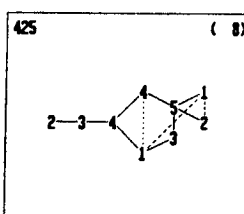
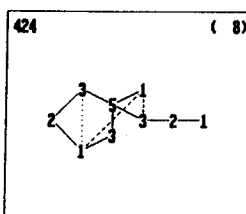
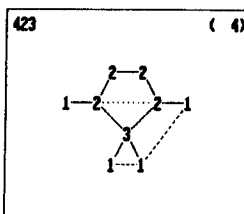
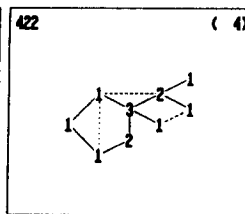
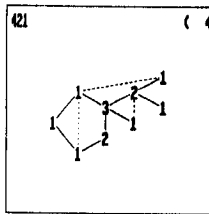
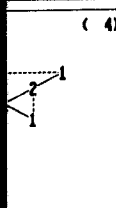
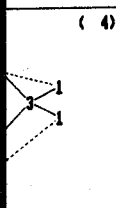
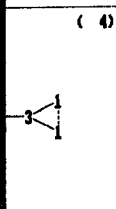
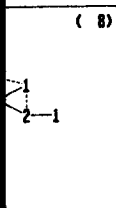
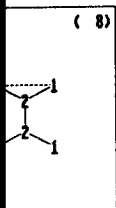
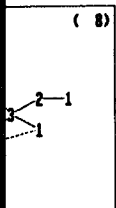


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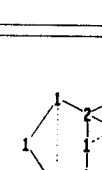
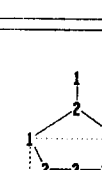
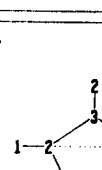
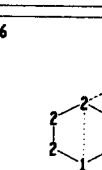
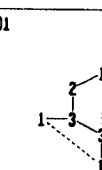
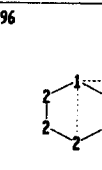
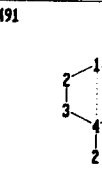
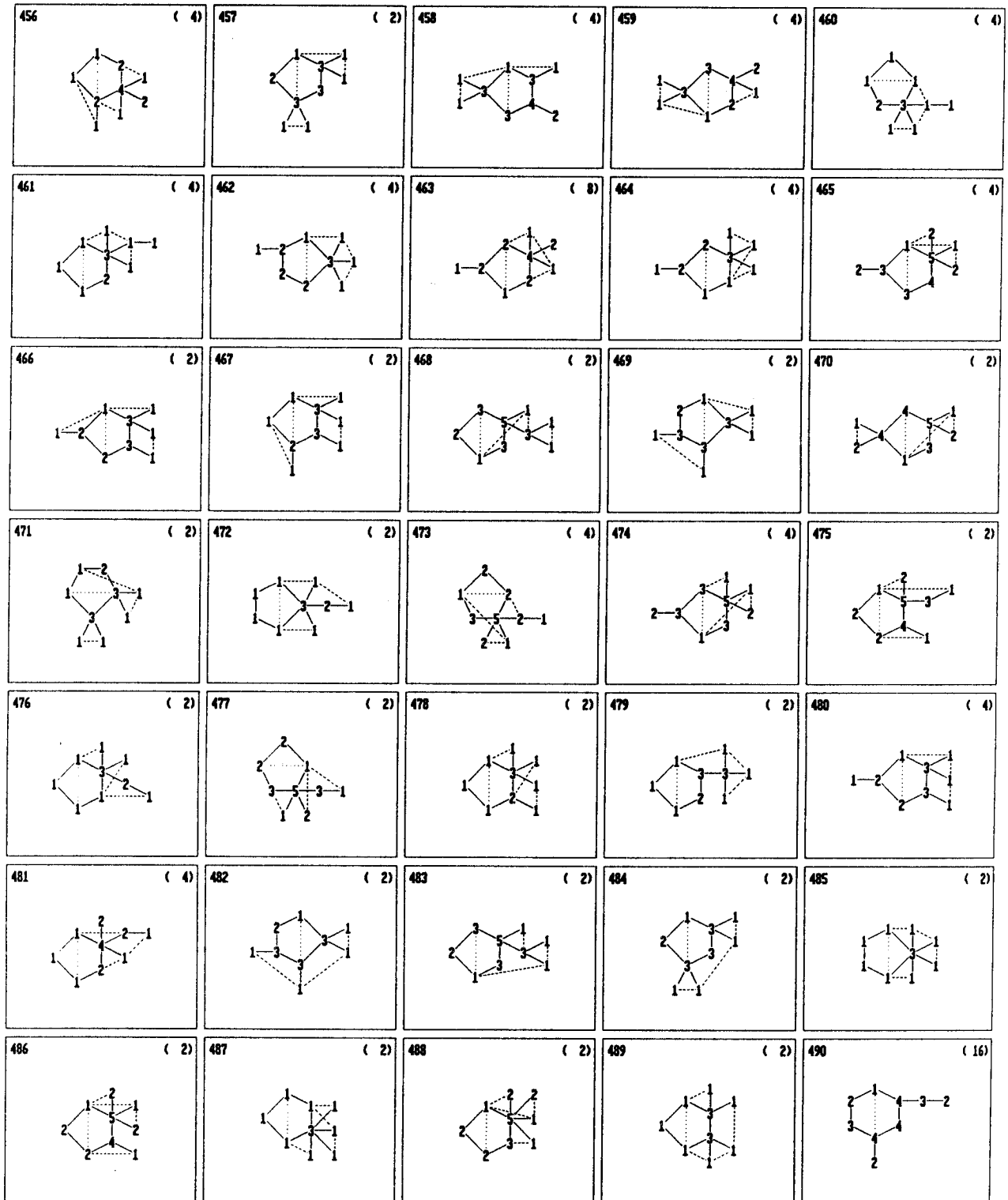


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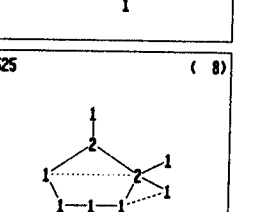
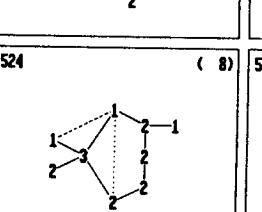
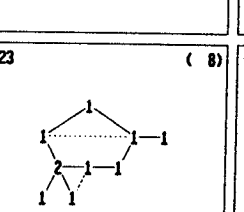
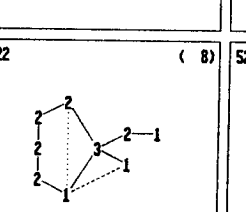
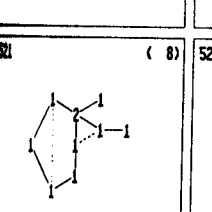
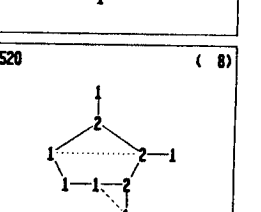
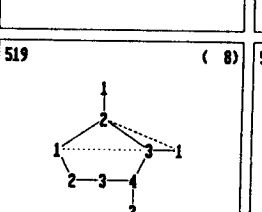
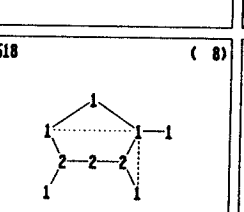
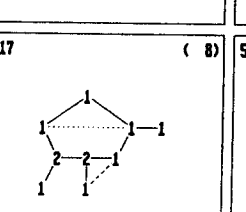
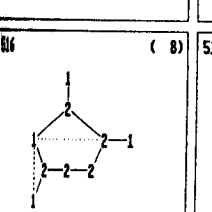
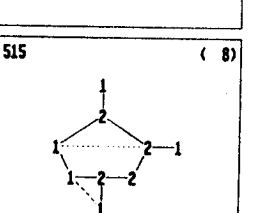
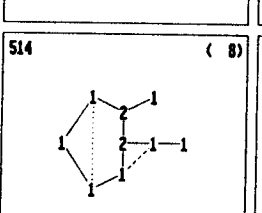
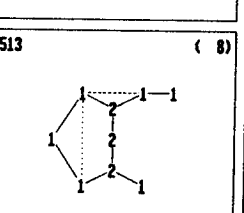
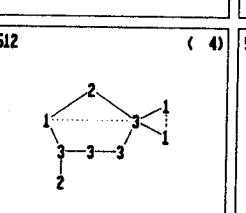
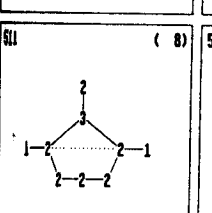
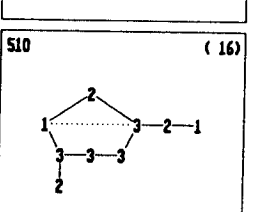
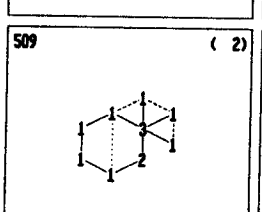
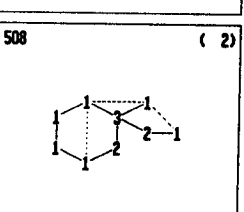
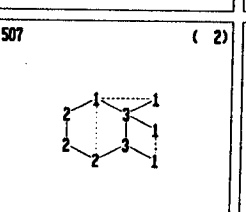
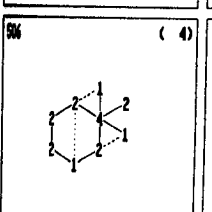
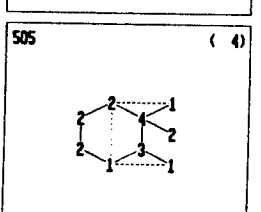
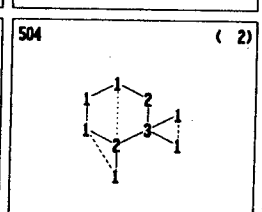
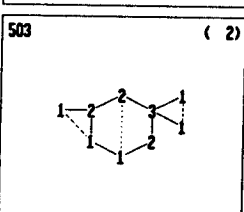
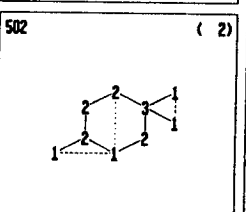
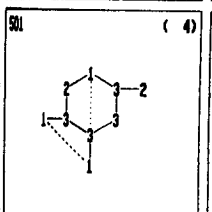
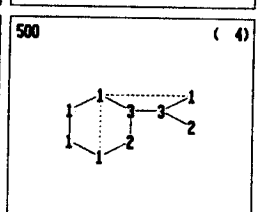
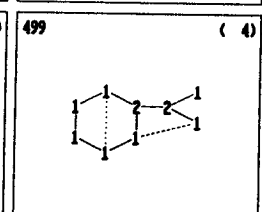
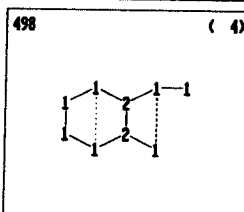
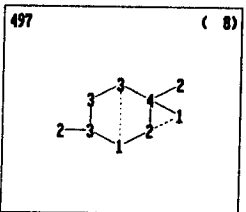
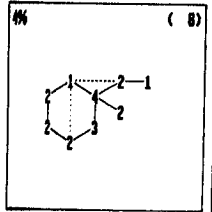
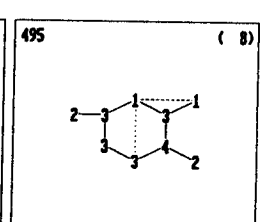
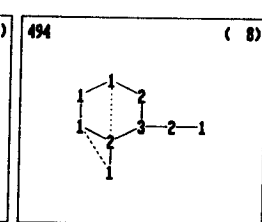
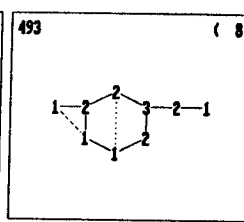
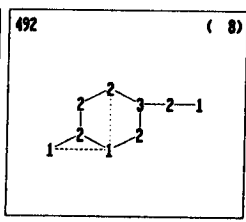
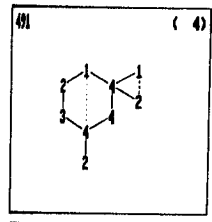
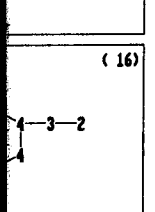
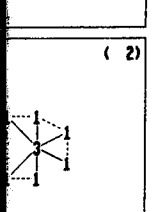
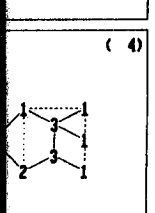
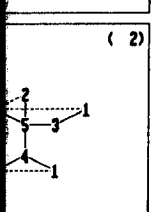
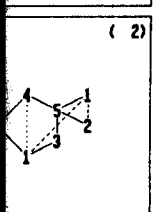
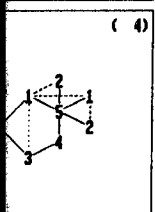
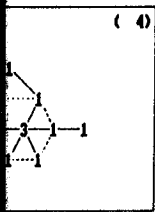


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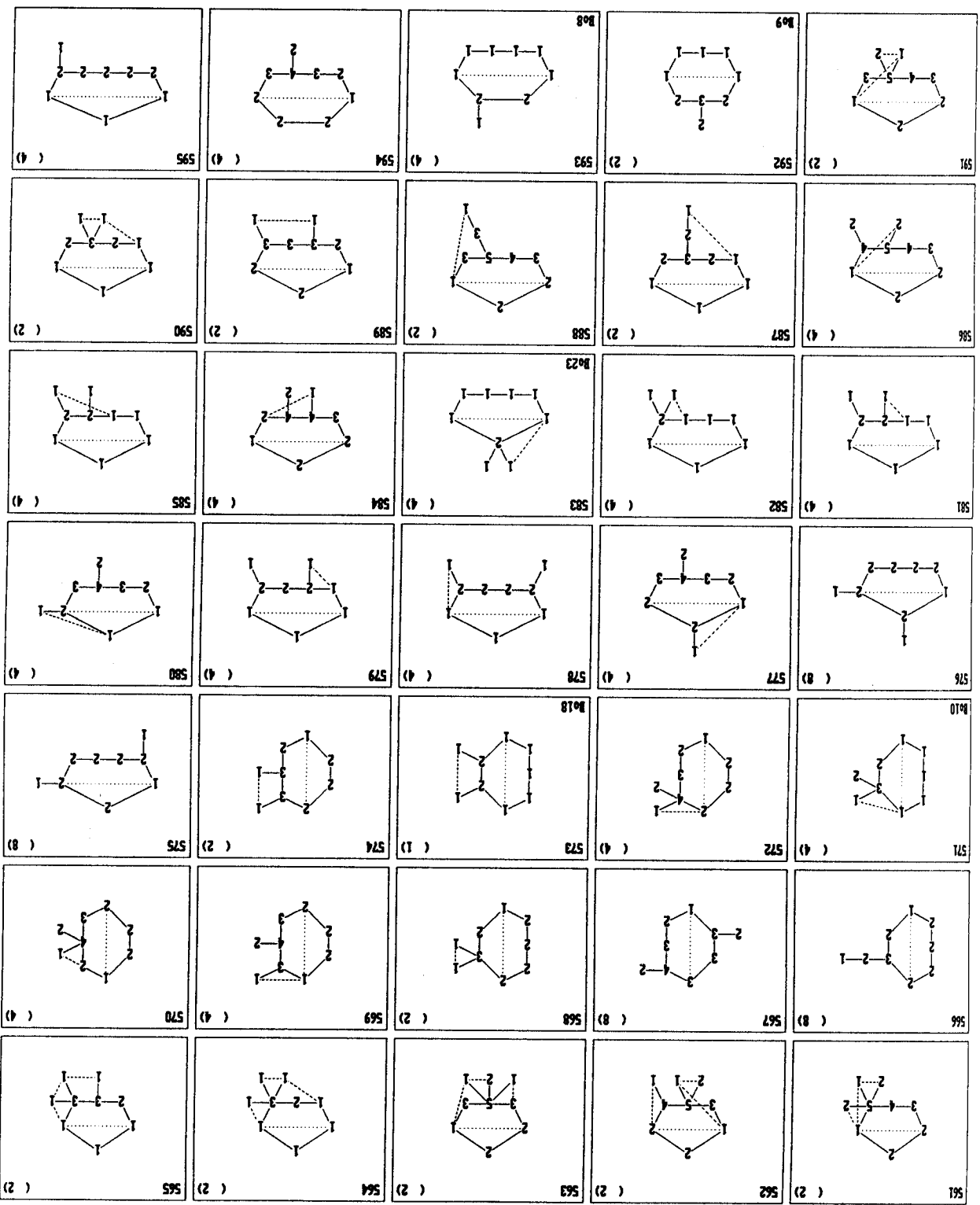


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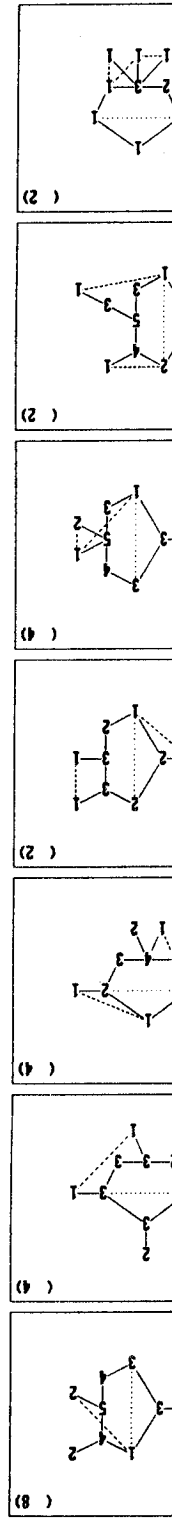
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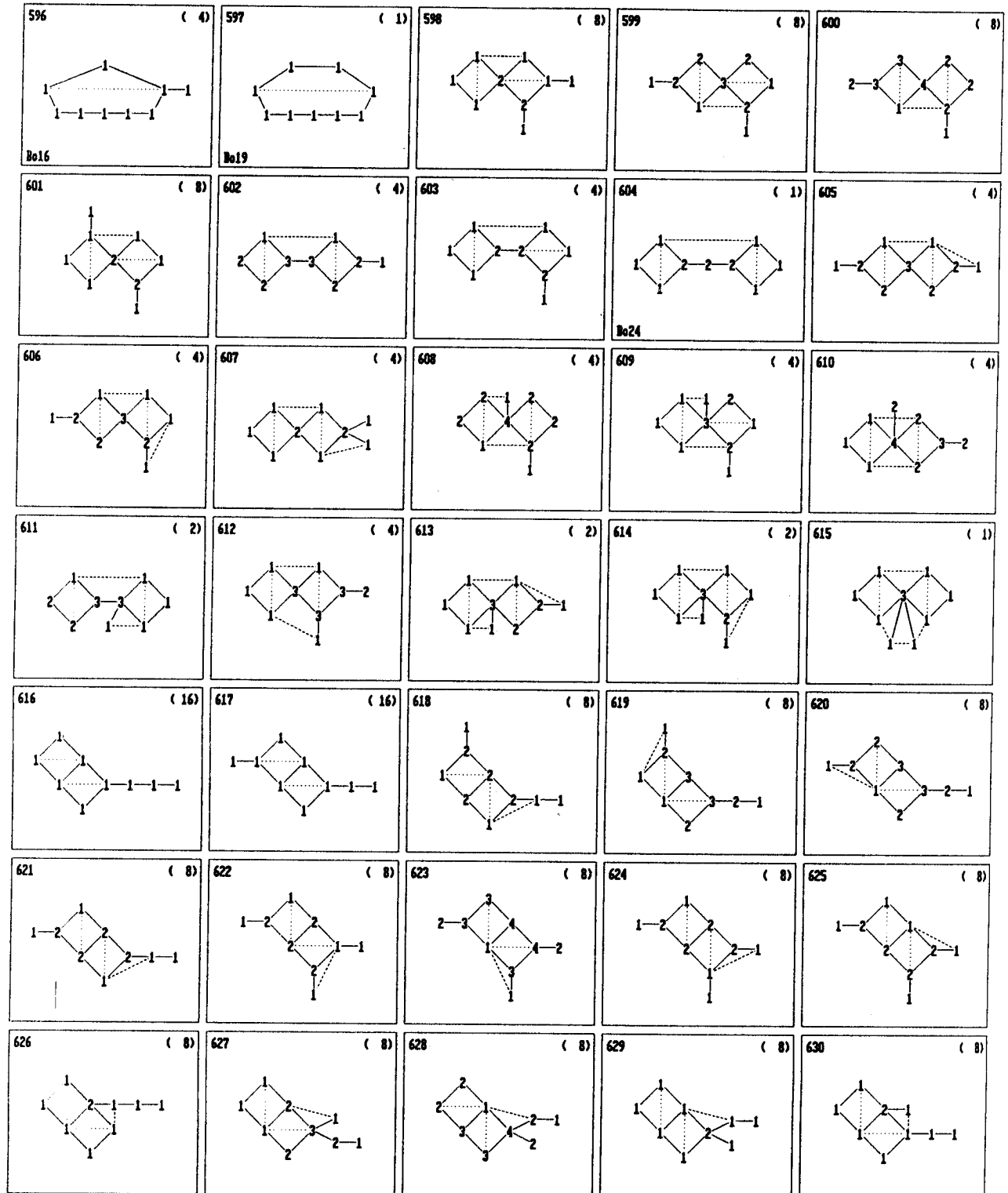
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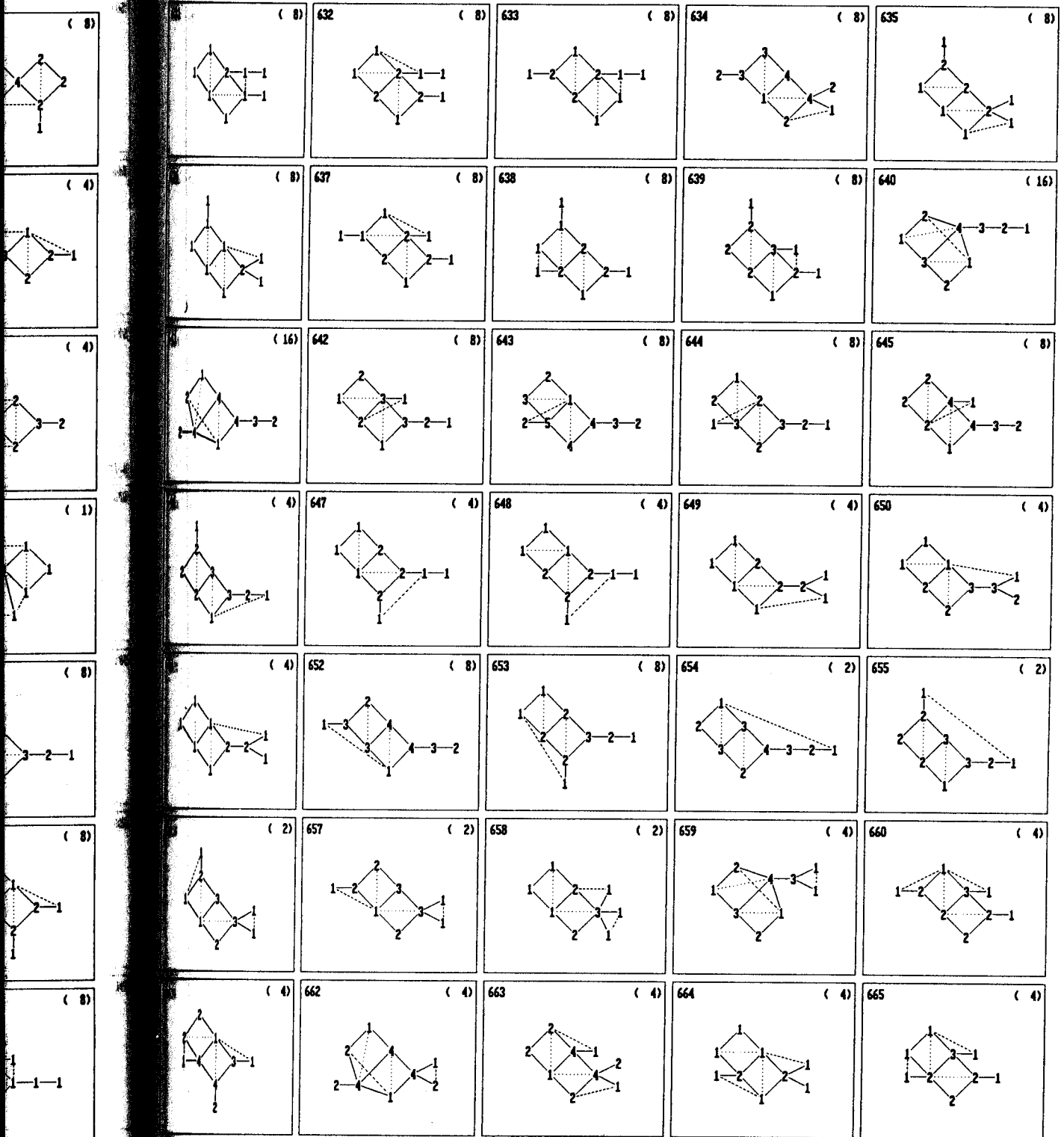
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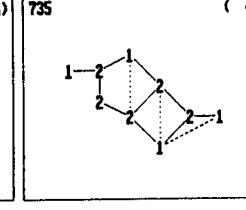
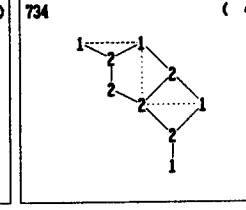
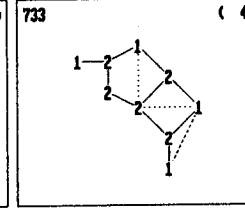
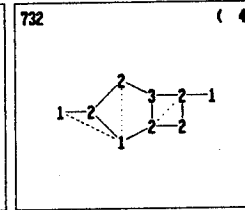
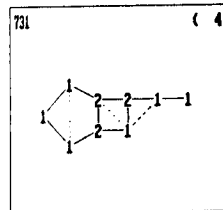
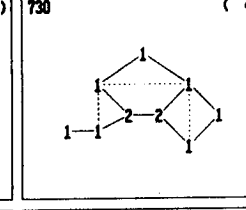
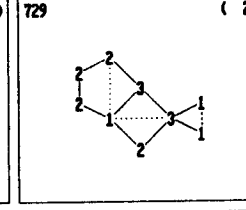
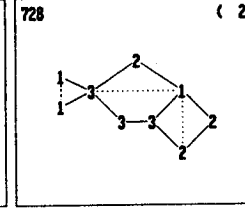
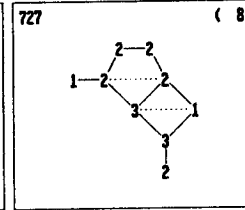
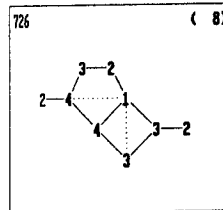
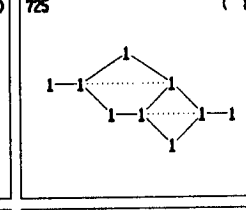
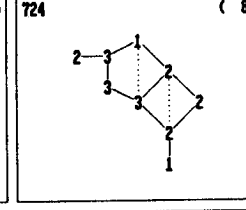
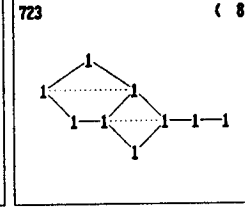
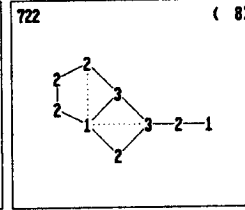
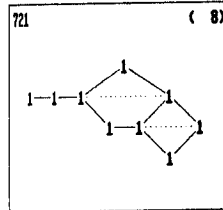
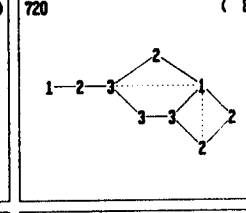
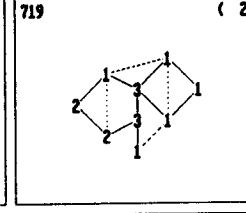
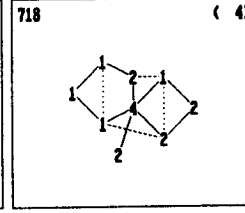
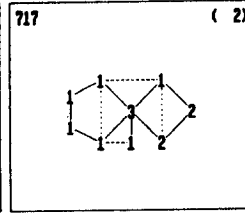
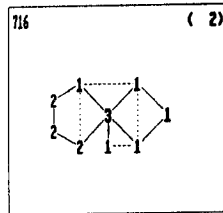
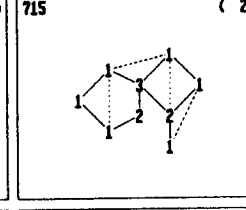
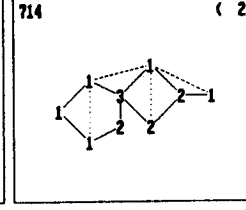
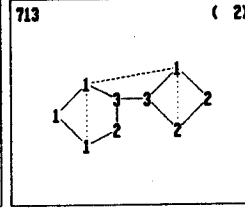
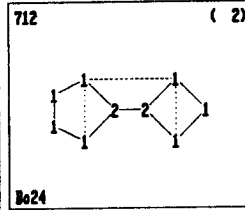
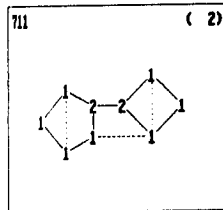
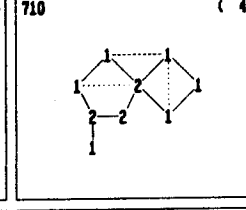
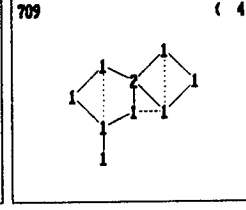
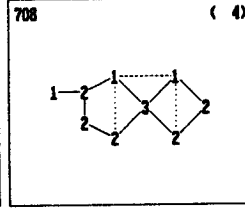
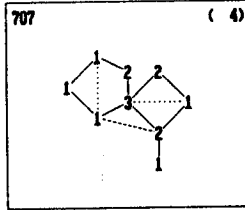
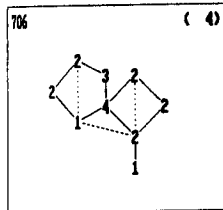
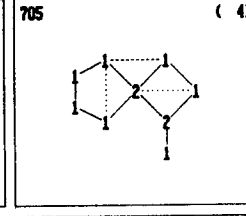
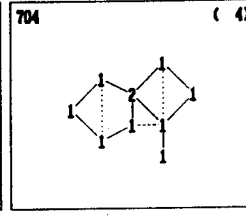
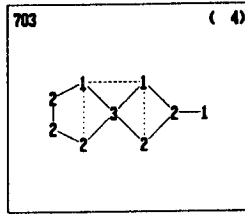
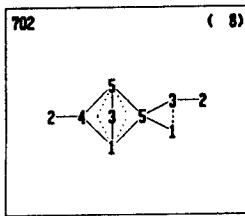
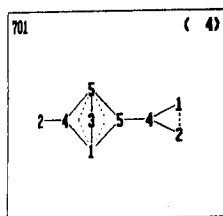
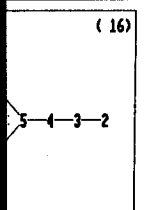
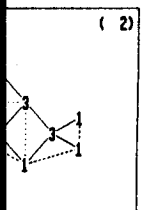
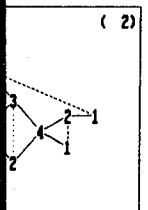
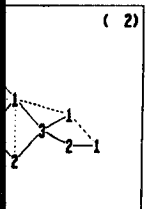
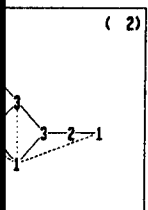
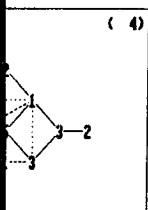
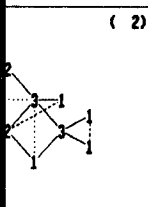


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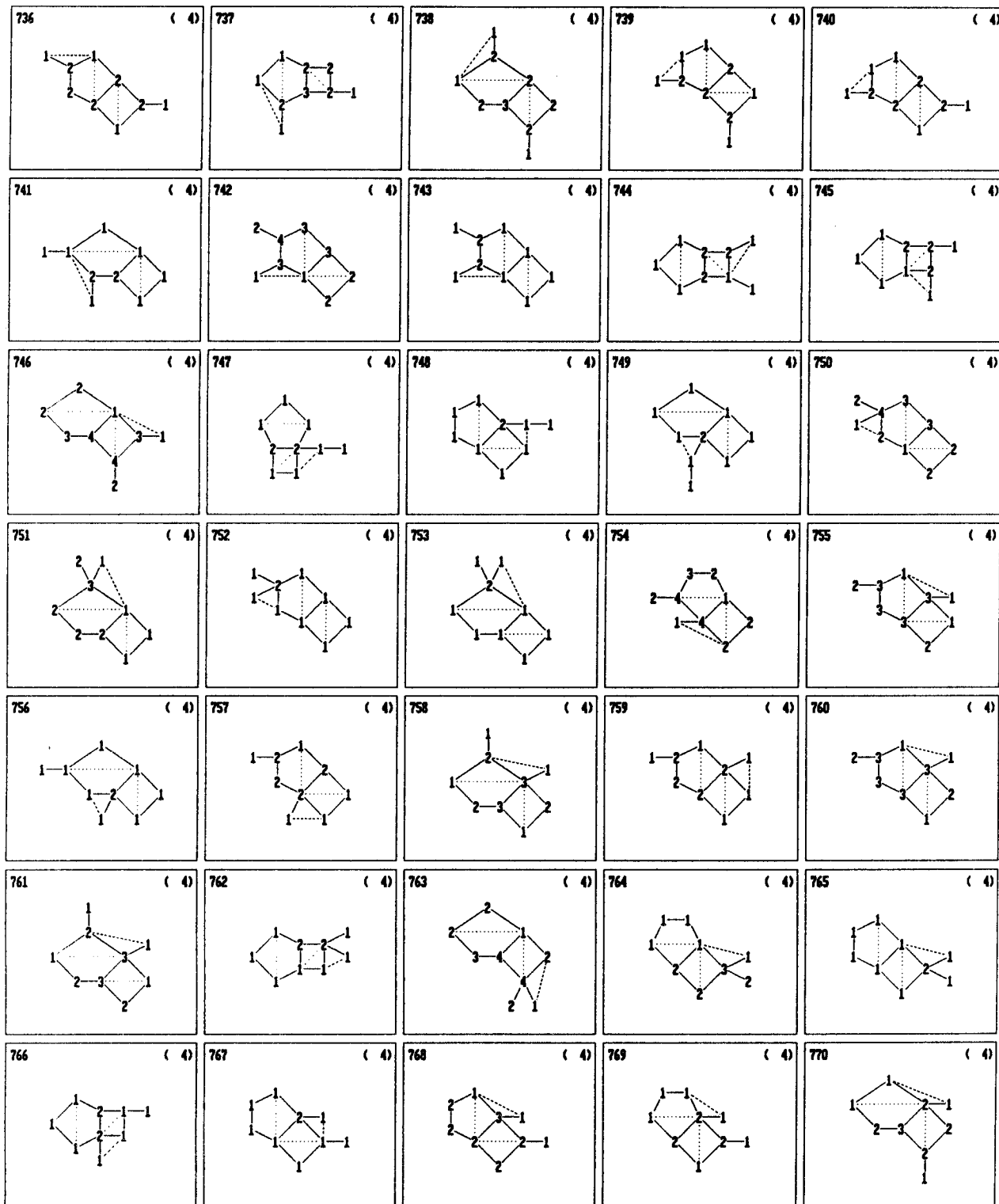




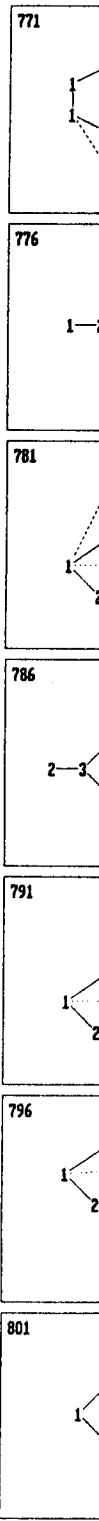
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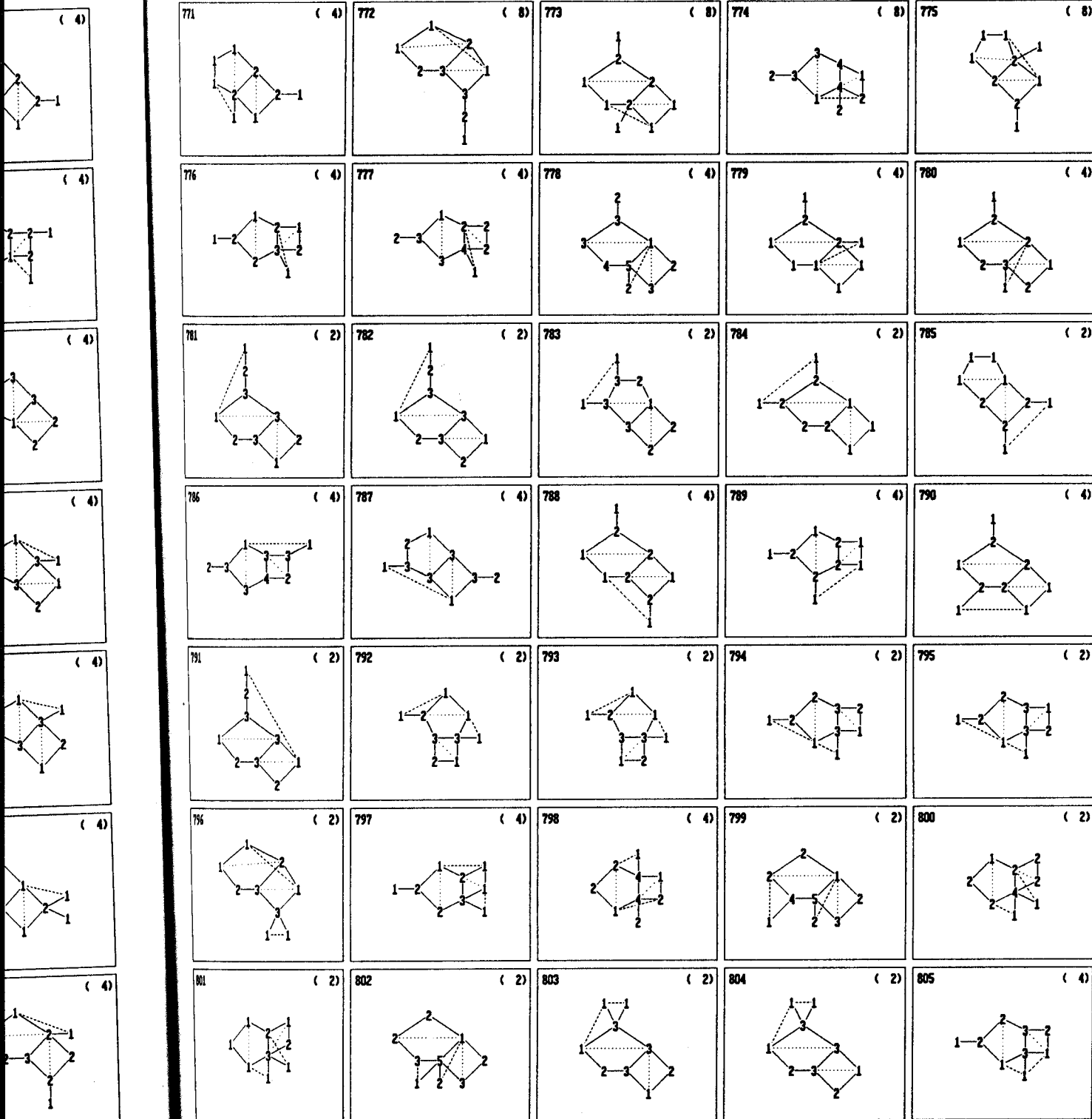
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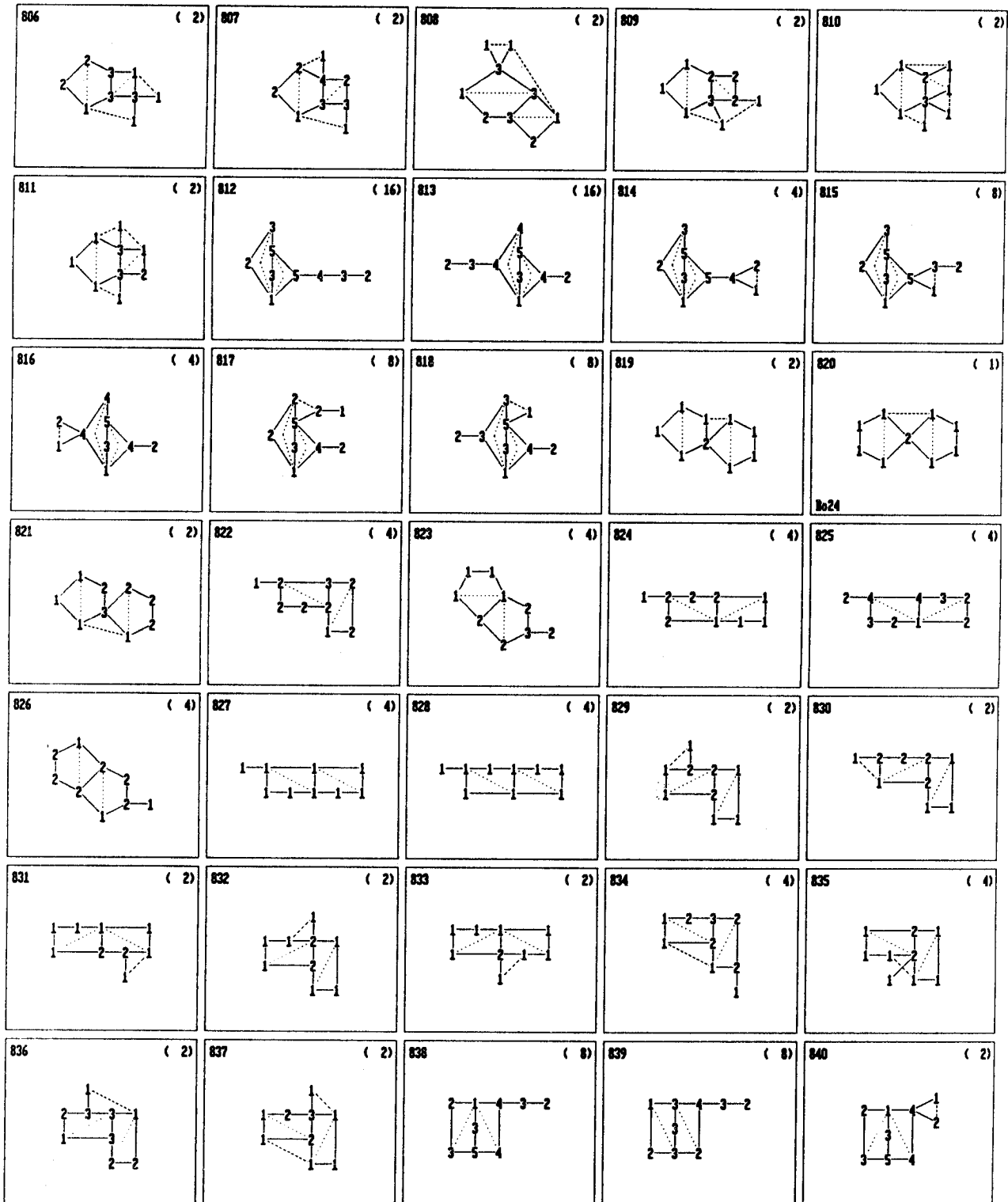


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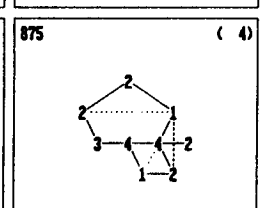
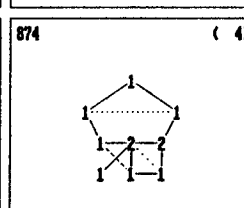
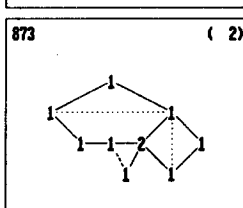
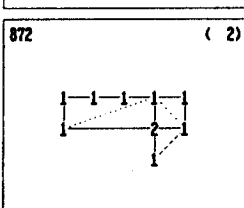
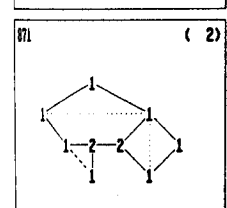
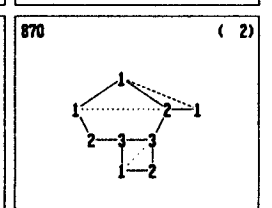
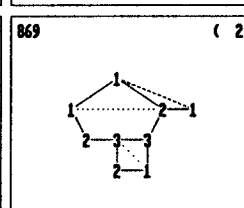
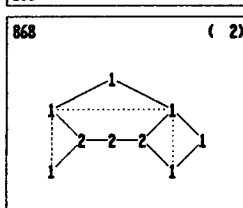
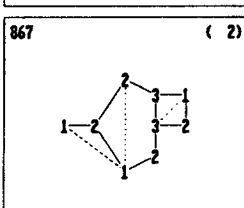
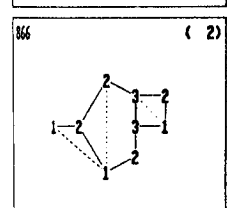
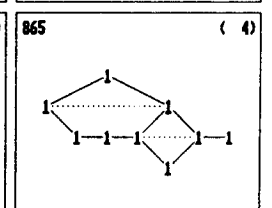
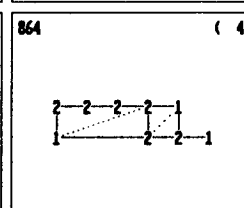
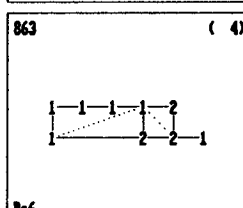
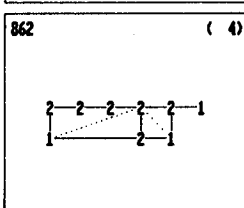
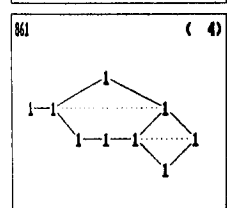
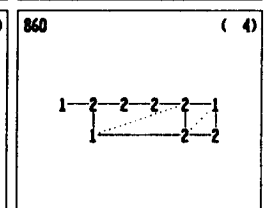
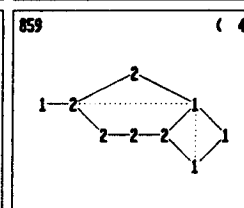
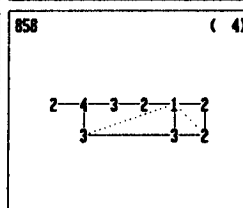
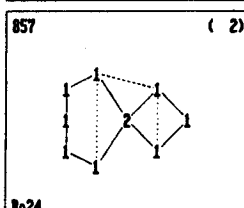
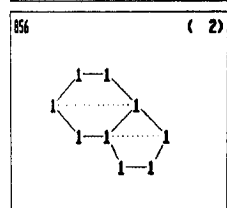
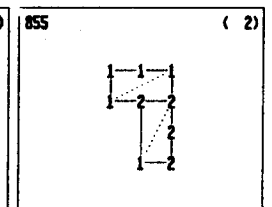
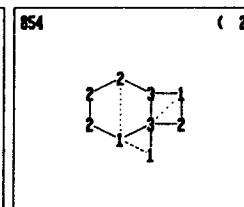
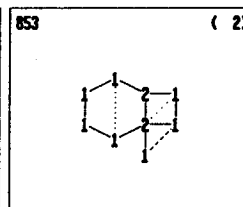
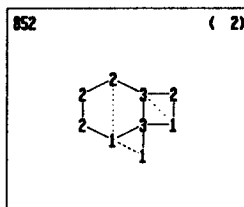
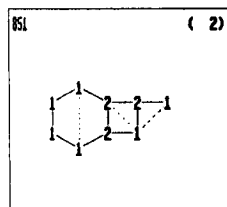
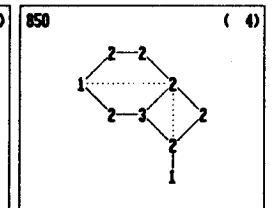
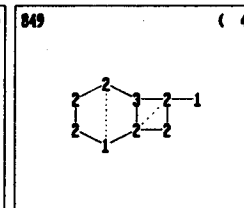
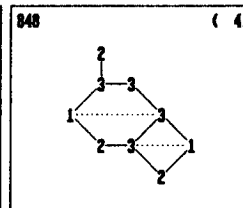
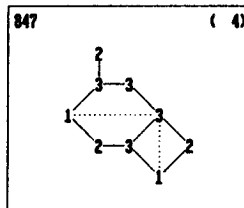
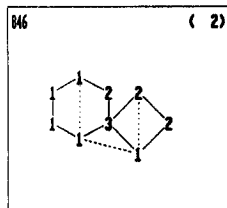
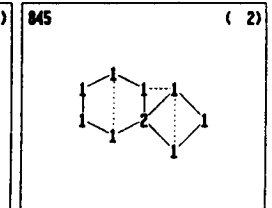
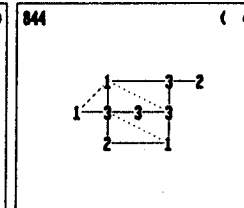
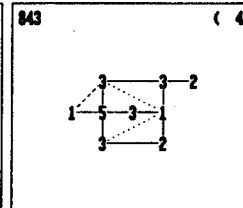
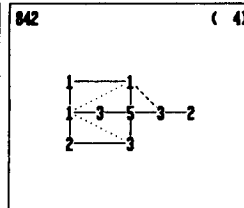
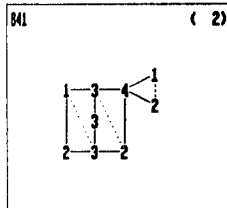
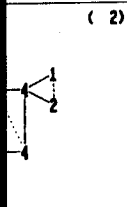
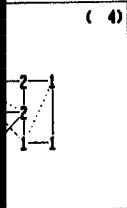
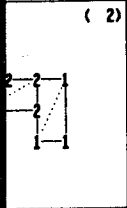
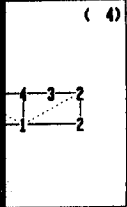
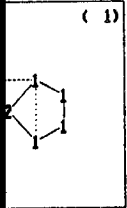
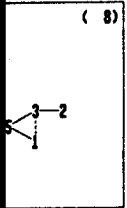
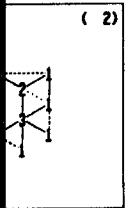
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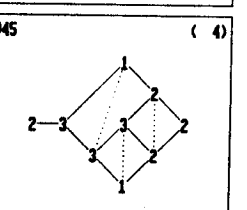
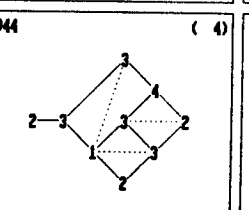
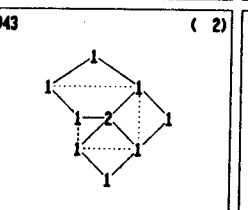
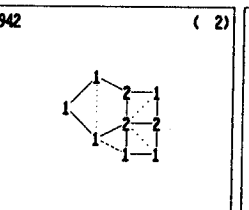
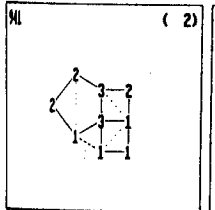
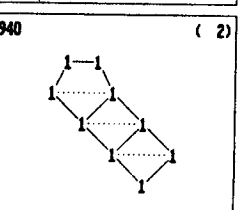
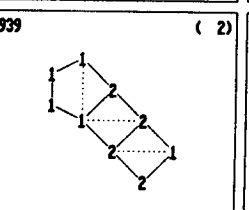
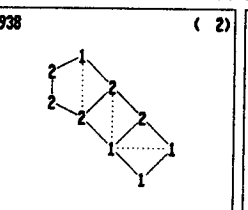
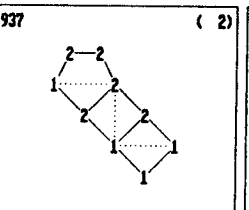
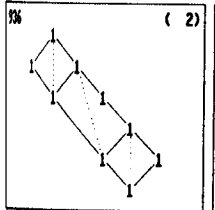
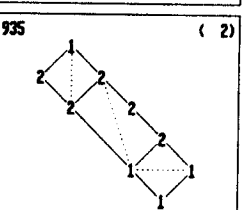
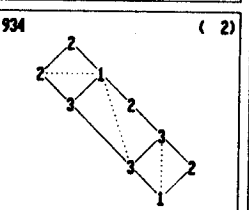
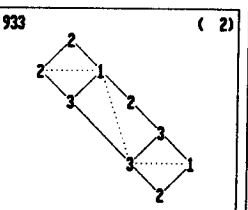
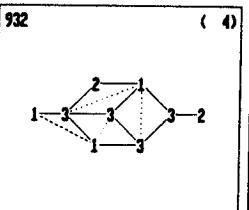
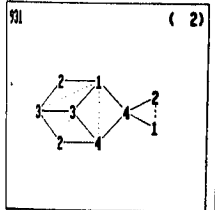
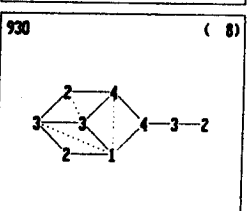
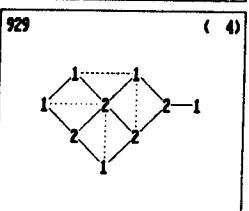
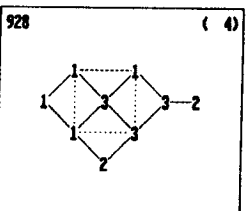
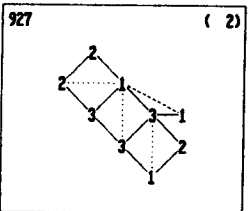
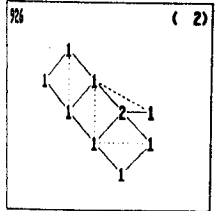
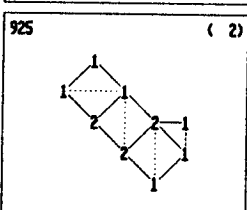
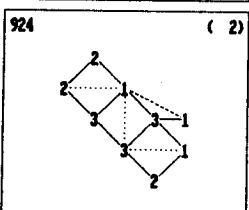
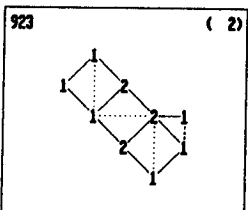
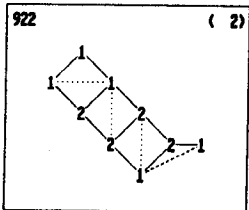
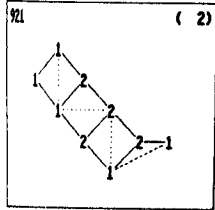
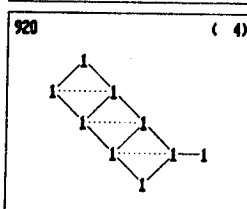
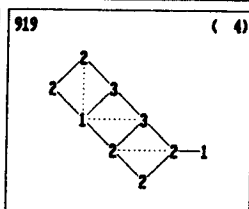
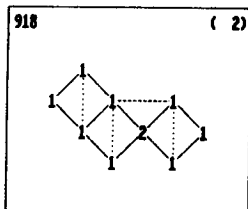
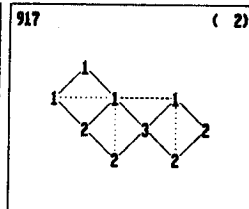
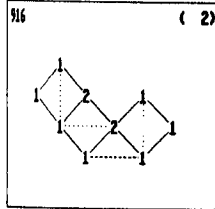
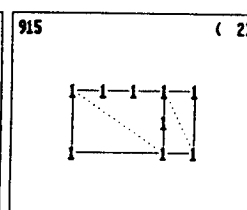
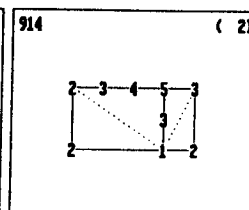
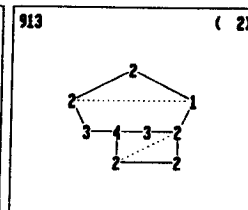
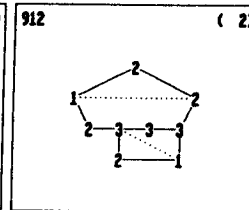
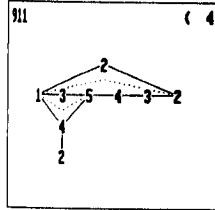
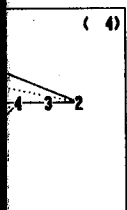
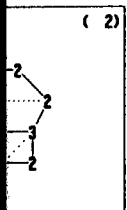
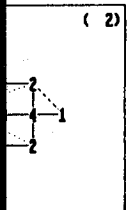
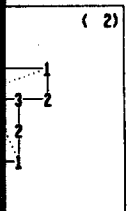
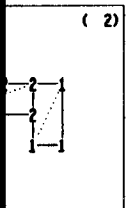
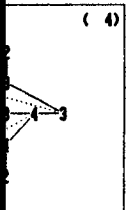
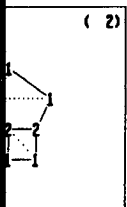
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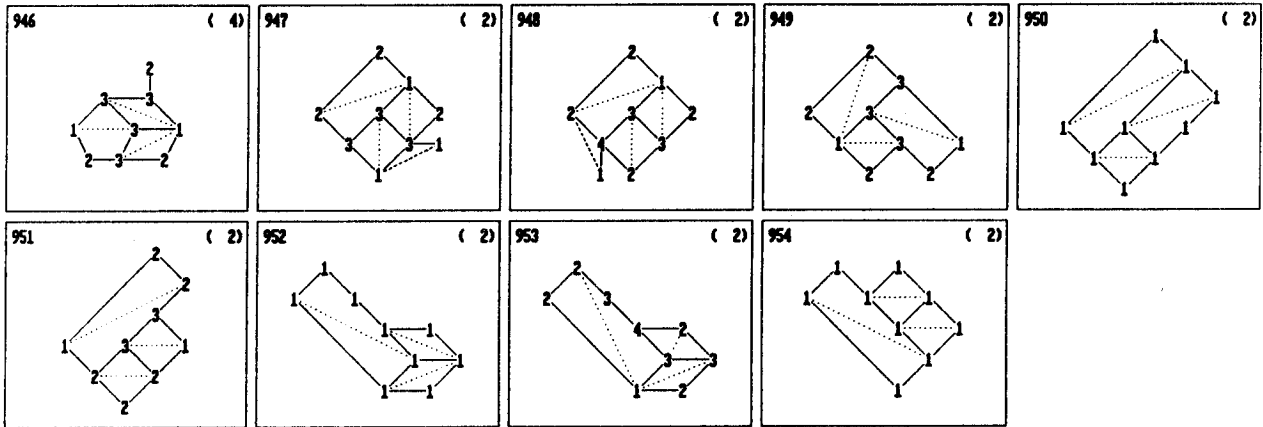
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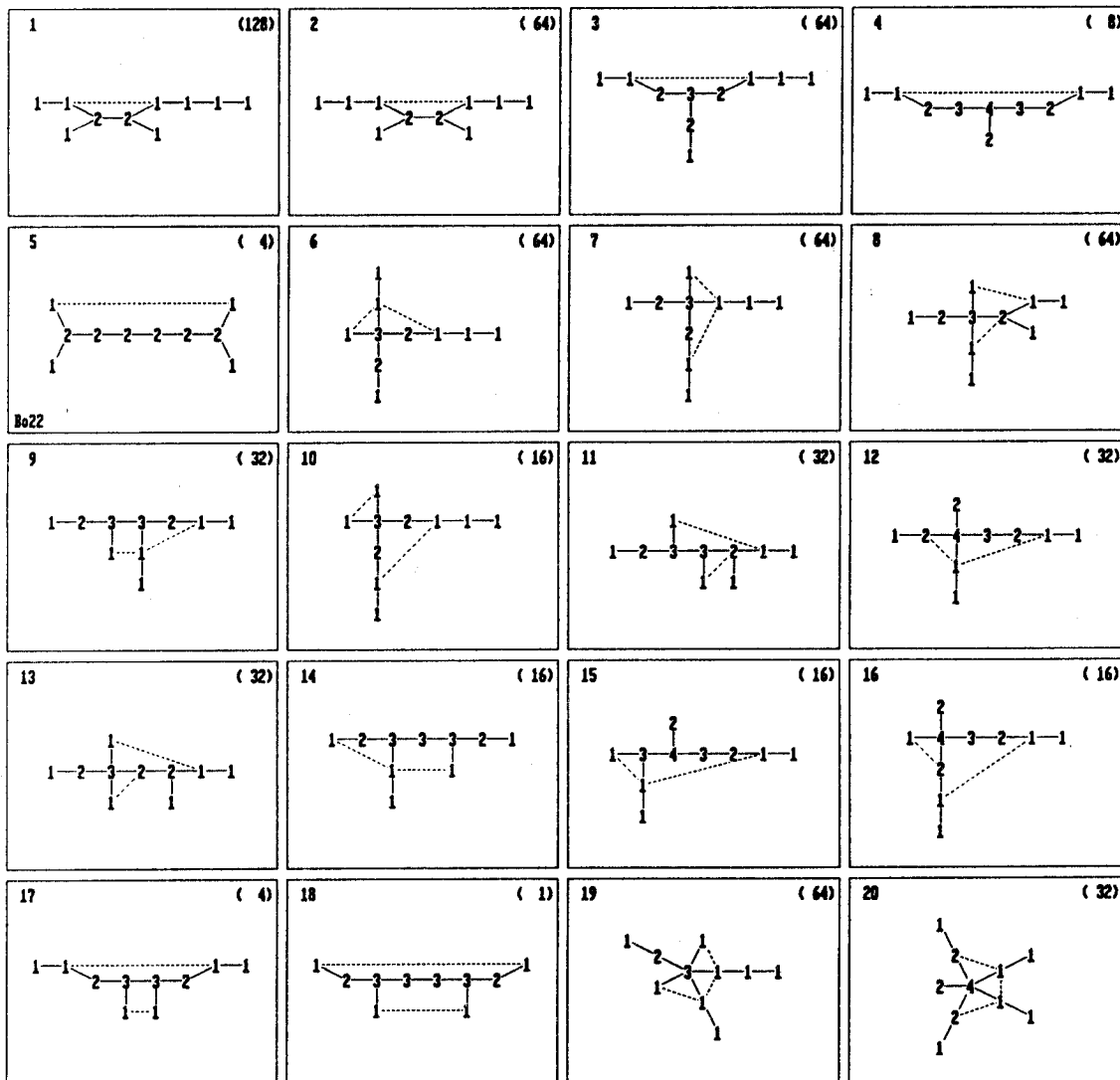
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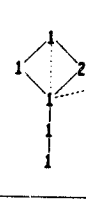
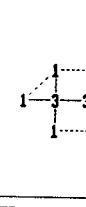
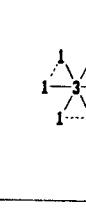
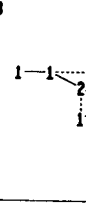
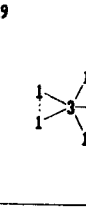
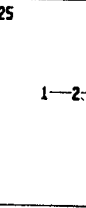
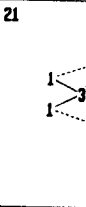
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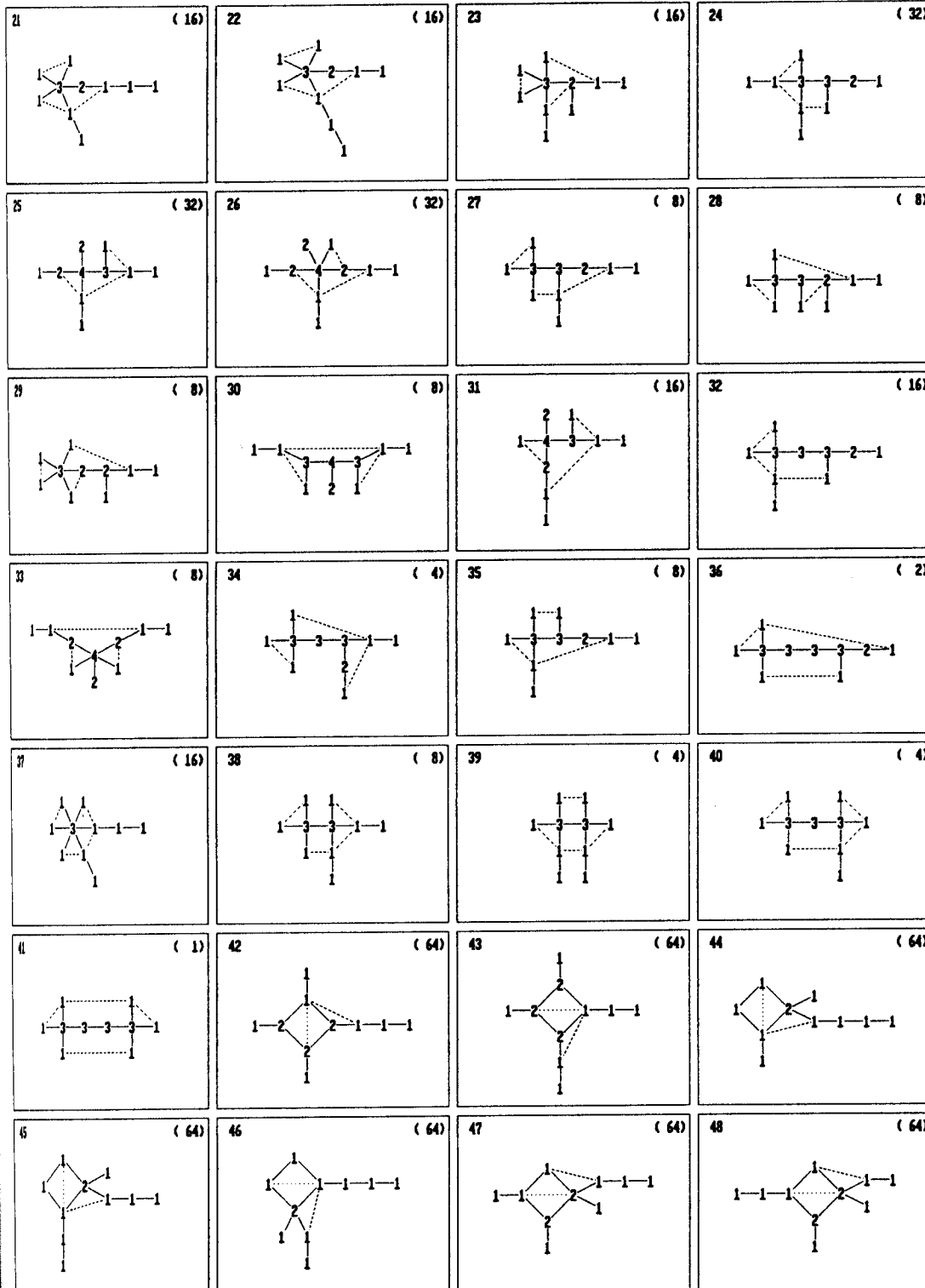
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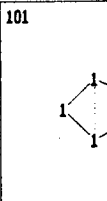
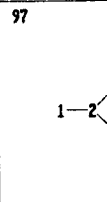
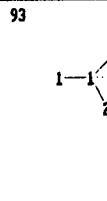
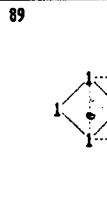
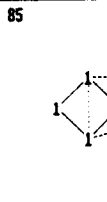
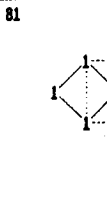
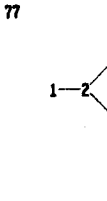
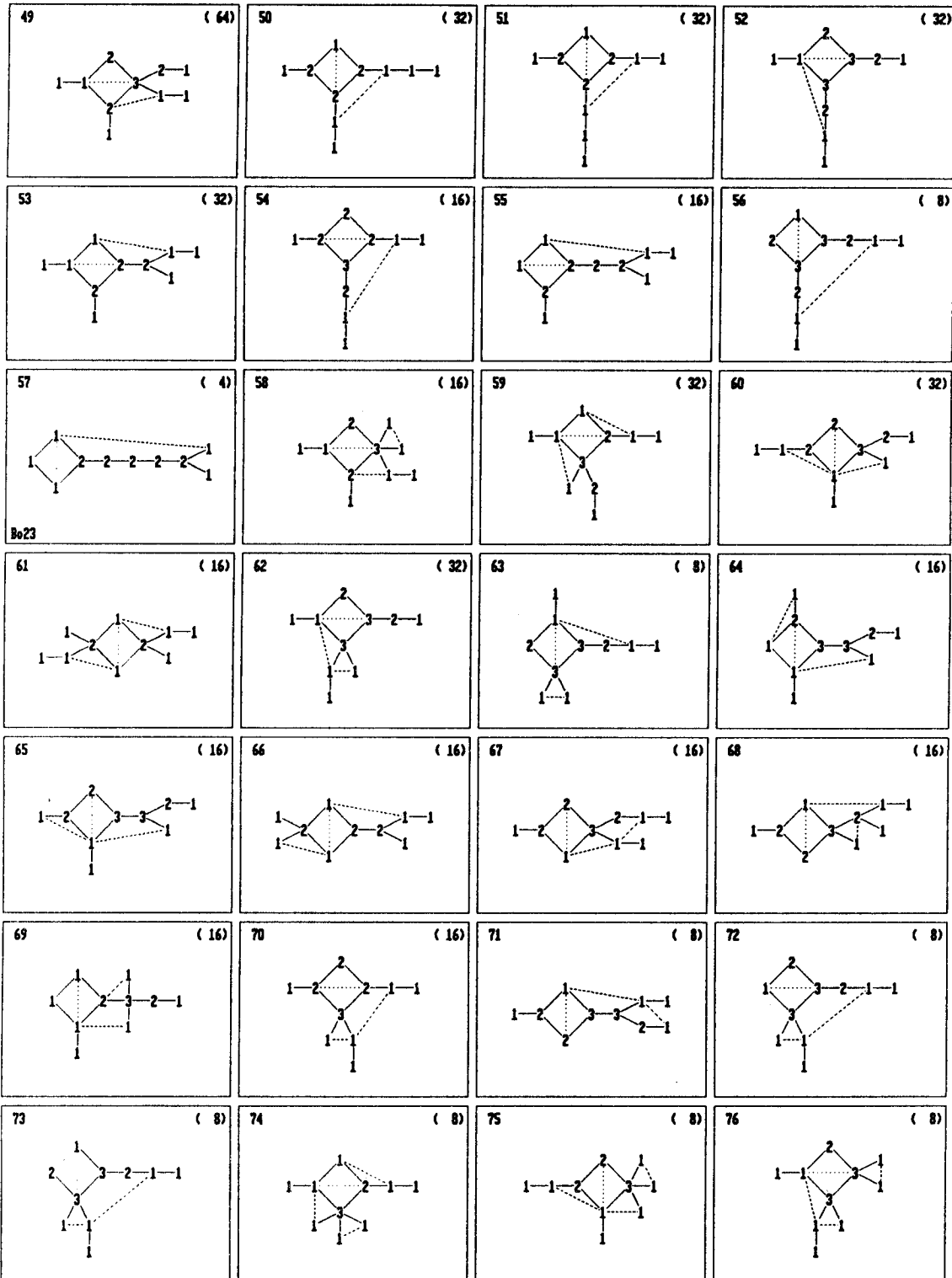


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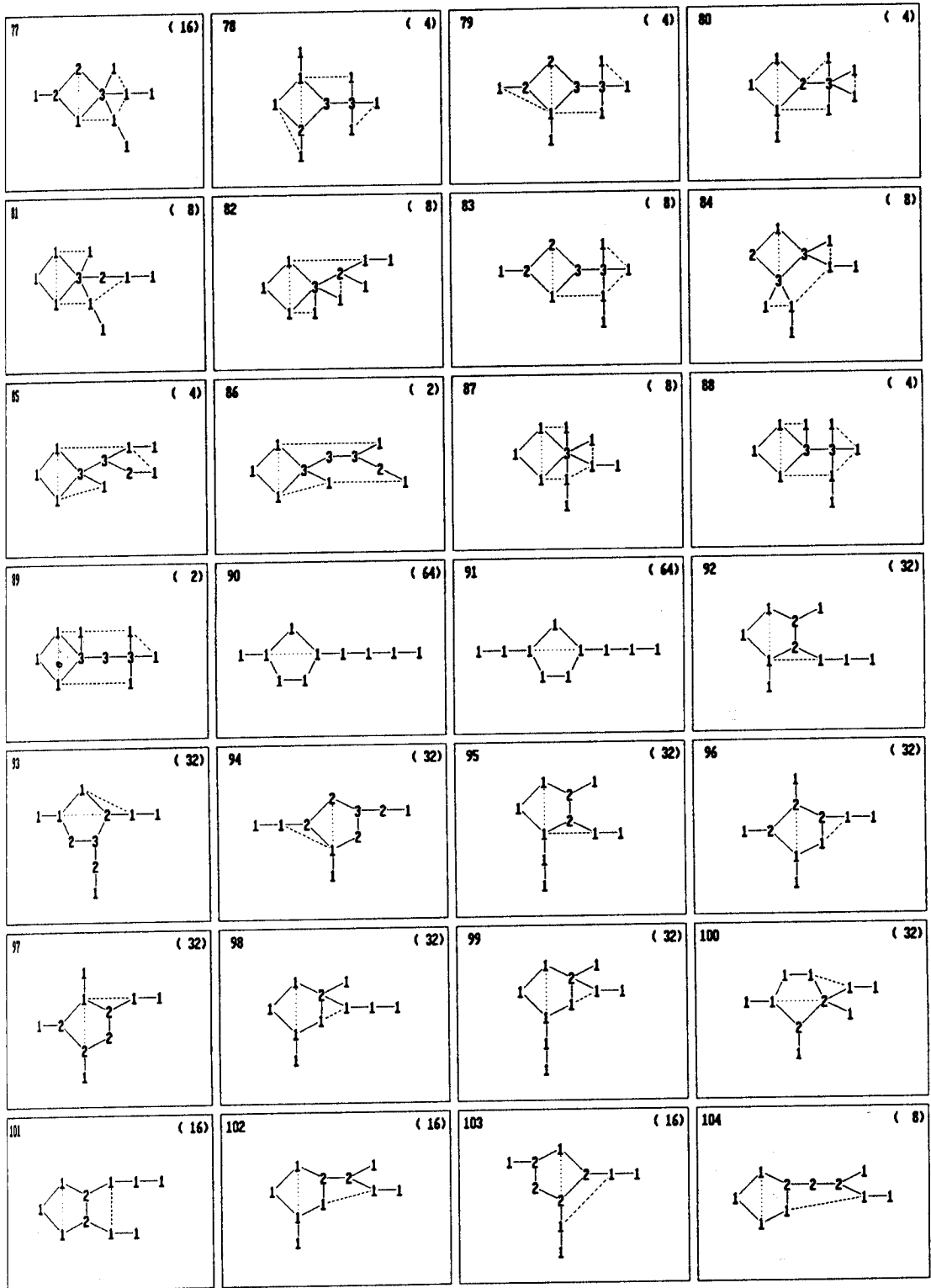


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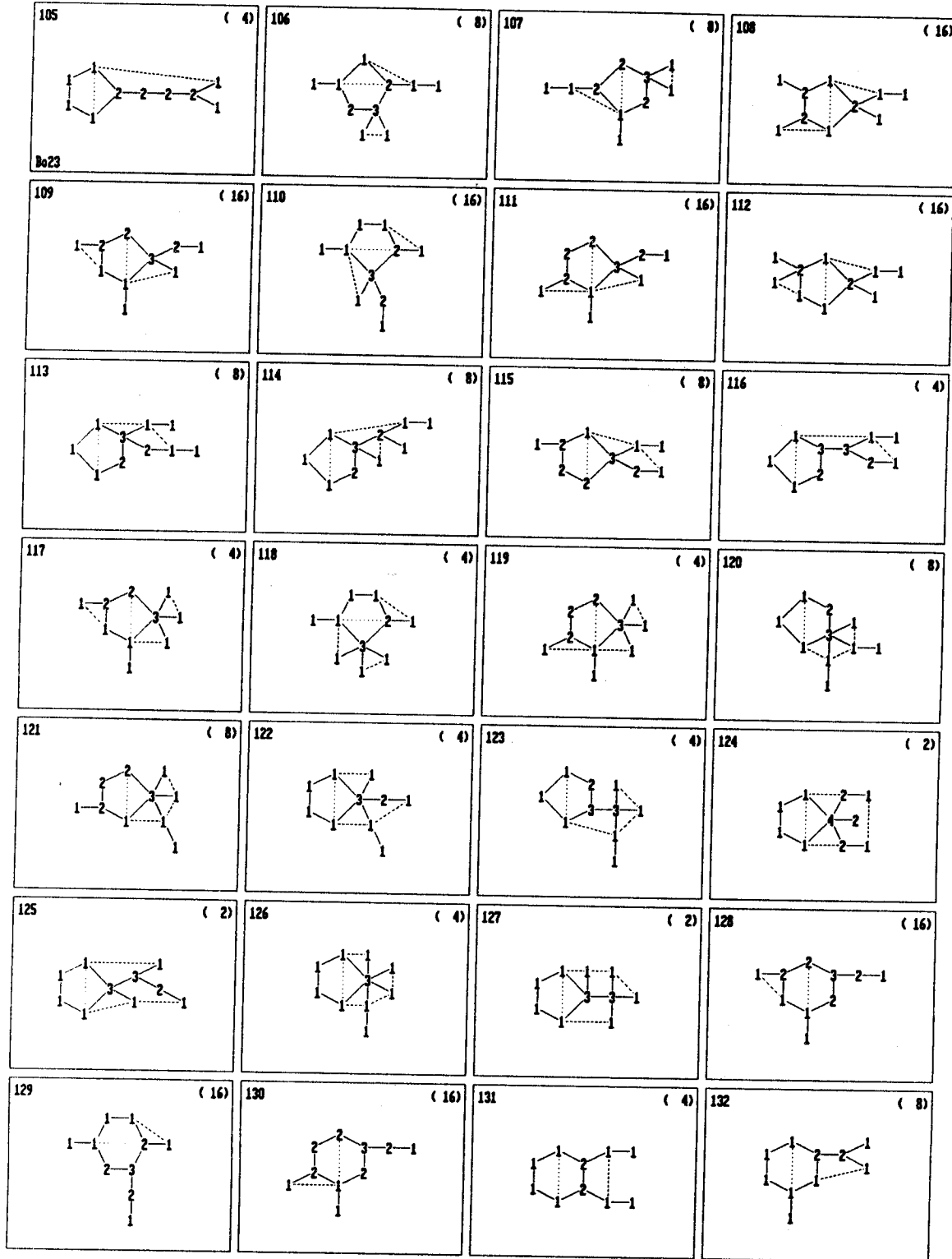
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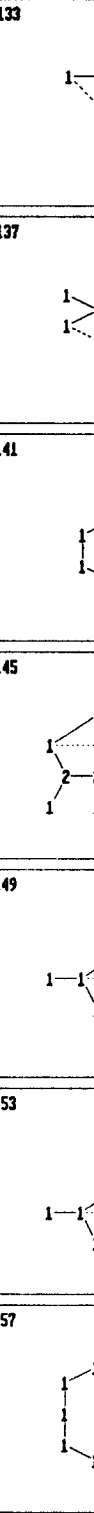
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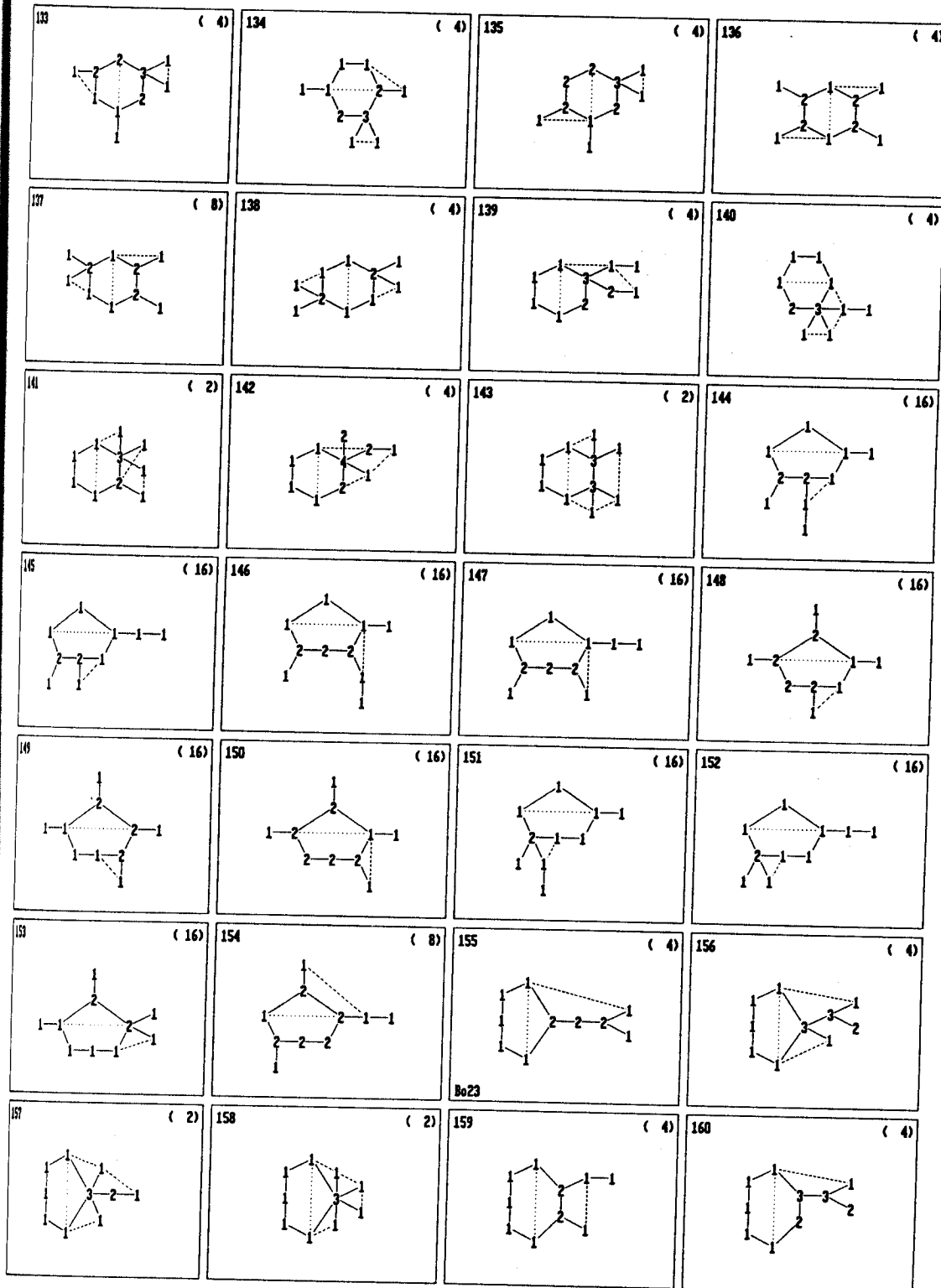
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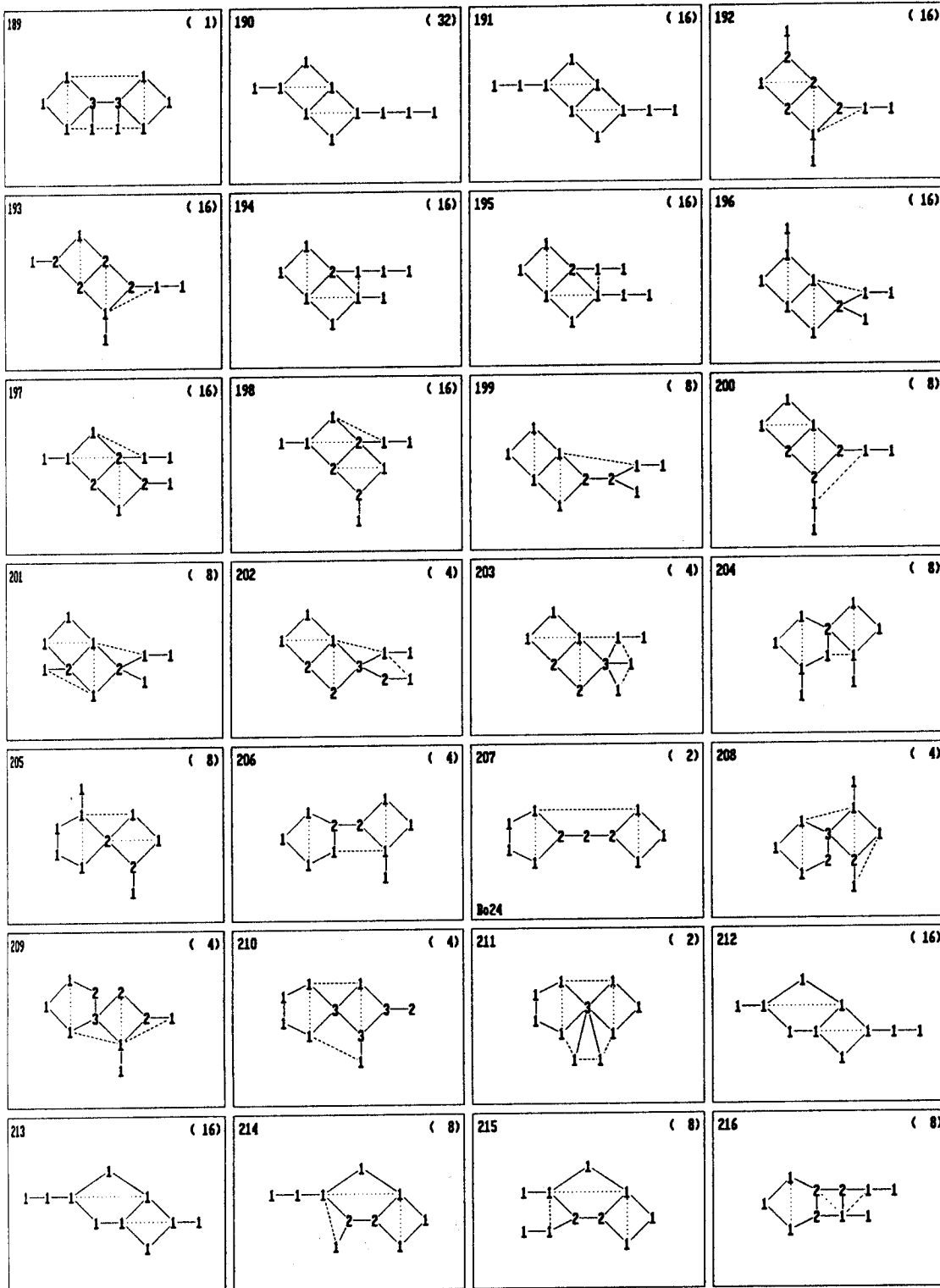
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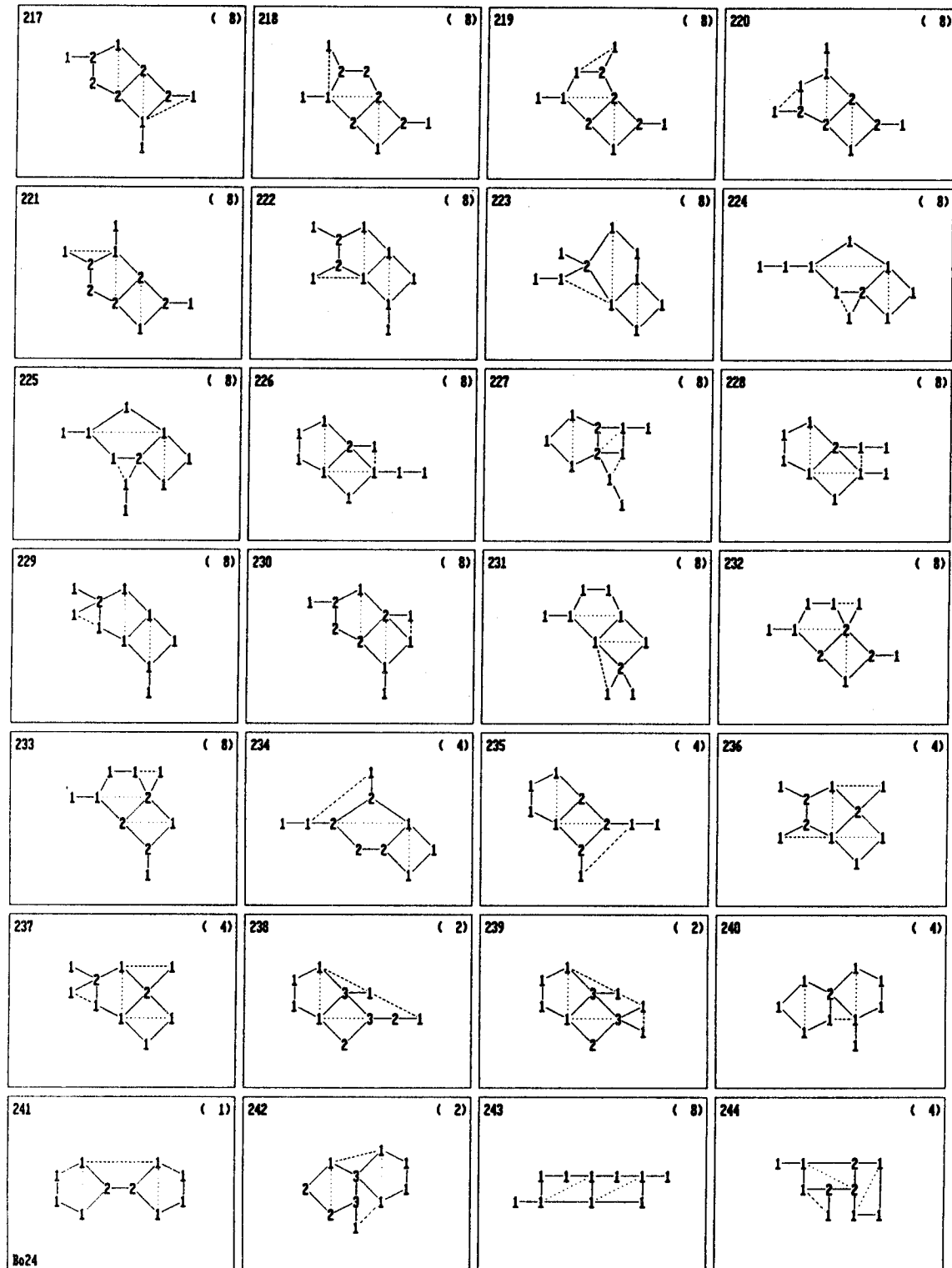
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Bo10 	Bo18 	Bo23 	Bo11 
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Bo9 	Bo8 	Bo16 	Bo19 
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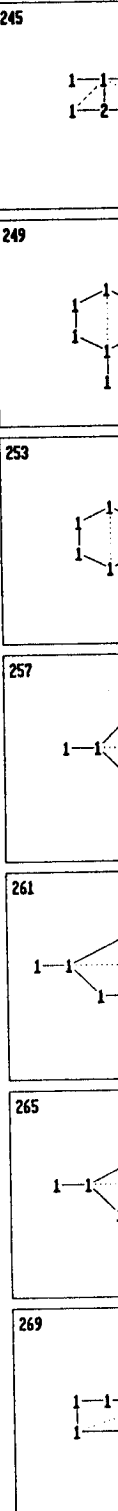


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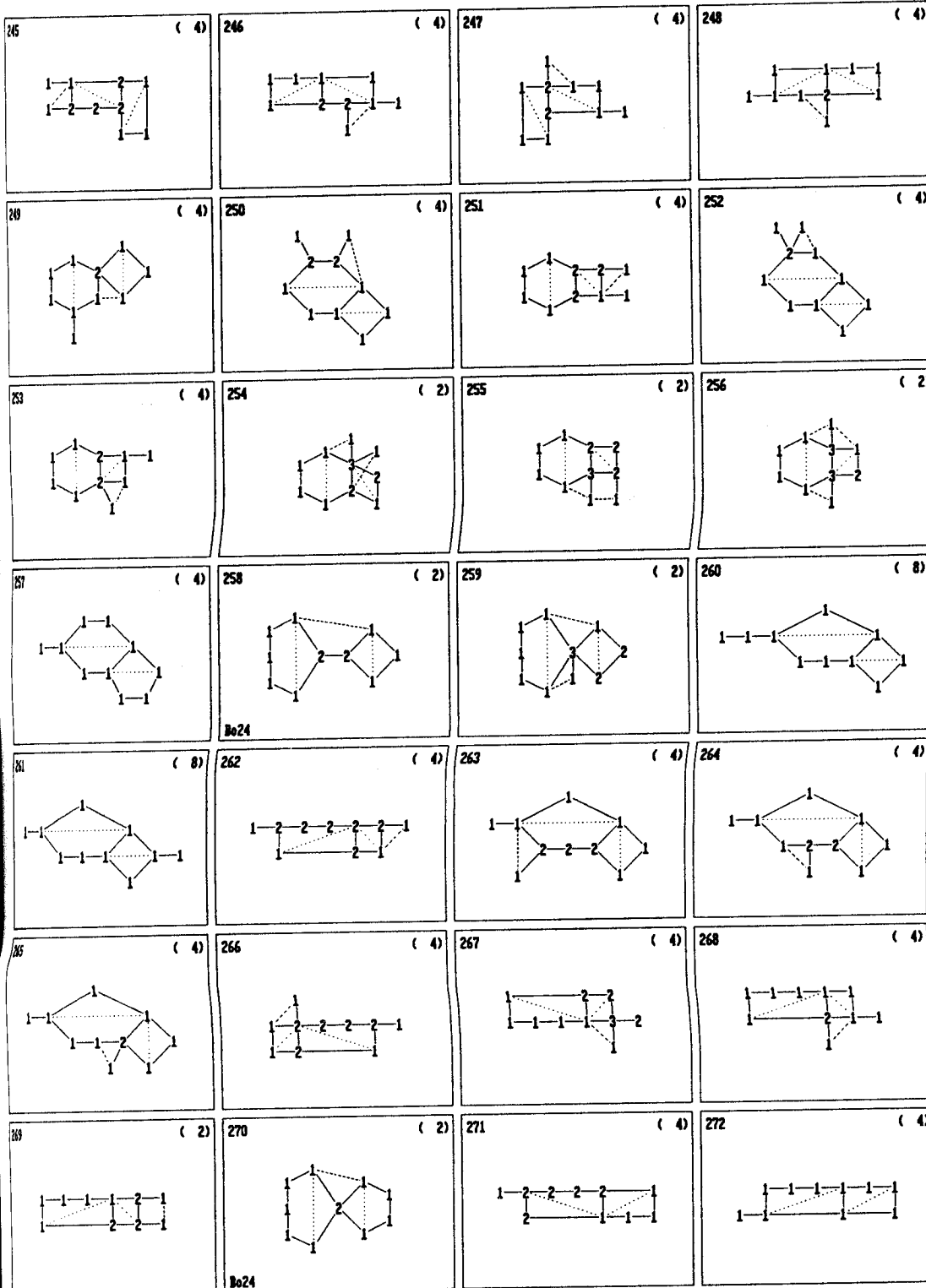


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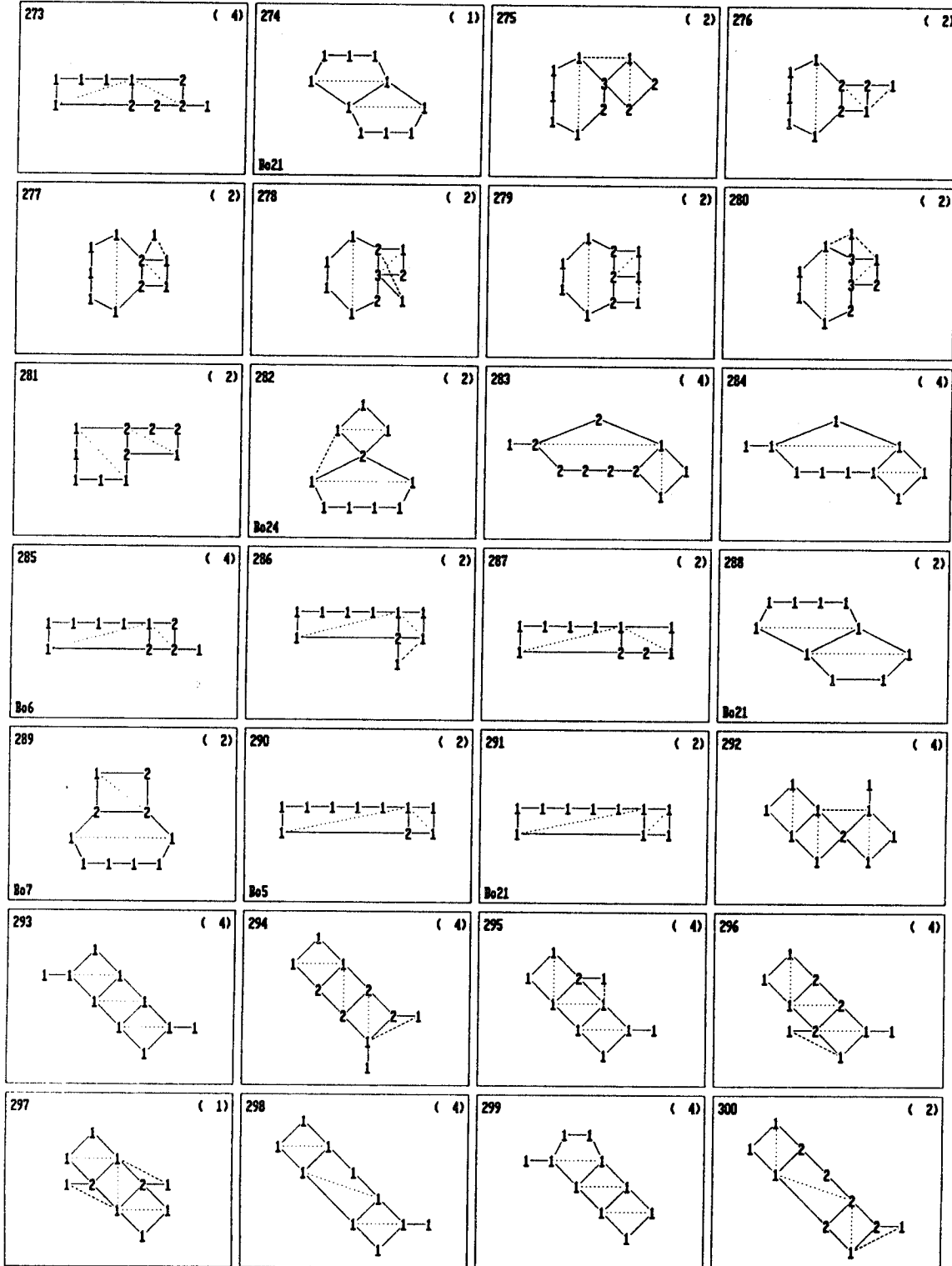
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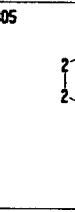
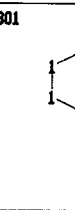
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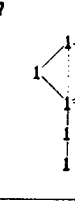
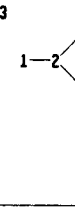
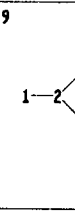
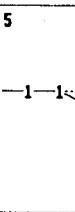
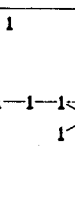
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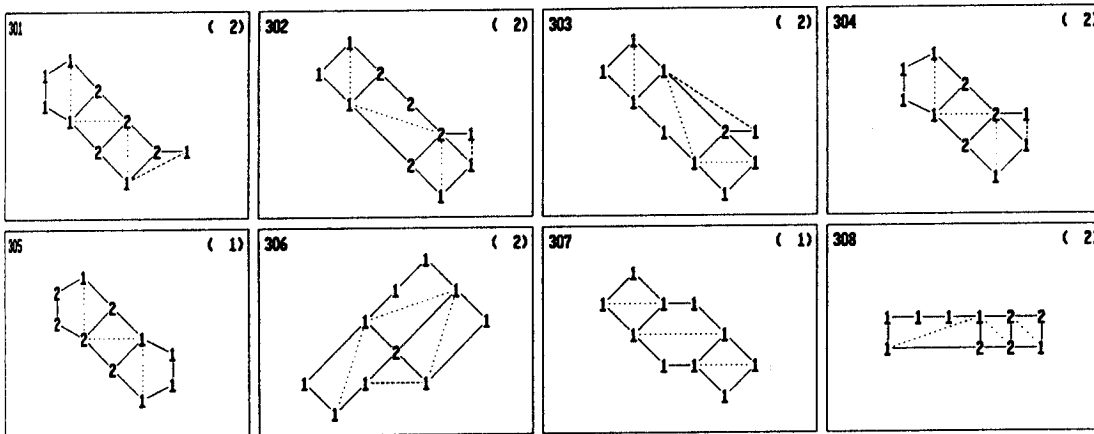
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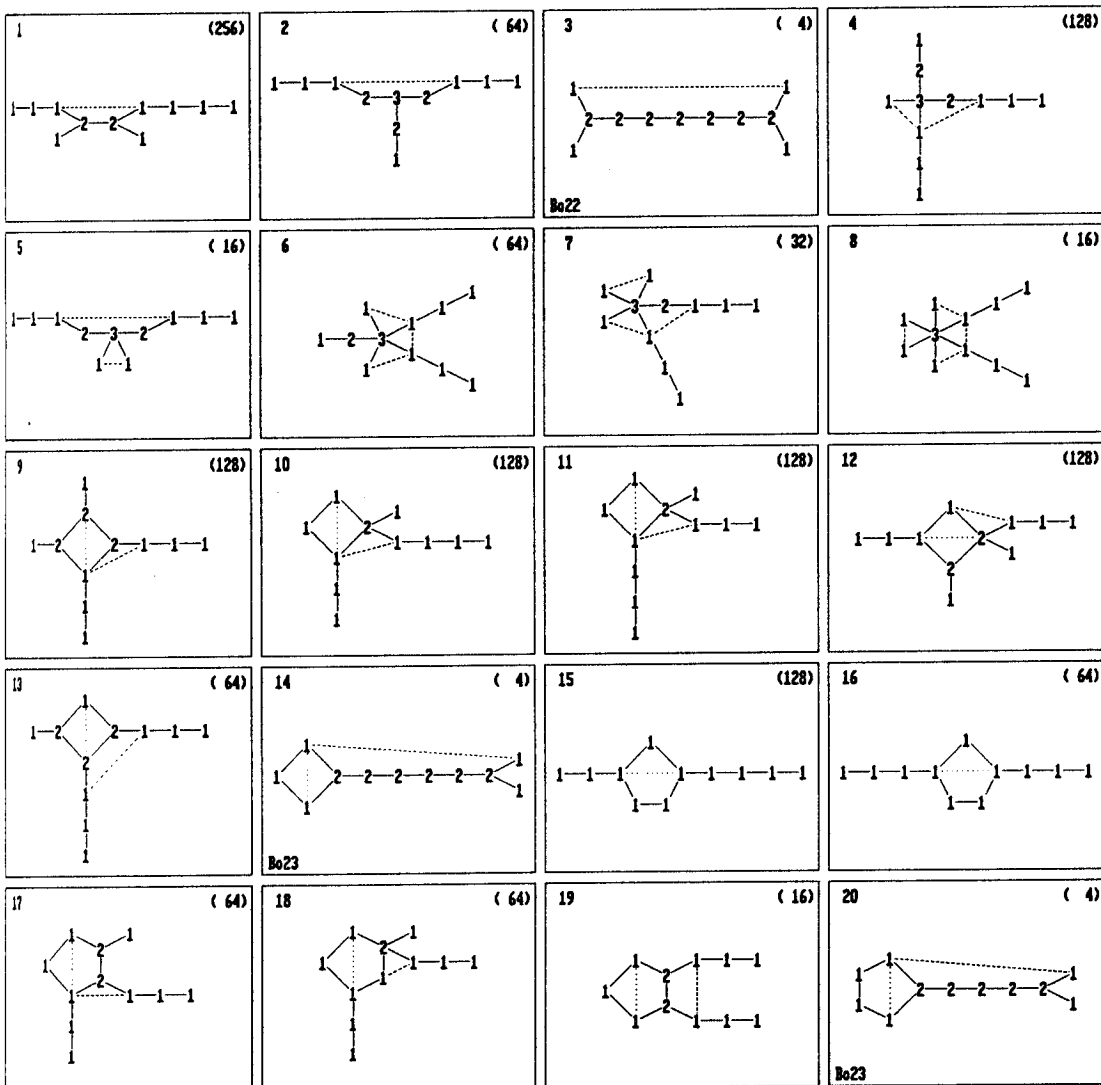
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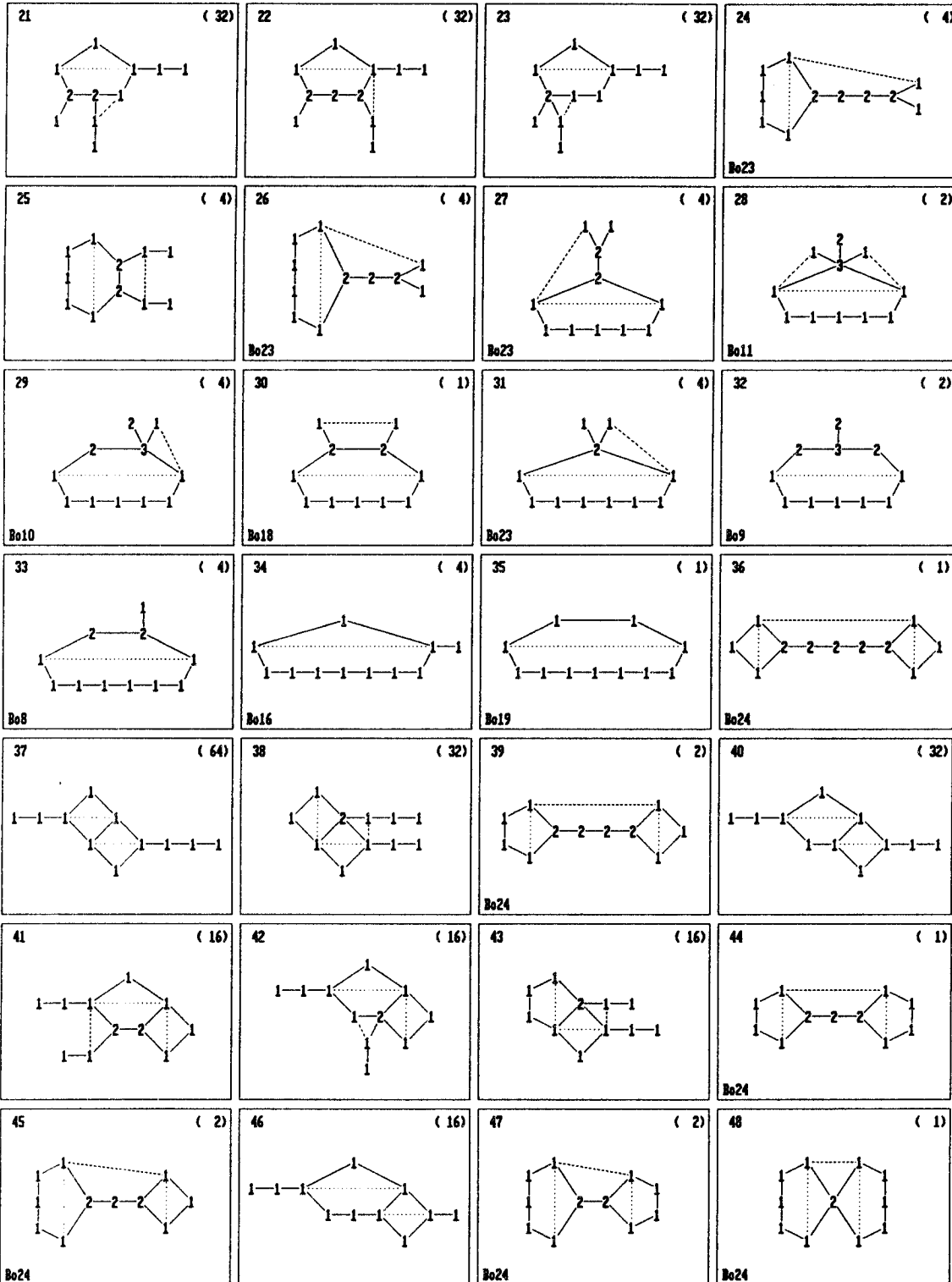
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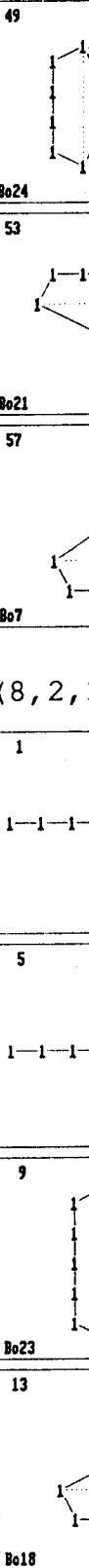
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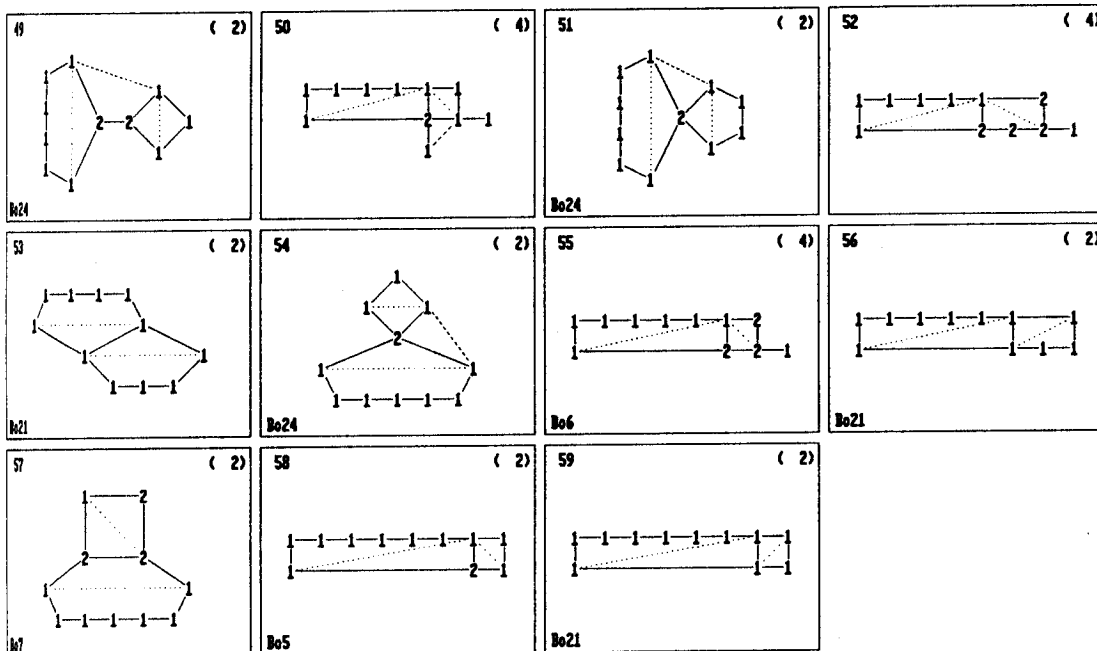
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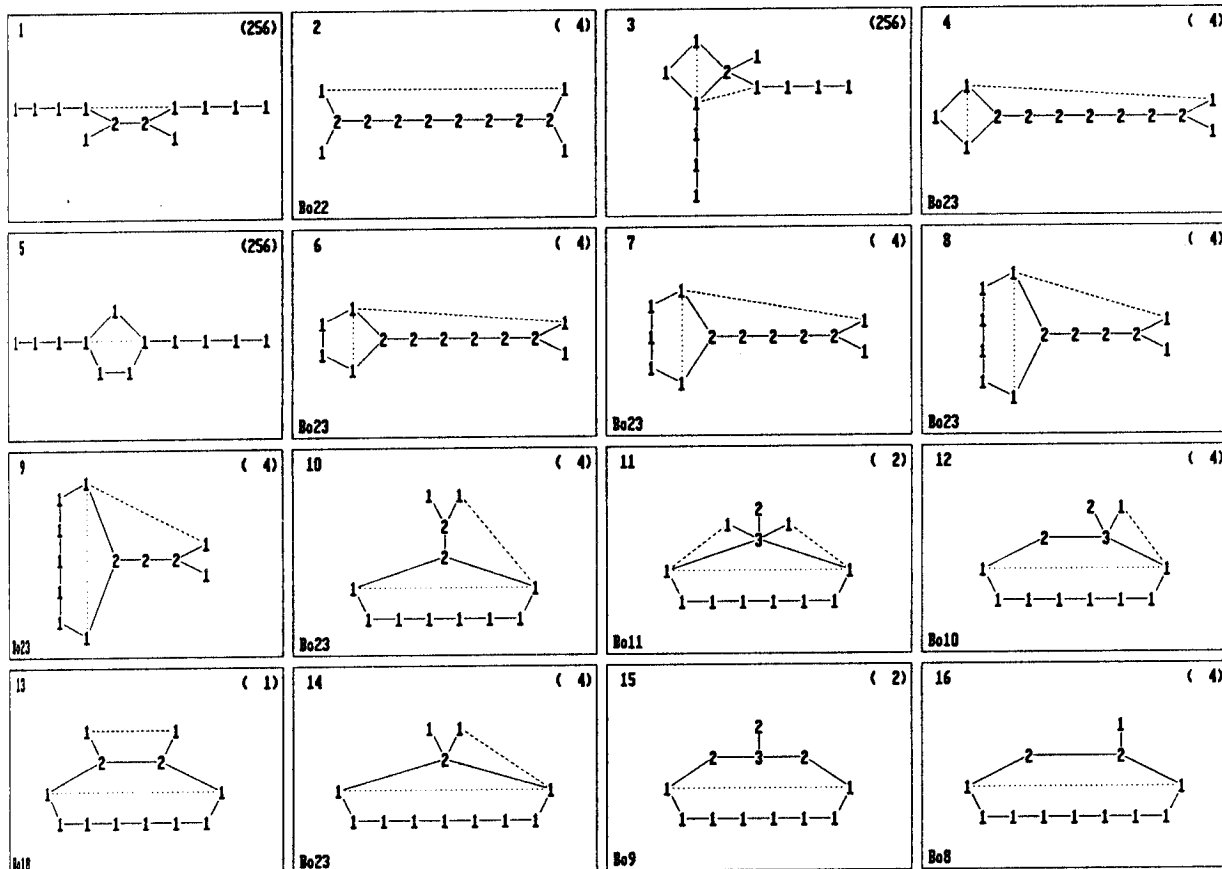
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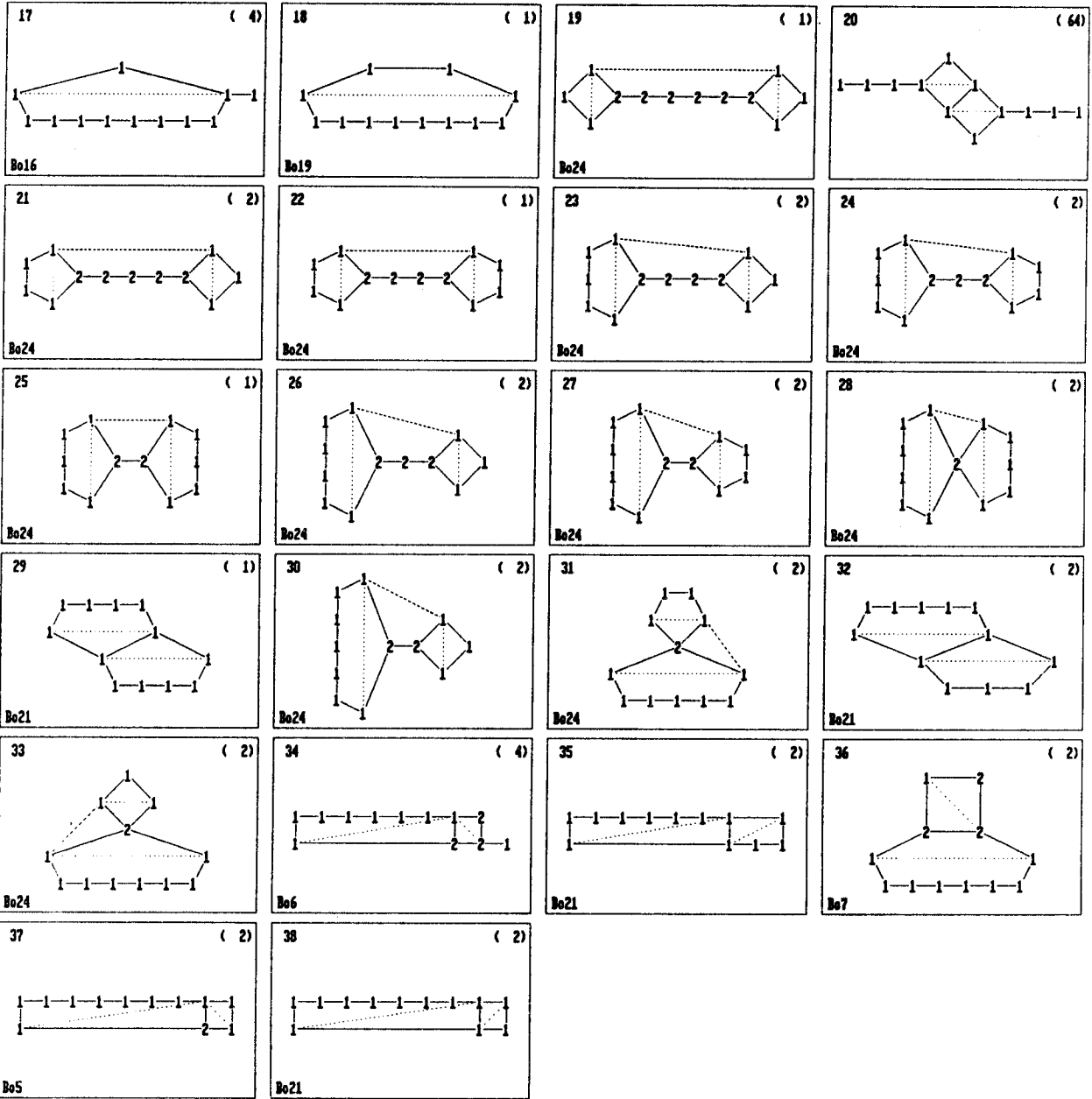
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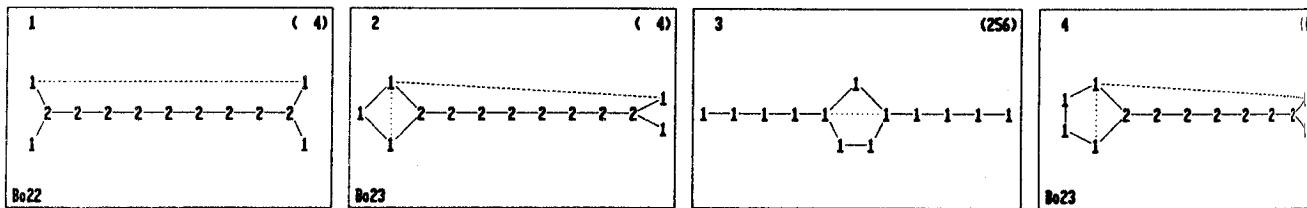
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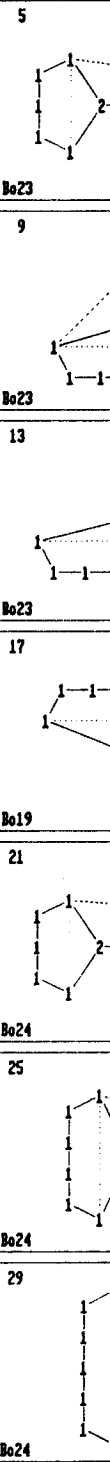
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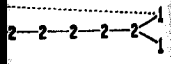
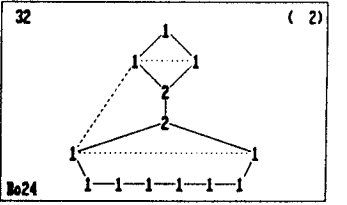
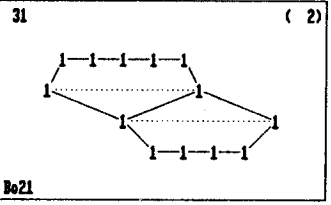
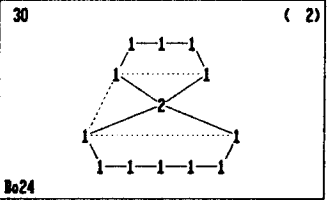
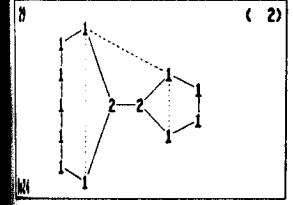
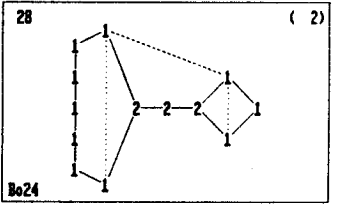
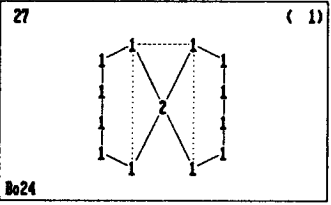
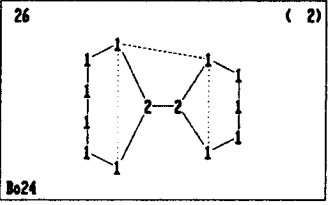
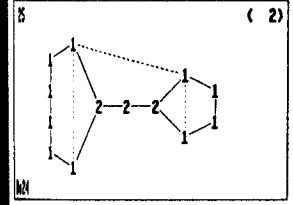
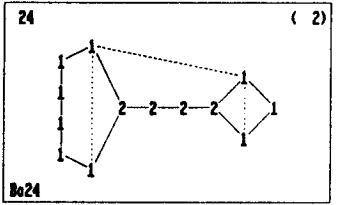
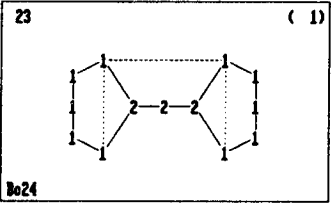
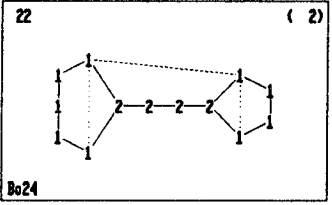
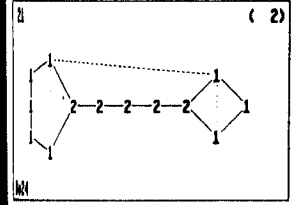
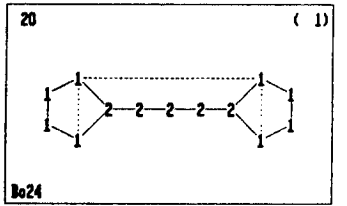
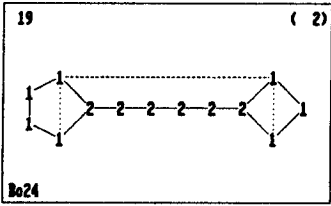
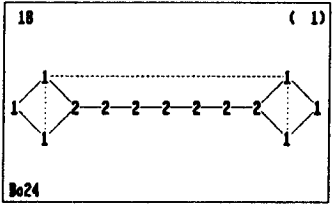
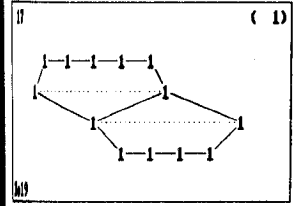
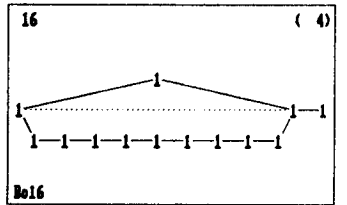
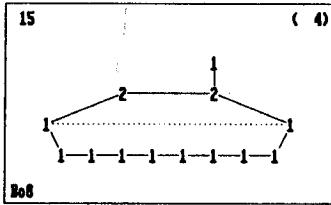
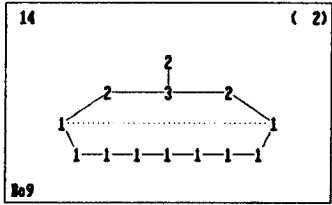
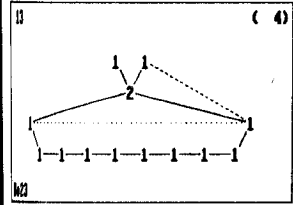
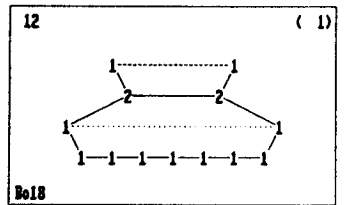
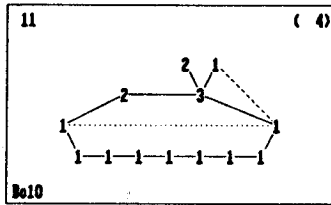
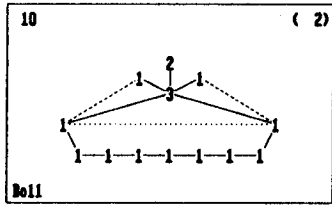
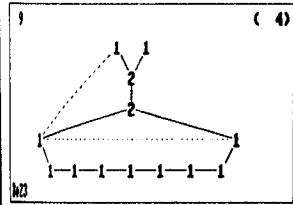
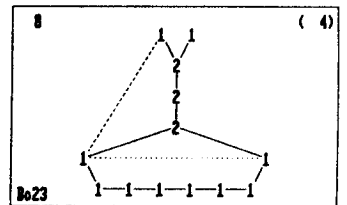
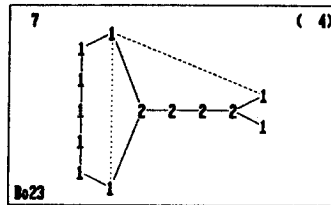
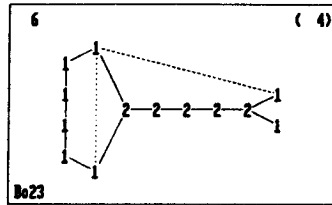
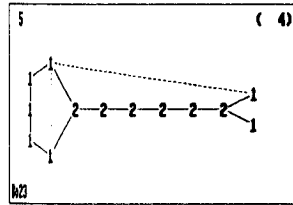
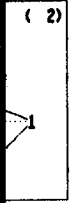
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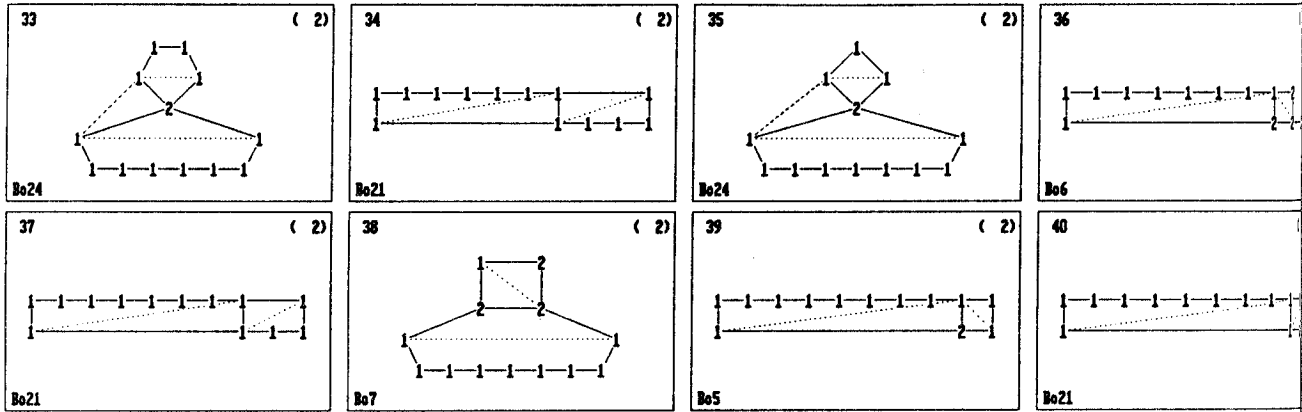
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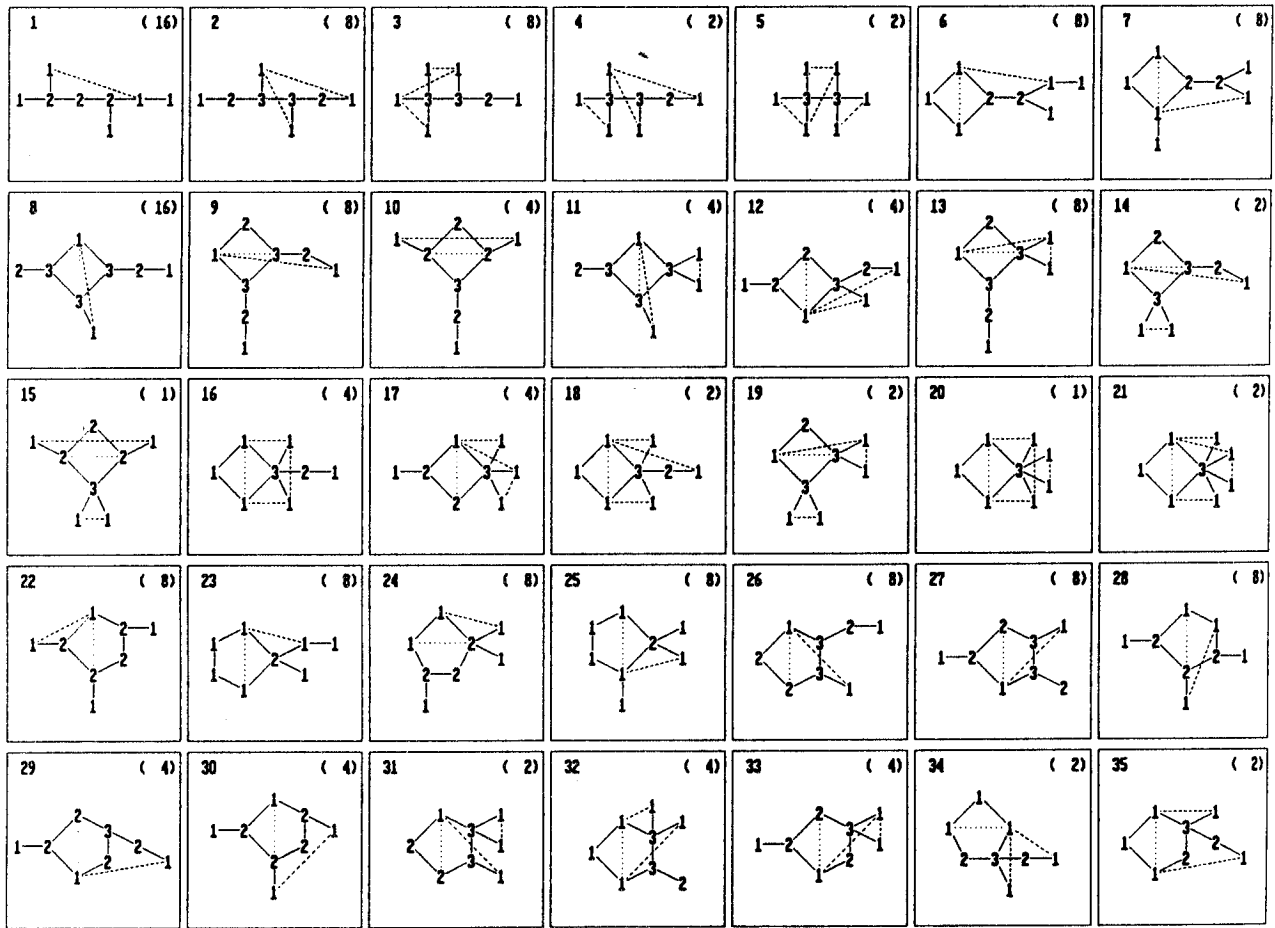
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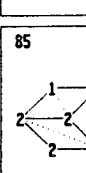
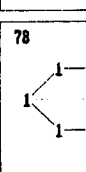
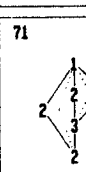
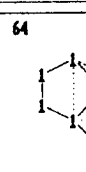
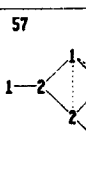
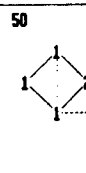
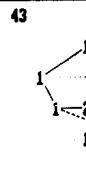
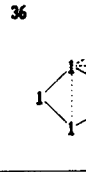
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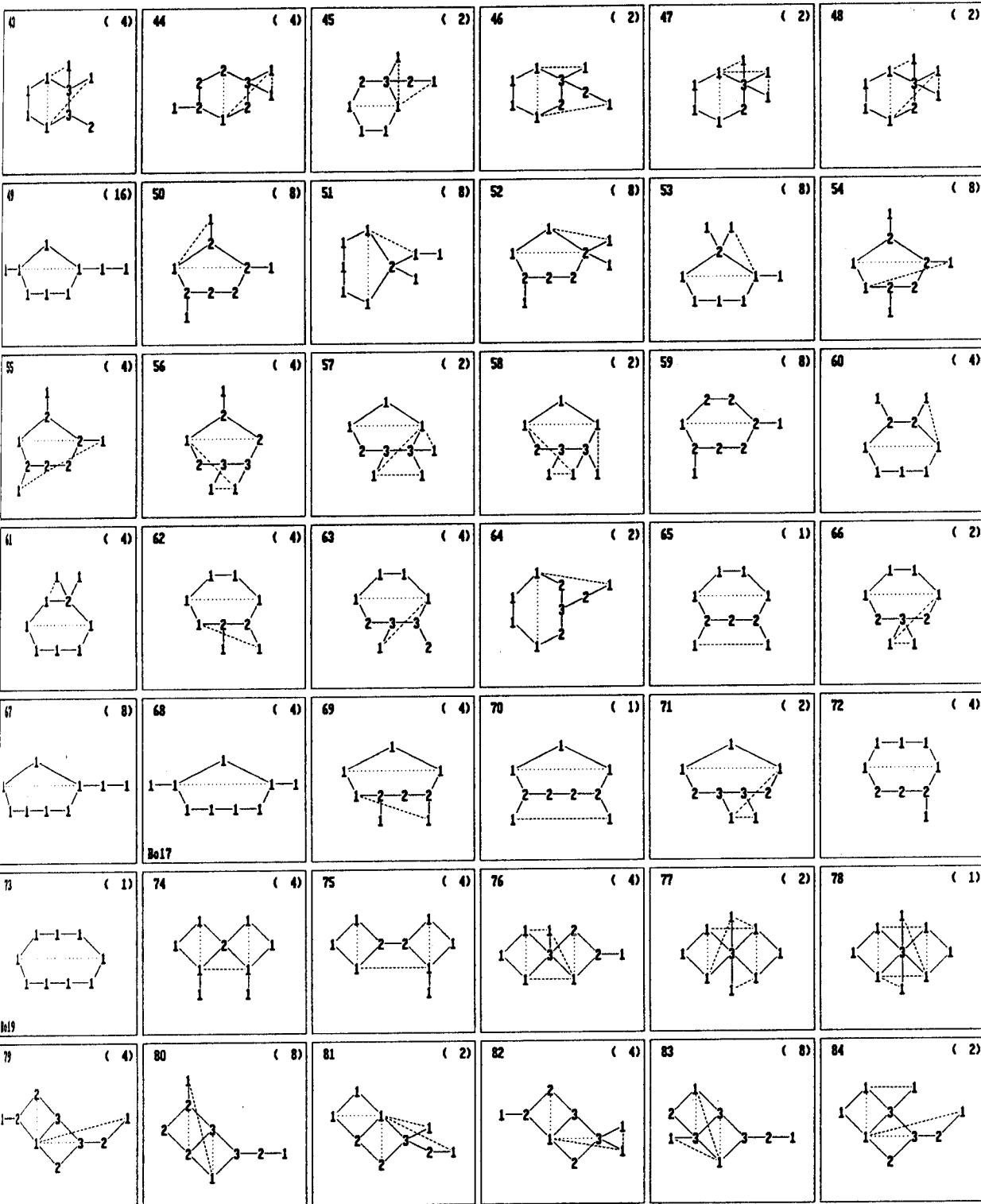
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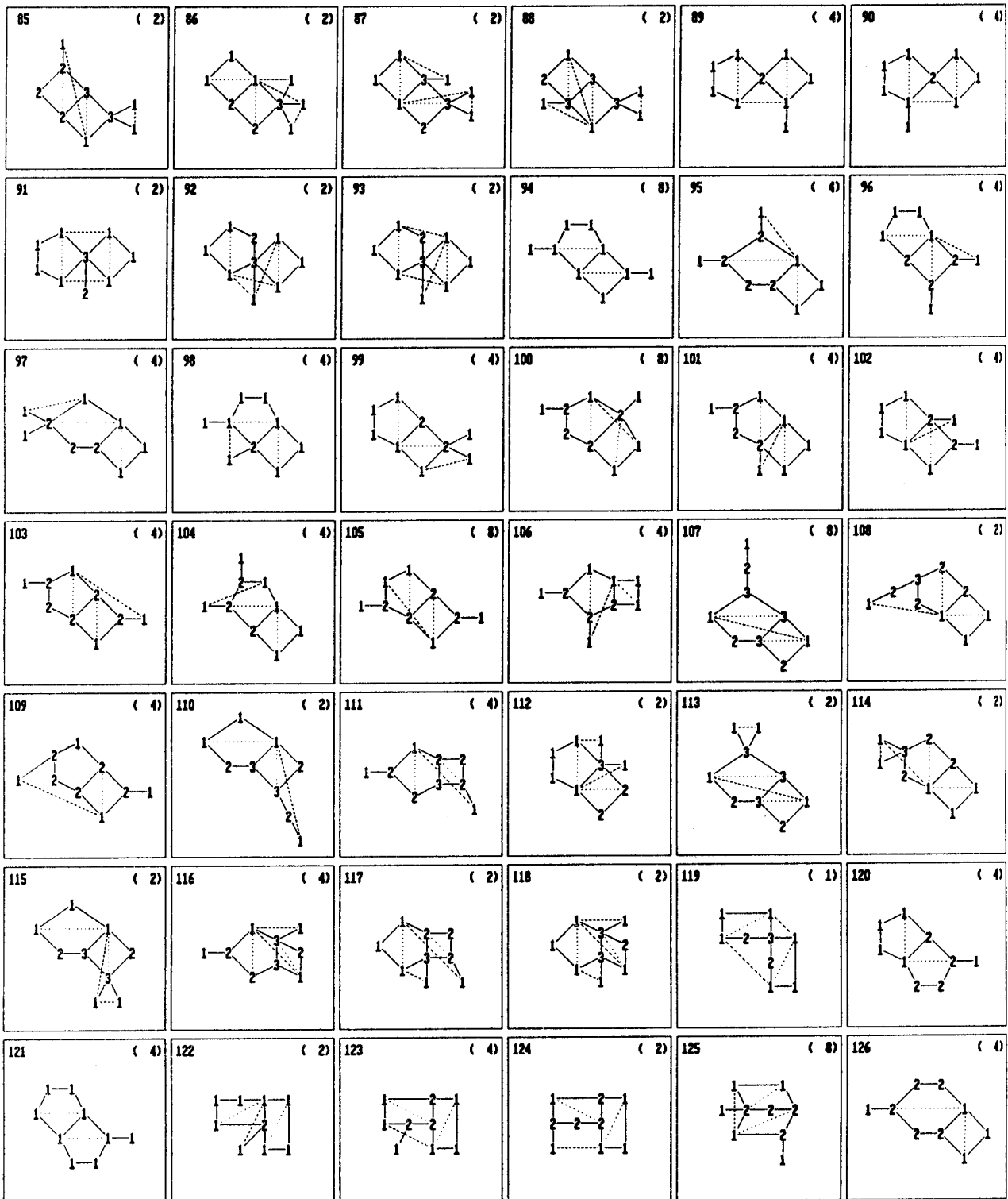




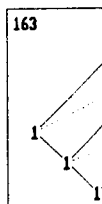
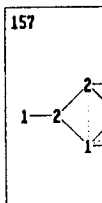
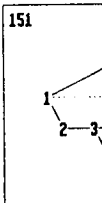
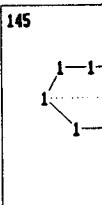
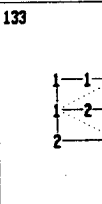
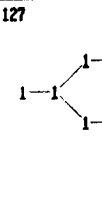
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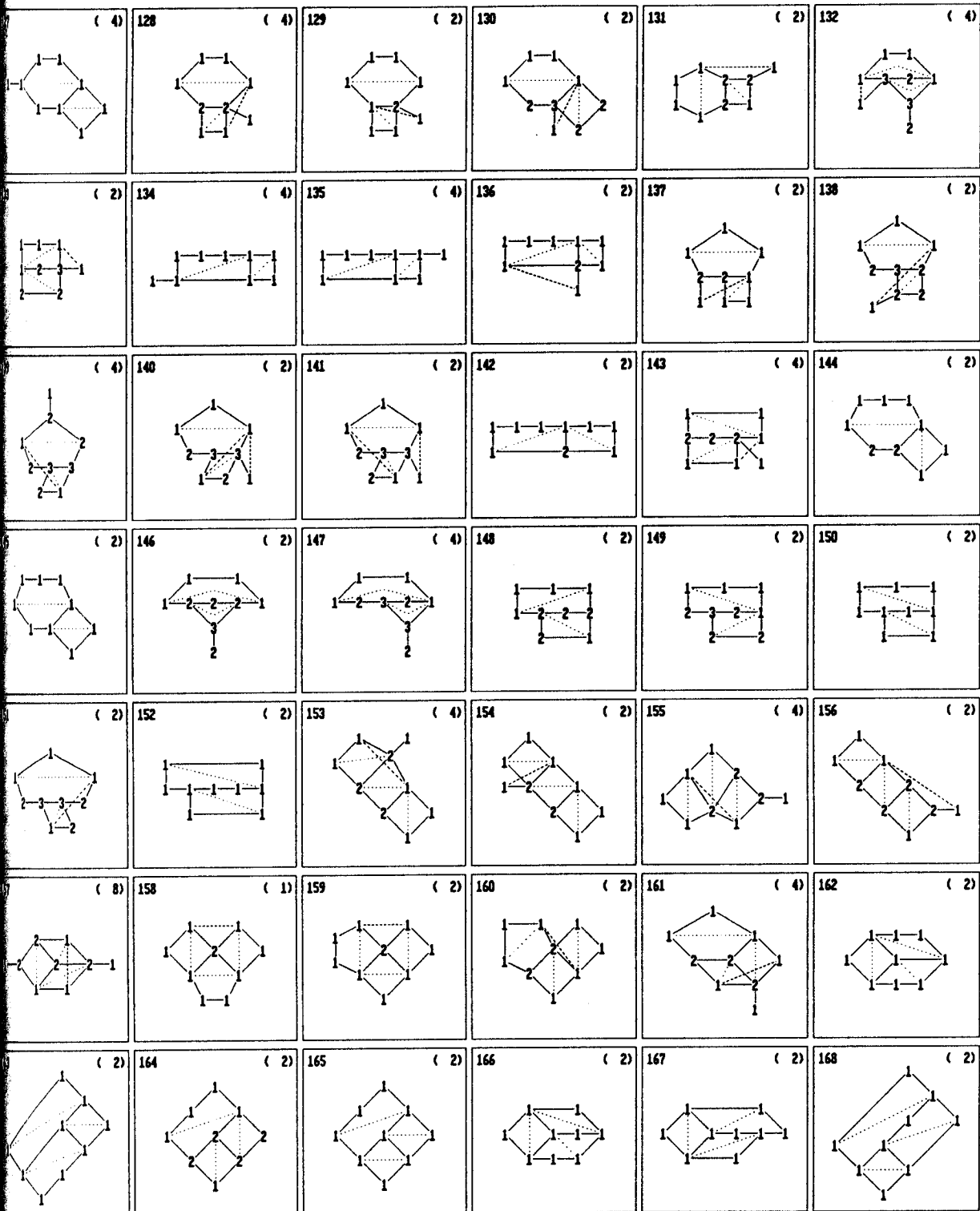
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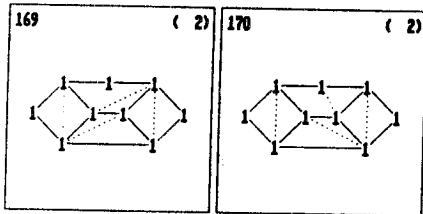
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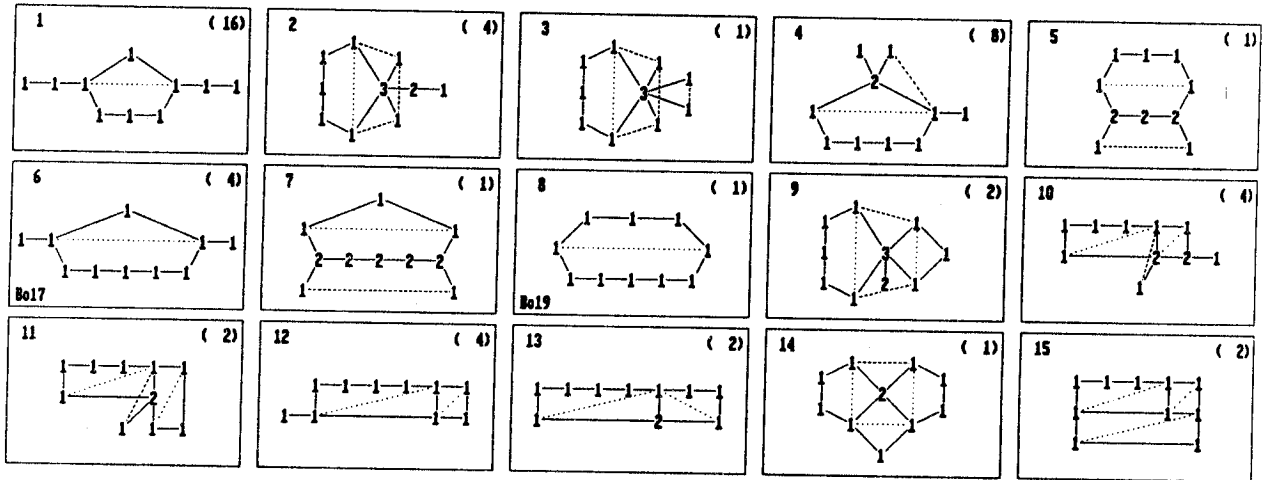
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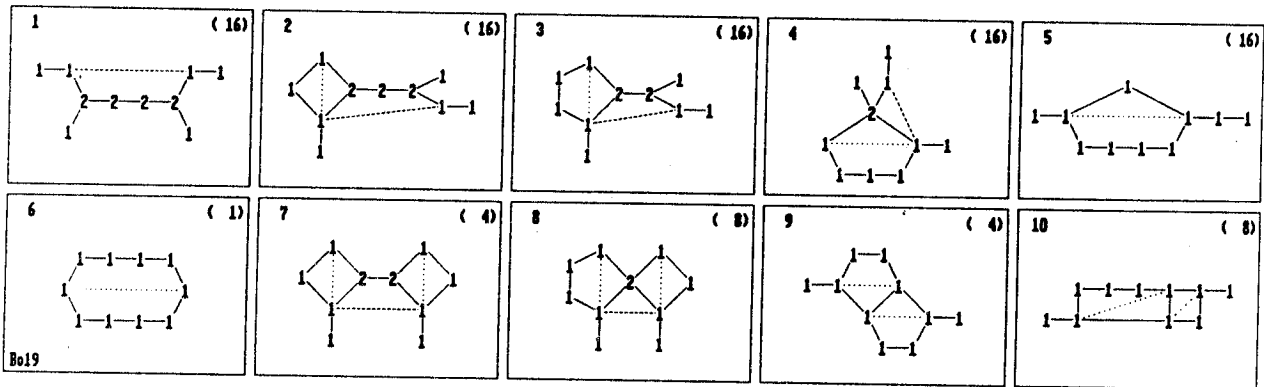
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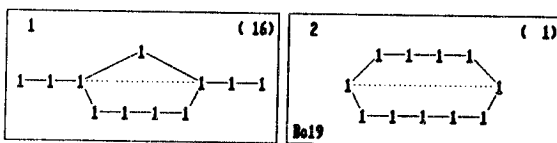
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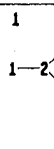
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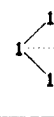
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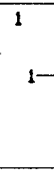
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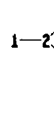
9



(3, 2)



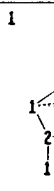
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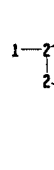
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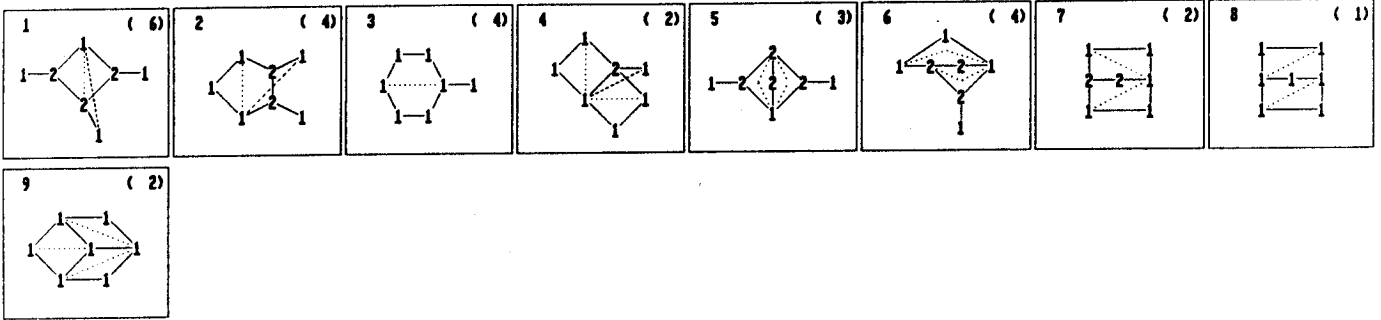
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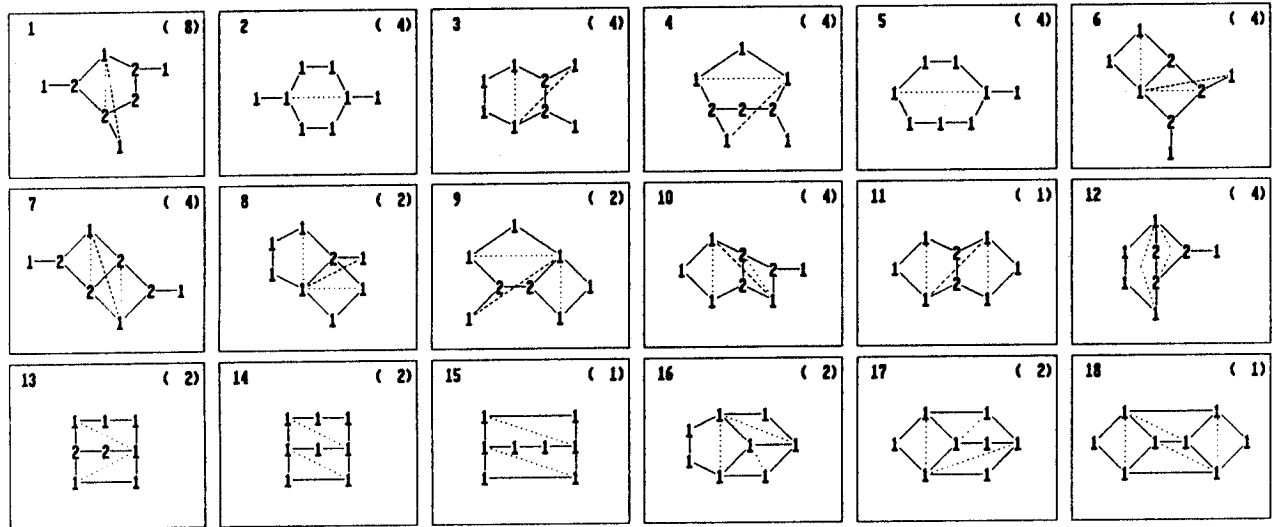
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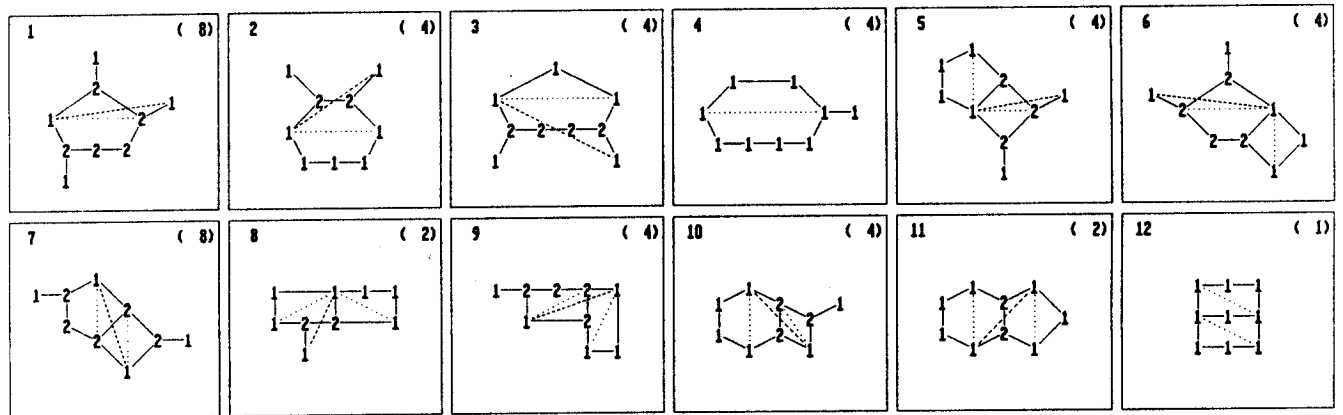
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(3, 2, 2)



(4, 2, 2)



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(1)

(4)

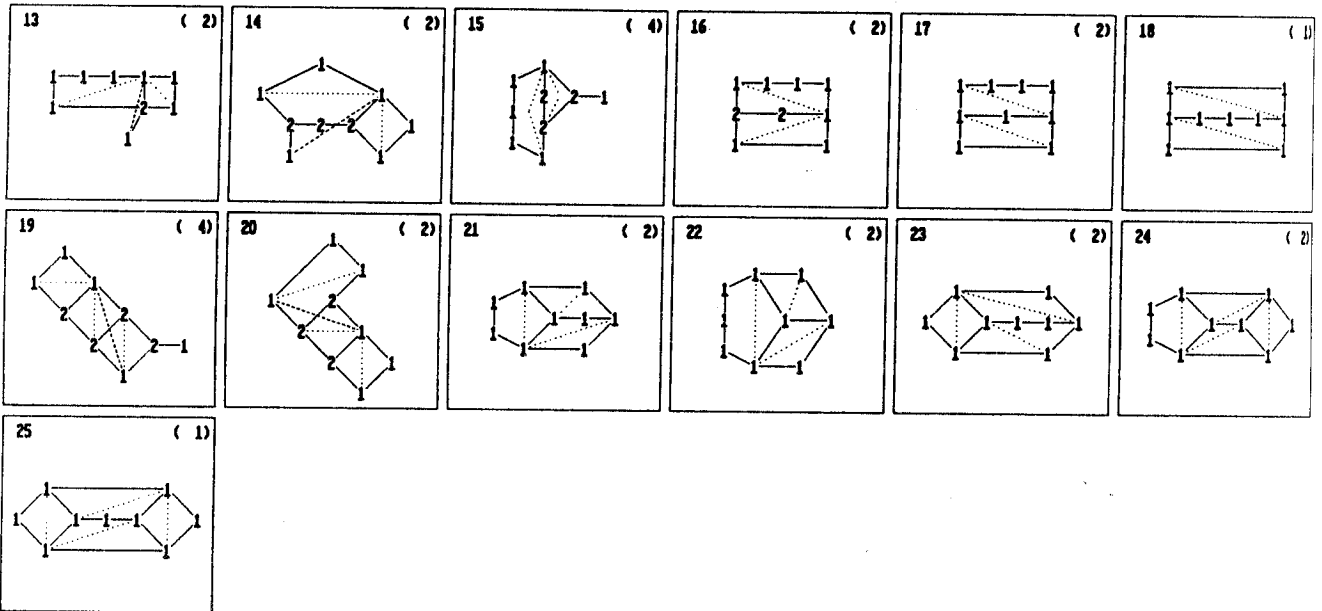
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(16)

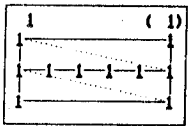
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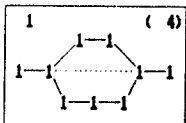
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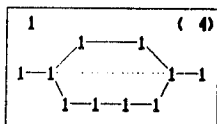
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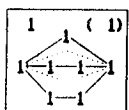
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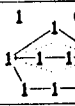
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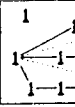
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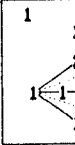
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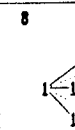
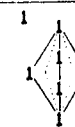
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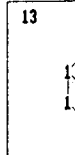
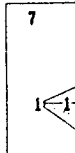
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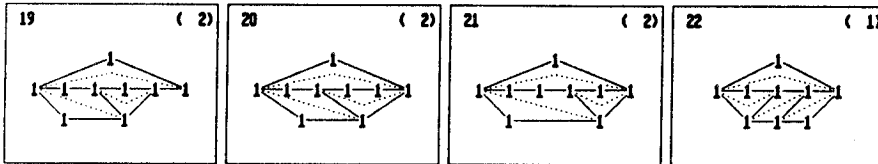


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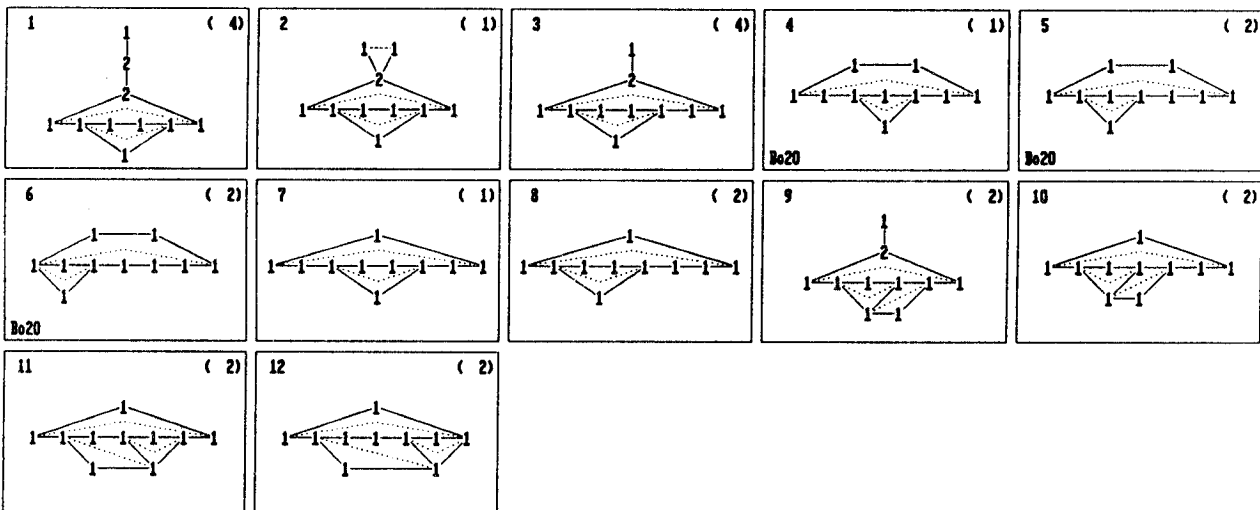




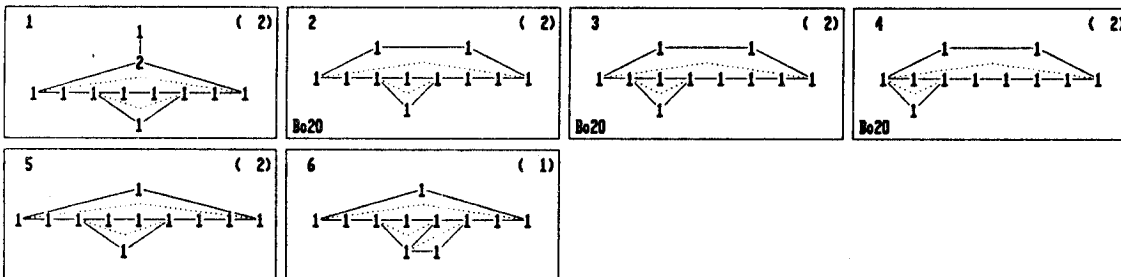
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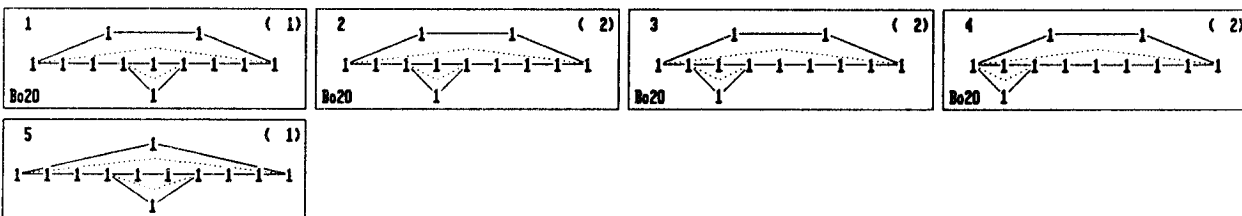
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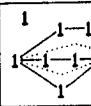
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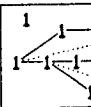
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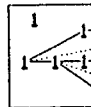
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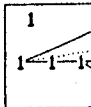
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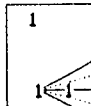
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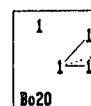
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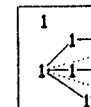
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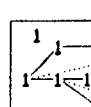
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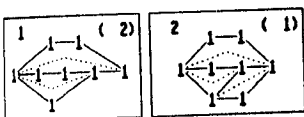
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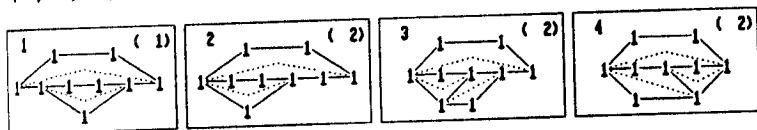
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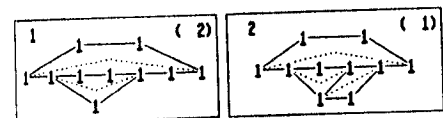
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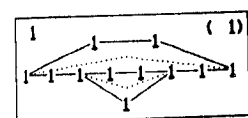
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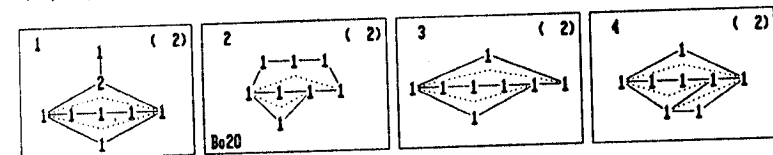
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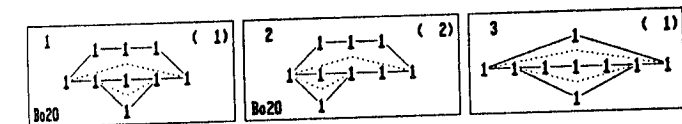
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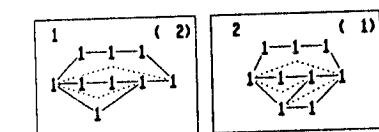
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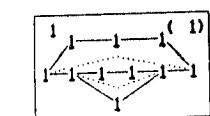
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6

REFE

[BGR

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[Bo2]

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[Dr1]

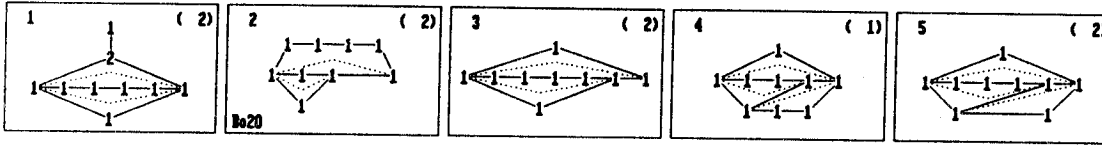
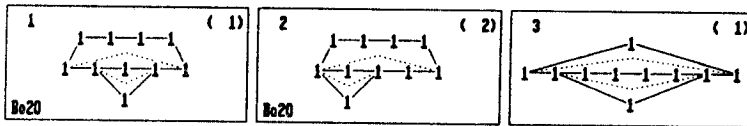
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[F]

[G]

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## REFERENCES

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